**Fluid Mechanics**
**Prof. Subashisa Dutta**
**Department of Civil Engineering**
**Indian Institute of Technology-Guwahati**

Lec 28: The Navier-Stokes Equation

Good morning all of you. Today we are going to discuss about very interesting topics, the topics on Navier-Stokes equations. That is what the basic foundations of computational fluid dynamics and many of the complex fluid flow problems also we can solve it by using Navier-Stokes equations. The derivations part of Navier-Stokes equations that what today I will cover it. The most of the part what I am covering it, it is comes from Sinzel Cimbala book, the derivations part. If you are not following the line by line the derivations, I just encourage you to go through the Sinzel Cimbala book.

If you look it for vector notation and related to the vectors, operations in fluid mechanics, the best course which is available in MIT OpenCourseWare which on fluid mechanics you can get it and also we are going through this F.M. White book on fluid mechanics. These are 3 books, you can always refer it.

to understand this the Navier-Stokes equations part. Now if you look go through its who is Navier and who is Stokes okay the way back the French physicist okay Claude-Louis Navier okay so it is almost 200 years back and also this George Stokes from as a mathematicians from England both have derived the Navier-Stokes equations independently. So that is what is we got the Navier-Stokes equations because of these two professors from Penns and the England to derive one is physicians another is mathematicians to derive the combinations of equations which we call Navier-Stokes equations. which they derive independently and they cross verified and that is the equations today we are going to discuss it. So, I will talk about start from a introductions part.

Also, I will talk about revisit Newtonian fluid and non-Newtonian fluid part which is quite interesting to look at how we can use for this Navier-Stokes equations and we will talk about the derivations of Navier-Stokes equations for with assumptions of incompressibles isothermal flow. Then we will be talk about in a two coordinate systems one is Cartesian coordinate systems the Navier-Stokes equations and mass conservations and similar way we can talk about cylindrical coordinate systems how we can have the Navier-Stokes equations. Now after that we can look it we can apply for a example problems with a what would be the appropriate boundary conditions to solve this Navier-Stokes equations that is what also we will discuss it. Let us go to the next slides as you look it from Cauchy equations what we get it just in vector forms. In vector form if you remember it which is the linear momentum equations deriving with the components like rho by rho t rho v is velocity plus is equal to del dot product of rho v is equal to rho g is a vectors plus we have the del product or divergence of the stress tensor.

This is what we have derived as a Cauchy equations which is a linear momentum equations for infinitely small control volumes. We have derived the basic equations for as a linear momentum equations for control volumes which is having infinitely small control volumes. We can derive where sigma ij is a stress tensor. Now if I look at the component wise before going to these derivations, I just want to look at what is the component wise of this part. If you go back very basic things, the the vectors divergence okay.

If you look at that del and V the velocity divergence it is talking about the net volume outflow from a control volume okay. So if you derive a simple equations as find out the velocity divergence what it says that what you will understand from that if you physical interpretations of that is a volume outflow per unit volume per unit control volume. okay. So if you consider a control volume is a unit then this is what it indicates the divergence of velocity is in this how much volume outflow happens is per unit volume. So we can consider the 0 for a simple cases where we are talking about So that is what is indicates the physical interpretations of velocity divergence.

The same way if I just put it the del into rho into v what is this is indicating is the mass outflow for a unit control volume that is what is indicating me. That same way this for volume outflow per unit volume if I multiply volume with a rho the velocity with rho that is what is mass outflow per unit volume. Same way if you can just interpret it that the divergence of rho v v okay is nothing but momentum outflux per unit contour volume. So if I try to interpret this physically this part nothing else that is what change in momentum okay. Here we have a linear momentum within the control volume with respect to time.

That is what with respect to time. This is what is a momentum outflux from this control volume. This is the momentum change within the control volume. This is what momentum outflux per unit control volume. These two can indicating for us two force component one is body force another is the surface force that is what body force and the surface force component.

So if you look at that what we are deriving this that we are not bringing a new physics to that. The same concept we are talking about that we know from Newton s second laws the force is equal to mass into acceleration. The same concept we are talking about but we are talking about a control volume. That is the difference the control value acted by at the surface as a surface tension as it has a body forces which is the gravity components what we are considering it. So if you look at this Chezy's equations if you have simple derivations alternative form okay which more derivations you can just go through the derivations go through the Senzel and Cimbala book.

I am not deriving one by one. If I look at that just modify that I will get this component this is vector is equal to v dot del into v that is what is equal to rho g factors del dot sigma ij okay. So just the same Cauchy equations if I just use the vectors relationship, I will get this part. As we discuss it, this is nothing as the local accelerations, this is what the convective accelerations. If you just split it, this is the local accelerations, this is the convective accelerations.

That means we can rho dv by dt is equal to rho g del into sigma ij. So this is what mass this is the mass into acceleration per unit volume that is what is this dv by dt is accelerations into the rho that means it is a showing it if I divide by this volume I will get this part and the force what I have I have a body force I have a surface force. So if you look at that even if you do a vector notations we are deriving the very basic Newton's second law is that force is equal to mass into acceleration nothing else. Please remember that we are not deriving a new hypothesis. We have followed the same hypothesis that force is equal to mass into accelerations.

Here in the fluid flow, we have a body force, we have a surface force. The surface force we are considering from the stress tensors. The body force we have the g components that is what we are getting a body force and surface force. Mass into accelerations that is what we have the mass but per unit the control volumes that is what we are applying it to get it these equations. So this is very easy the same the great equations of Newton's second laws the force is equal to mass into accelerations only we have a change of the formats okay for a control volumes.

we are writing this form which is having a two force components the body force and the surface force component and the accelerations part okay that is what we are deriving it. If I consider it that same concept as that is not in the difference between Cauchy equations or Newton second laws both are the same. The Newton Cauchy's equations we are applying for the fluid of a control volume V which is having a surface tensors the gravity force component we are doing that part where you all you know it how to apply the Newton second layer for a solid particles or solid motions that you know it that is what we are deriving for this. Now if you consider it that I have this big control volumes which is movable control volume okay it is moving it. the moving control volumes which is having velocity v which is having accelerations a and we have the sum of the force components.

So it is a movable control volumes having the velocity v and also have the accelerations a. I can apply the same concept as if you look at the derivations part is nothing else. that force is equal to mass into accelerations and the accelerations component I can write this and followed by I can get it dx dy dj nothing else is the volume of control volumes okay. So if we consider a parallelified control volume which is simple to derive the equations not going for surface interval volume integrals, it is easy for us to do it. So if you consider this type of simple control volumes, the Cauchy's equations help us to derive the simple form of equations, vectorically you can remember it.

both way that how the equations are there. Even if you have a doubt over these equations many of the times always we have a doubt about the equations how it is there. That is a common things when you have big equations and always we put a doubt over that how these equations are the all the components of equations are correct or not correct. what I can suggest you please look the dimensions okay. So the basic equations what I have rho dv by dt is equal to rho g del dot sigma j okay.

What is the dimensions of this side rho is m by lq into accelerations okay l by t square. So you can just cut it okay you can see that this is what force per unit volume that is what you will commit okay that is what you can see that this is a force per unit volume. Same way m by l cube g is a accelerations unit you know it l by t square plus what is the unit of this 1 by L sigma or stress is a force per unit area. So force you have a m L by T square unit area is L square. So if you look it just put the dimensions once you put the dimension you can understand every the del is the dimension by 1 by x that is what 1 by L is here.

So, if you put the dimensions you can see these are all the component force per unit volume, force per unit volume, force per unit volume that is what this is. force because of surface force, this is a force because of body force, this is the force because the body is moving with a velocity v which is having the acceleration component is a. That is the basic things when you look at the basic equations of Cauchy's equations if you derive from Newton second law concept. Always you can tell it that if we can get directly from this simple control volumes with a movable control volume, what is a necessary to derive Cauchy equations considering infinitely small control volume? What is a necessary? The necessary is to you to demonstrate it that we are discussing the velocity field, we are discussing about density field, we are discussing about the pressure field. All this field we derive it for a small control volume.

The basic equations is this part whether you can have this. Just to demonstrate it that these are all the field That is the reasons we have considered infinitely small control volume. Otherwise, I can use the same Newton's second law to derive directly these equations. No, we follow to show you the velocity, density, the pressure, the tensors all are continuous functions you apply through a control volumes using the Reynolds transport theorems and the Taylor series. We demonstrated that the functions what we are getting it which is as close to as we are getting from simple So basic idea you to demonstrate it all this field the velocity field, the density field, the pressure field, the trace fields.

all are having a functional dependency of the space and time for a infinitely small control volumes. If I derive it with a Taylor series with a Reynolds transport theorems the finally we will get this form of these equations nothing else okay. That is reasons I want to say that please do not be fear about the equations okay. Try to understand the physics which is a very interesting and that is what will make it force is equal to mass into accelerations nothing else the same we have the different formats. To demonstrate you that in a fluid flow we have to consider these are continuum functions with a space and time the trace the velocity density and the pressure as I demonstrated in the first class itself.

in fluid mechanics what we look it we just look it what will be the velocity field, what will be the pressure field, what will be the density field, what will be the acceleration field, what will be the trace field all we need it that is the reasons we try to solve these equations okay. So if you look at that the same concept we are talking about do not have a fear about vector notations of these equations which is looks very difficult but I do believe it today is what you have a computational efficiency as well as the materials what is available in many platforms. I think you can understand very easily all these vectors components okay. Let me I go to the next level where I will be just telling it that many of the times I do not need three-dimensionals the Cauchy equations okay. We are enough look at the one-dimensional part but always you can say that how do we can remember that equations which is looks a very difficult.

It is not a difficult, it is a very easy equations to remember it. First what we can do it always draw the axis okay. So draw x, y z. So we have a friendliness whenever you solve the fluid mechanics problems you draw the Cartesian coordinates or cylindrical coordinates or polar coordinate systems. The fluid mechanics problems before solving you have to scale the coordinates okay that is the axis that is what is necessary once you put the x, y, z you can write the scalar component u, v and w okay.

So that is we know it okay. Then you can write other components okay which I am not looking now. I have to look at the stress component which is acting to parallel to the x directions okay. So I am to so what I will do it first sigma xx so we can understand it is acting over this surface and perpendicular to that which is one yx sigma zx which is all 3s are acting along the x directions okay. So you can take the surface you can look at that this is a sigma yx and take this surface you can look at what is the sigma zx that is what you can notate it. whenever you sketch the figure coordinates as soon as you put the coordinates write u v w.

Same way if I am to write this part, I will write sigma yy, sigma will be the xy sigma zy. So, these type of notations you just have a practice or you can easily find out that these are all in x directions component. So I will putting the seconds of crits in the x directions then x, y, z the same way I am putting it away the this is what y directions I am putting it x, y, y, y, z, y. So I can put it in this way the z direction it is a very simple but please case the coordinates write down the scalar components of velocity then you write the stress components in each directions and try to understand it which plane it works which is the directions of acting of this stress components. Now if you look it if I draw this part now this same quasi equations if I write in x moment x directions.

It is easy as I know it so my component will be du by dt this is total derivative okay I can put a capital D also but okay rho gx the g components which is commit here we can put a gx. okay do not have a confidence that it is act vertically downwards or u y that please do not consider that the g can be a vector so we will have a g x g y g z. So this is a scalar component of the g sigma by sigma x rho x x that is what is acting in the x direction y rho y x delta by delta z is rho zx. So, you can look it. If you just look at this part of the component, always you can write this part.

It is not a difficult. The same way if I write for the y directions, dv small v dt is rho gy delta by delta x that is the components what I am looking at this way sigma xy del by del y sigma yy del by del z sigma zy. So same way you can write for the z components. Most of the fluid mechanics when we talk about the spacecraft we talk about biological science we cannot make it the y axis verticals because we have to simplify the fluid mechanics problem. So that times it is very easy for us to make a gravity force component in three directions x, y and z directions then the solve the problems that is the reasons we many of the advanced fluid mechanics books they follow the gravity components having the gx gy gz that is the difference. Now let me go for the next part which is Navier-Stokes equations okay.

What the Navier-Stokes equations are did it and what is the problems was there from Cauchy's equations if the problems or the mathematical challenges okay. If you look at this the equations what we have in Kaji's equations we have a stress tensors okay which is having 9 components as look at the stress tensors is having 9 component even if you look at a symmetric cases so it comes back to a 6 components okay so if you so 6 are unknowns are there then we have a unknown functions we have a density we also unknown having the velocity scalar component. So that means we have a 10 unknowns for 4 equations. So 3 equations from Causes equations, one equations from mass conservations. How can I solve the 10 unknowns for 4 equations? We need to have the approximations here.

The Navier stoke equations does that approximations. to reduce this unknowns the 10 unknowns to the 4 unknowns. So that with help of the 4 equations we can solve the problems because we have a 3 equations from causes and 1 equations from mass conservation equations. So we have a 4 equations 4 basic governing equations with us. But at presence we have a trace components are 9 components but if you consider the symmetric then even if the symmetry cases I have a 6 components.

6 plus 1 is density and the velocity is a 3 scalar components. So that way I have a 10 unknowns for the 4 equations. So basic idea comes from the Navier-Stokes equations okay which looks like that is almost 40 to 50 years works to reduce this the equations which is in a 10 dependent variables okay 10 dependent variable equations can make it to a reduce the variables okay dependent variables from 10 to 4. So that we can solve it with certain assumptions there is no doubt about that since it is a very basic level of the fluid mechanics I am not going to advanced level those are interested for the advanced fluid mechanics level they can have a advanced fluid mechanics course or There are the lot of books are there and you can follow it. So, the basically our objective is here to reduce the dependent variables which is the 10 dependent variable to the 4 dependent variables and the 4 equations and we can solve the problems.

That is the idea for Navier Stokes equations. First approximation what they have do it is very interestingly they consider it that if you assume it the fluid at the rest. If you are assuming the fluid at the rest at that point there will be no shear stress that is what you know it. So there will be no that is what the shear stress components becomes 0. Only what we will have through the Pascal's laws that I will have the pressures which is the equal amount of the pressures will be acting in all the directions. that is what I will get it when you have a fluid at the rest.

That means if I consider fluid at the rest my stress tensors which I defined earlier that becomes comes out to be again I am just writing for you the stress tensors what we are define it That is what will be becomes minus P 0 0 0 minus P 0 0 0 minus P. That is what will become it because there is no shear stress component when fluid at the rest. That is what the basic definitions of the fluid at the rest condition. when the shear stress 0 as equivalent the stress tensors will be a functions of the pressure. The negative sign we have considered as if you look it the stress and the P is a opposite directions that is the reasons we have used the minus P component.

That is what the basic concept has started it that we can split the stress component into two components. One is having the pressure and another having the shear stress component. That means if fluid start moving it from rest to a moving conditions then the pressures component what will be build up that is what will come out to be this. So it will have a two components. It will have the two component one will be the pressure functions as similar it remains at the rest okay.

As it is moving it this is what you have a viscous stress tensors okay that is what viscous stress tensors what you will get it which will have a same notations if you just look it if are like stress stentors it is also easy to write it okay do not we have a confusion much sigma xx first you write it okay. Next will be in the same x directions it will act it but the plane which is perpendicular to the y axis then z axis so this is what all x direction. So same way you put it for y directions that is what will come with x and y, this will be sigma yy, this is what will come with zy. So you remember it, the second subcrits will be the same, it will move xyz, xyz. So same way I can write it for this is y direction, this is for z direction.

Sigma So please have a always have a practice to write this component the subscript form components and this is the x direction, this is the y direction, this is the z directions. So, this is in the x direction, so second substrates will be the x. So, as going from up to down x, y, z, x, y, z, x, y, z that is all. So, if you can remember these notations which is not difficult, always you can remember these notations because that is what and always things that is the stress acting on the surface.

That is what you have to visualize. This is the trace components acting over the surface. The surface which is a parallelepiped or very simple way you can understand is a cube. over that cube surface I have a internal resistance force for unit area that is the and here the viscous components plays the real. So very technical way we divide these stress components into for a moving fluid in two components. One is the pressure components another is viscous stress tensor or more details I am not going it because you can look it detailed derivations of any advanced fluid mechanics book.

But try to look it the Navier-Stokes equations try to separate the fluid trace in the two components one component for the pressure one component which they get it they define is that because of internal resistance which is the trace component because of viscous components. So more details I am not going through it but if you are interested always you can look it advanced fluid mechanics book. Now for just to revise it the basic things what we discuss lot in very beginning classes about Newtonian fluids and non-Newtonian fluids okay Newtonian fluid. So this is what is established the relations between the shear stress Okay stress tensor stress tensors and strain rate tensor or Newton's laws of viscosity established between the shear stress and the shear strain rate that is what you know it the tau is equal to mu dv by dy that is what the Newton's laws of viscosity and mu as you know it, it is a dynamic viscosities. So if you look at that the same way we can always put a relationship between sigma the stress and the strain again I am hypo strain rate okay that is the reasons basic difference between the fluid mechanics and the solid mechanics.

As you know it has a relationship between stress and strain in the fluid solid mechanics but in fluid mechanics we talk about strain rate the change of the strain with respect to time okay. So this relationship if it is a linear relationship okay having a direct relationship like most of the normal fluid okay which air, waters and all. It follows basic relationship between stress and tension linear relationship that what makes us a validity of Newton's laws of viscosity as well as that fluid flow we define as Newtonian fluids. In this course, we will talk about Newtonian fluids but today's world we have a lot of the flow does not follow this Newtonian fluid concept. For example, if you talk about blood, biomedicals, you talk about even if slurries you talk about the suspensors, the polymer solutions they do not follow a linear relations between stress tensors and the strain rate sensors okay.

That is the reasons mostly we are combining it with air, waters, the kerosene and gasoline. So please have a knowledge over that the Newtonian fluids the Navier-Stokes equation see what derived for the Newtonian fluids that cannot be used for non-Newtonian fluids. We need to do a lot of other things but let not be discussed this. This is what the Newtonian fluid flow is defined by this what is the relationship between shear stress and shear strain rate that is what is the Newton's laws of viscosity. Same way we can have the relationship with the stress and strain rate That relationship also give us the new tanner fluids.

Before going to the next levels, I just to show you, we discussed that in chapter 17 on fluid kinematics. This is what repeating that. that we will have a many times will have the fluids will have a subjected to the shear stress and it will go for rotations okay that is a shear strain rate okay. So now what is our idea is that if I can write the shear stress components viscous shear stress component in terms of velocity field that is what we are trying to look at that can we write the shear stress components in terms of a functions of velocity components okay. That is the basic idea to derive in Navier-Stokes equations what we try to look it to go back to the chapter lecture 17 where we discuss about a fluid element subject to shear strain, the linear strain all we derive the equations I am not going more details as already we discussed in lecture 17.

What we got it that is what I am highlighting that the strain rate sensors in Cartesian coordinates we get it as a functions of u v w, no doubt it is easy. partial derivative components are there. So we can get it these derivations and these derivations already we derive it if you look at the fluid kinematics part okay that is I am not repeatedly derive it. But that is what is giving me the stress strain rate tensor is a functions of a partial derivative functions of the scalar velocity of u vw that is what we derive it you can see this the relationship between them okay. Now if you go for the next one which is very easy for us now because what we are looking at we first look at certain assumptions okay.

First is incompressible flow okay. That means density more or less fairly okay does not change with space as well as time. That means it is a constant. It is a constant. That is the flow is called incompressible flow.

Next is that iso thermal as flow. So thermal means you can indicate it is a temperature. Isothermal means within the fluid domain, temperatures remain more or less constant. That means within the fluid domain, we are not considering it that there is a temperature change. That is what a significant order of change of the temperatures.

So that is what is isothermal. Because the temperature does not change it, the dynamic viscosity is more or less is a constant. That is what it happens as the temperature remains constant. We are considering a fluid domain where we solve it. We consider that temperature does not change significantly. okay in the fluid domains which we are considering to solve the problems that means the dynamic viscosity that is also remains more less constant value.

If I have a these two assumptions okay this great simplifications of fluid flow problems okay. So if you look at that and go through very details book is there but it is quite higher level book okay of fluid mechanics by Kundu, Cochin and Dolling which is really it is a higher level book of fluid mechanics. If you go to it they have establishes for the stress is a functions of 2 times of mu of sigma is del the stress tensor rate tensor. okay so with these assumptions and the derivations which is available as you can get it a relationship between it the two factors always you can look it why it is there I am not going details for that so if you look at that I will getting the stress factors now in terms of the gradient of the scalar components of UVW. So that is the advantage of this that viscous shear stress we have written it for incompressibles, isothermal flow, with a certain assumptions with relationship between stress and strain rate.

We derive it the functional relationship as a shear stress is this as well as the stress tensor is like this. So both the derivations you can do by step by step which is there in the Zimbala book. Also you can see in FM White book. But basically it is a relationship what we have derived from fluid kinematics with certain assumptions as the incompressible isothermals.

We are deriving the relationship between stress tensors and the strain rate tensor. As the strain rate sensor is a functions of the gradient of scalar u v w that is what the advantage we have. If I have this if I use this relationship in the basic quasi equations then I am just expanding this is what the pressure components okay if you look it this is what pressure this is what gravity force component these terms is because of viscous stress components. So if you look at that you just substituting that you will getting all these viscous stress components which are the functions of mu and u and vw okay mu u vw that is what is the in the x directions that is what is a in the x directions you can derive similar way for y directions and the z directions. Now if you look it that is what is the derivations what is given it here that the pressure is a normal stresses only okay that is what is this and viscous stress can see both normal and shear stress component it contributes the three terms. if you look it there are certain assumptions which I am not going details like if you look at this the functions of when you had do the two different sessions okay x or z or x z that reverse things does not matter it as it is a smooth functions of x y z you can equate the both are the same values if I consider that rearranging these terms I get very interesting equations okay.

This is if you look at this is a continuity equations okay. Del dot v is 0 for incompressible flow that is what if I substitute that values okay. Now what I am getting it again I am just to rewrite it because we retain these things the accelerations components okay this is a total derivative minus dP by dx. So new terms has come because of pressure gradients this is a force component because of pressure gradient rho gx then we are getting it mu times of Laplace operator of u okay. So that is the mu is a constant for us as we consider is isothermal case. So we are getting it The additional two components which is a stress component sorry the stress components were there we split into two parts.

One part is a pressure components which is getting a pressure gradient component here. We also getting it the components because of viscosity is that mu rho square Laplacian operator of u. If mu is equal to 0, so you can easily say that this is equal to 0. That means your basic equations is du by dt is del p by del x rho gx which is a form of Euler equations. This is nothing else.

If there is no viscosity components, we can reduce to a Euler equations. If you consider the fluid at rest, so u is equal to 0 dP by dx plus rho gx is equal to g0 which is the equation for hydrostatics. So if you look at that the equations what we have derived the Newton Navier Stokes equations if we simplified it we will get it in different forms Euler or we can get it basic hydrostatic equations. So we need to remember not to remember it is okay I can say it you know it mass into accelerations in the scalar directions of u, x, x directions I have u component that what total derivative will be a force due to the pressures gradient because of change of the pressures, okay. That is the pressure gradients will give a force to that, the force due to the gravity, force due to this the viscosity that is what we can simplify it as a mu times of Laplandian operator of u.

that is what it is very interesting. And if you look it for other directions I think you can derive it for y directions and z directions as I just emphasize is that you just look at the notations okay here gx, gy, gz, This is gradient in x directions the equations what you are deriving for this is a gradient y way some gradient z direction showing that what is the pressure force is acting on okay per unit volume. This is the gravity force component and this is the three force components is coming it for us as a Laplace's operators of scalar field of u, v, w representing us discuss terms. So now if you look at these equations which we derived now as the basic equations but I derive it as the Navier-Stokes equations. This is the three equations what I have. The Navier-Stokes equations gives me three equations as you can look it that these three equations we are getting it plus we have a one equations is a mass conservation equations.

But these equations if you remember it, it is the equations what we have derived for the incompressibles isothermal cases okay. That is the assumptions you have to look it. For that derivations we have a three equations what we got it which is a nonlinear secondary partial differential equations one equations we have from the mass conservation. So that means again we have a four equations no doubt about that. But if I just look it how many dependent variables now okay how many the number of dependent variable.

If you look it as this flow is incompressible, so density is not there. It is a constant value. I have three velocity components are unknown and I have pressure components are unknown. So, Navier-Stokes has made it very simplified with certain assumptions, no doubt about that. We make it the equations with a 4 equations, 4 dependent variables, 4 unknowns to be get it which is a function of space and time which is the scalar velocity components u, v, w and the pressure. So, we have a 4 equations and 4 unknowns. So, we can solve it but still it has the problems is comes it here is it is a unsteady time component is there nonlinear partial differential equations.

So we may not have a exact solutions okay unless whether is very simplified case okay unless otherwise we need to do lot of approximations to solve these equations because still it is a 4 equations and the 4 dependent variables density is not there because flow is incompressible. So, if you look at that you have a 3 u v w and the pressures that is what 4 unknowns for me 4 dependent variables I have to derive it from the space and time that is my solutions for a particular fluid flow problems by solving these 4 equations. The problem is it is unsteady there is a time component the equations are non-linear second order partial difference equation if you look at this is a second order. So that is the reasons we work for approximate solutions that is the reasons we discussed from very beginning it that we look at the fluid mechanics problems still it is in this century is a challenging task for is a million era problem still we have not solved it. That is the reasons we try to look for a approximate solutions because there is no exact solutions we will get it unless otherwise very simplified cases like plate movements moving of two cylinders where this is a simplified cases we can get the some solutions analytical solutions but we cannot get a exact solutions of these four equations which is a four dependent variables because it is a nonlinear partial, secondary partial differential equations this nonlinear things makes us a very difficult to get a exact solutions that is the reasons we look for a last almost 50 years we are looking for approximate solutions of navistic equations in different forms that what is given birth of computational fluid dynamics which we discussed many times.

So this is the reasons we have the CFD now because there is no analytical solution otherwise there was there were no CFD or no computational full dynamics in subject of anybody because if you have a exact solutions of Navier-Stokes equations even if for incompressible isothermal cases. With this I think today I will just to make it this part to show you it is a big form of equations okay always you expand it it looks very difficult equations but please try to understand it is nothing else is a mass conservation equations del dot v that is what easily you can remember it for incompressible flow. This component as I try to explaining it step by step. Always you can expand it the local accelerations, the convective accelerations component local and convective, local and convective, the pressure force, the gravity force and the Laplace operator of u, v, w and mu times.

So the equations when you expand from the vector form it looks very complex. As I said it is a nonlinear second error partial differential equations what we are getting it that is will be there. But many of the times we go for cylindrical coordinates which easy to solve it. I am not going the derivations of the cylindrical coordinates just show the this is what the mass conservation equations for the cylindrical coordinates we discussed about this of how we will define this coordinate axis for the cylindrical coordinates. And one of the equations components looks like this okay if you look it is really complex as equation looks like that okay and it makes a fairness but I do not think it is that if you really look it the same way the convective acceleration and local accelerations and look at the Laplace's operators in the cylindrical coordinate systems you will get the same components what we will be talking about that. So, more details if you look with the theta component and z components okay, these equations are more complex.

All these equations are available in Sins and Cimbala even if FM white books okay, I am not encouraging you to remember these equations as it expected not expected in many problems okay. I do believe it look it positive way these equations what we have derived from Cartesian coordinates try to understand in a vector forms it is easy to under interpreted at the vector form as compared look it each components as in a cylindrical components. So in the next class we will talk about what are the boundary conditions and some of the basic features of the Navier-Stokes equation as well as we will try to solve few of the problem. Thank you. Thank you.