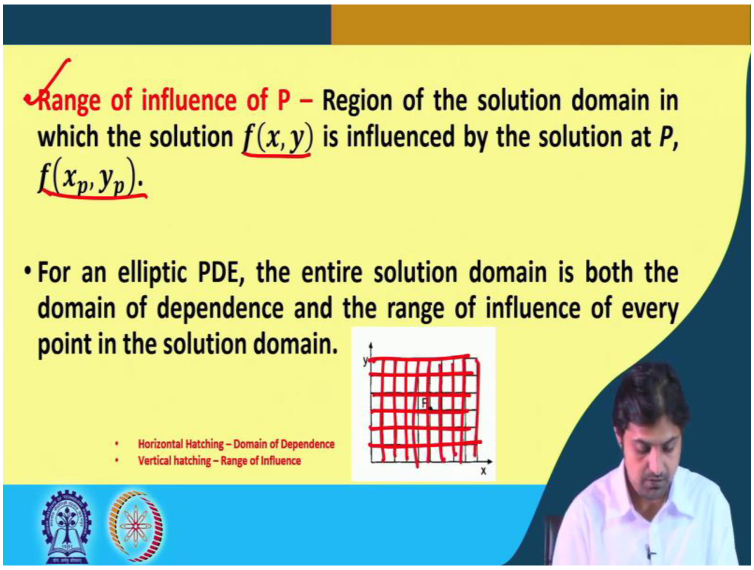

So, region of influence of P the region of solution domain in which the solution of x, y f of x, y is influenced by the solution at P which is f x p, y p. So, for an elliptical partial differential equation the entire solution domain is both the domain of dependence and range of influence of every point in the solution domain. So, there are 2 things domain of dependence of P and secondly, there is a range of influence of P which we have defined as the domain of dependence of phase the region of solution domain upon which x p f of x p, y p depends whereas, the range of influence of phase the range of solution domain in which the solution of except of x, y is influenced by the solution at p.

So, applying this to an elliptic partial differential equation the entire solution domain is both the domain of dependence and the range of influence of every point in the solution domain. So, the horizontal hatching here these ones shows the domain of dependence whereas the vertical hatching the shows the range of influence.

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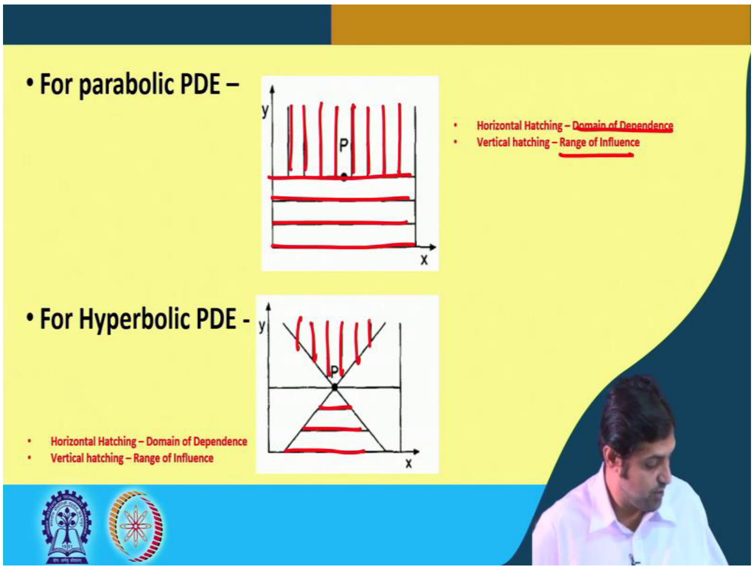

Now, for a parabolic PDE this is the domain of dependence so, horizontal hatching whereas the vertical hatching like these ones these are the range of influences. For a hyperbolic partial differential equation, the horizontal hatching shows the domain of dependence this one whereas the vertical hatching shows the range of influence. So, for 3 different type of solutions we said profiles, elliptic PDE, parabolic PDE and hyperbolic PDE.

For elliptic PDE we have seen that the domain of dependence is the entire solution and also the domain of dependence. Whereas, the parabolic and hyperbolic PDE the horizontal hatching as shown in the figures here shows the domain of dependence whereas the vertical hatching shows the range of influence.

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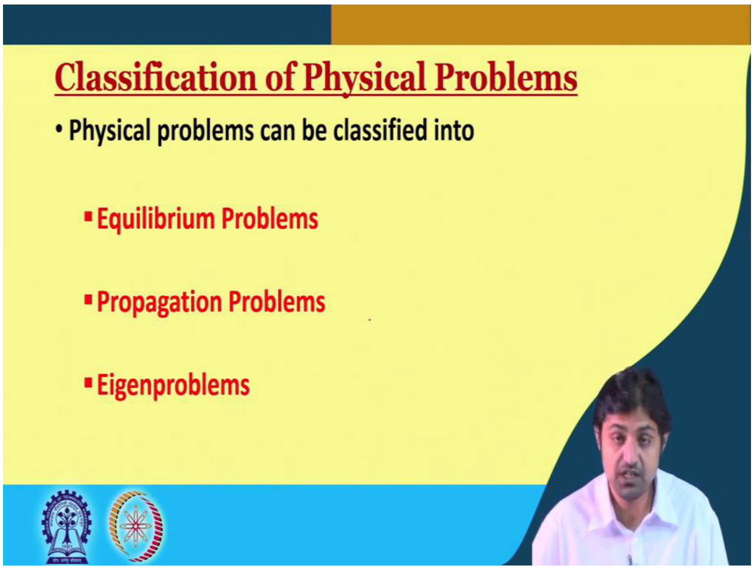

Now the classification of the physical problems can be classified into equilibrium problems or propagation problems, the third is Eigen problems. So, any physical problems can be classified into 3 different forms.

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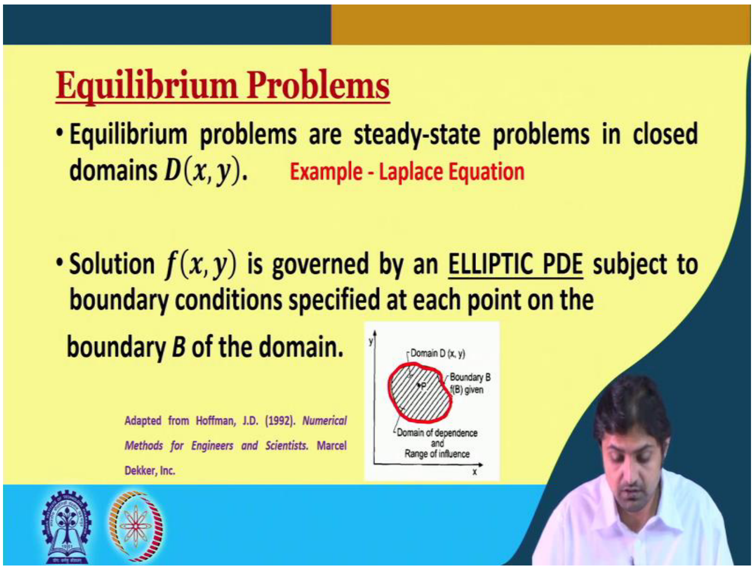

What are the equilibrium problems are the steady state problems in closed domain steady state means, there is no dependence on time there is an equilibrium. So, example is Laplace equation of such type of problems. So, here the solution f of x, y is governed by an electrical partial differential equation subject to boundary conditions specified at each point on the boundary B of the domain.

So, solution of Laplace equation is governed by an elliptic partial differential equation is an important thing to remember. And of course, the solution will depend upon the different type of boundary conditions that has been specified at each point on the boundary B of the domain. So, we have to specify suppose this is the you know, so, we have to specify boundary here also at all those points something like this.

So, if this is the figure you see you see, because this is an elliptical PDE the domain of independent dependence and range of influence are same all the domain and this is the boundary. So, we should be able to supply the boundary conditions at all the points, which is denoted by B here.

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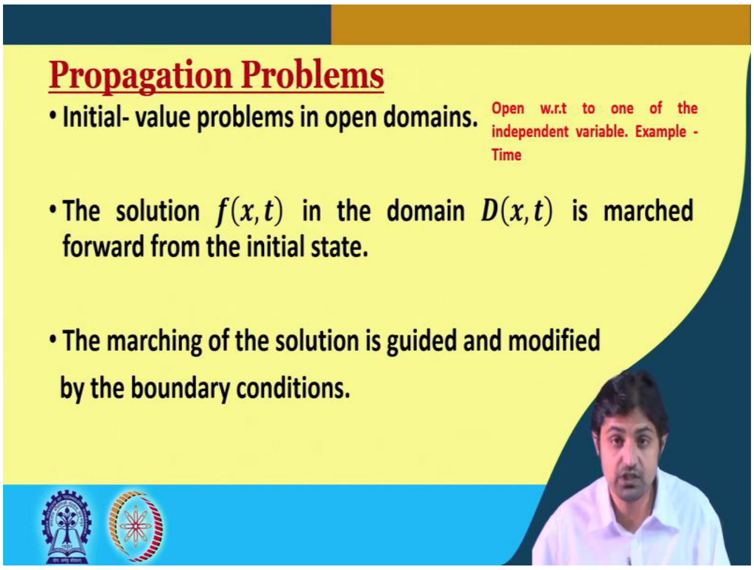

The second type of problems or propagation problems. So, an example is initial value problems in open domains. So, open with respect to one of the independent variables example time the solution f of x, t in the domain is marched forward from the initial stage. So, we know things that time t = 0, then we go from time t = 0 so let us say t = 2 seconds then time t = 2 seconds and time t = 3 seconds.

So, the marching of the solution is guided and modified by the boundary conditions. So, if we have a different boundary conditions, we still have to specify the boundary condition at time t = 0 initial at initial point, and we will also have to specify the initial problems at initial values at all the points in the domain. So boundary conditions and also the value at all the points in the domain initially to.

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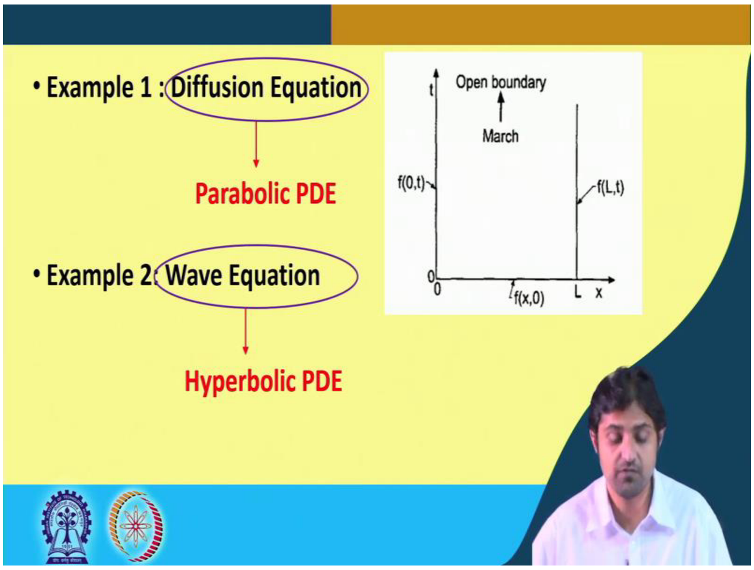

Example 1 here is the diffusion equation, you see this is an open boundary and we have to go from it is in x and this is in time t, this is x direction. So, this are solved by the parabolic partial differential equation parabolic PDE diffusion equation, second example is a wave equation. So, the wave equation is solved by the hyperbolic PDE hyperbolic partial differential equation. So, as we have seen 3 different type of equations have different partial differential equation profiles 1 was elliptical, Laplace equation the diffusion equation has parabolic partial differential equation and a wave is hyperbolic partial differential equations.

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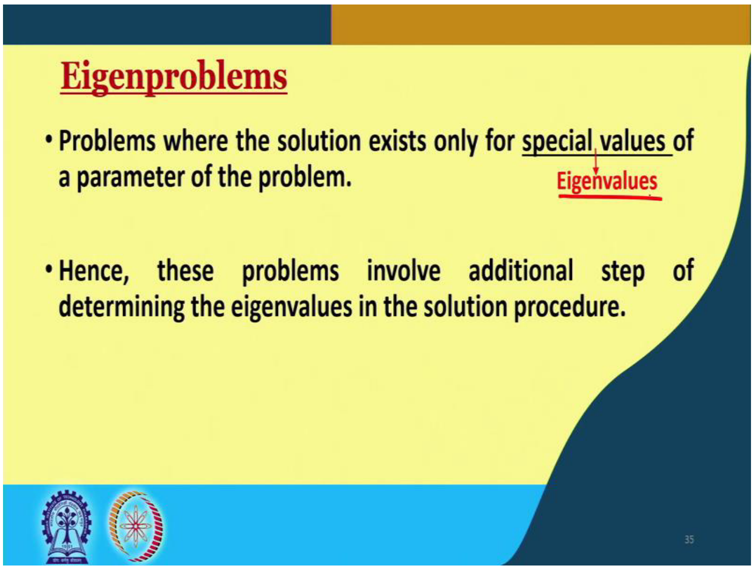

Now the last one in this set is the Eigen problems. So, problems where the solution exists only for special values of parameter of the problem. So, it the solution will not be there for all the values of the parameters. So, and these special values are called the Eigen values hence, these problems have involved additional step of determining the Eigen values is the solution procedure.

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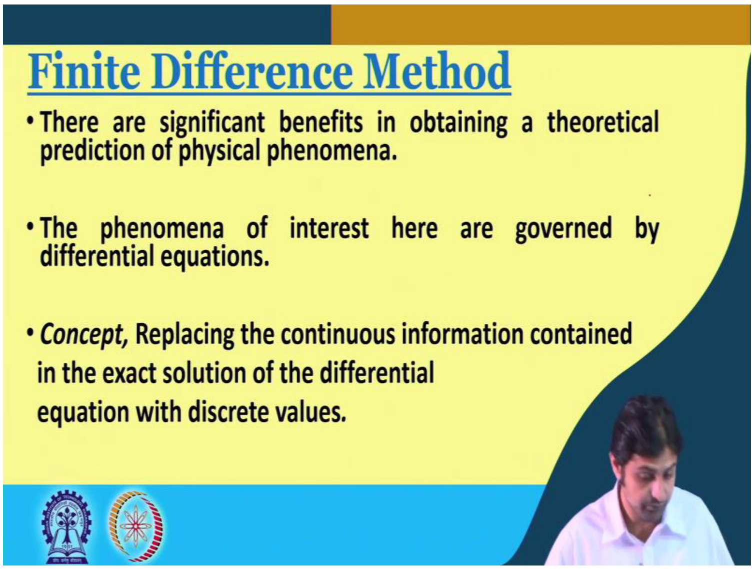

So, this is about the 4 type of problems that we were talking so, now, from this point onward we will proceed to the discretization technique that is the in we start with the finite difference method first. There are significant benefits in obtaining a theoretical prediction of a physical phenomenon. So, the phenomenon of interest here are governed by differential equations concept that is replacing the continuous information contained in the exact solution of the differential equation with discrete values.

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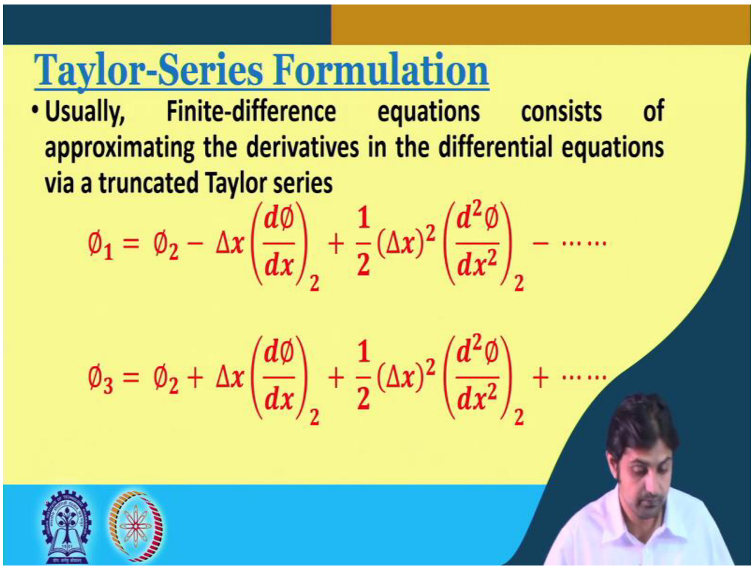

There is something called the Taylor Series Formulation which we generally use, usually finite difference equation consists of approximating the derivatives in the differential equations via a truncated Taylor series. So, how do we approximate the derivatives in the differential equation using a truncated Taylor series, which looks like this? I am pretty sure you have read that in your math class.

So, phi 1 is written as phi 2 - delta x del phi by del x at 2 + half delta x whole squared del squared phi into del x whole squared at 2. So, at series like this goes on with alternate - and + signs. So, this is called the truncated Taylor series or phi 3 can be written as phi 2 + delta x del phi del x + half del x squared and it can go on like this.

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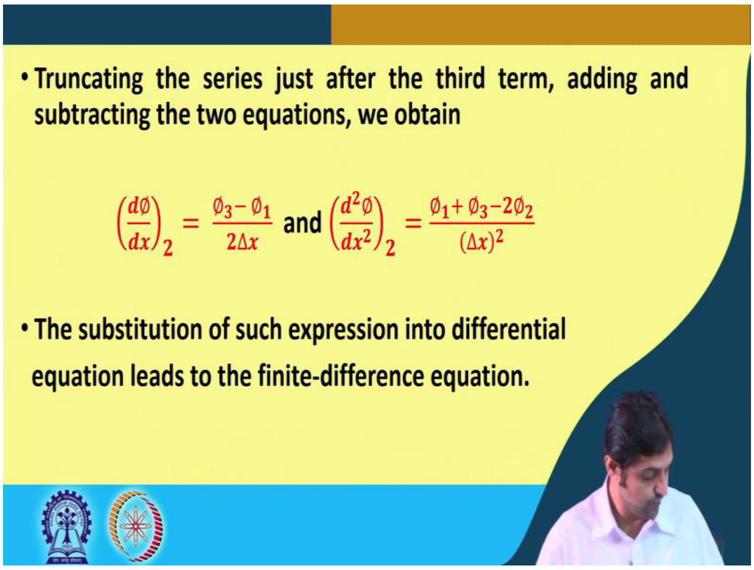

So, truncating the series just after the third term adding and subtracting the 2 equations so, you see there were 2 equations phi 1 and phi 3 both were return in terms of phi 2. If what we do if we just do you know if we first stopped the series just after the third term and add and subtract the 2 equation then we obtain del phi by del x at .2 will be phi 3 - phi 1 by 2 delta x and also del square phi by del x square can be written us phi 1 + phi 3 - 2phi 2 by delta x square.

So, what we have seen here, we have used the truncated Taylor series in general to write these values phi 1 and phi 3 and phi 2 and in once we add those and once we subtract those we get terms like these so, the substitution of such expression into differential equation leads to finite difference equation.

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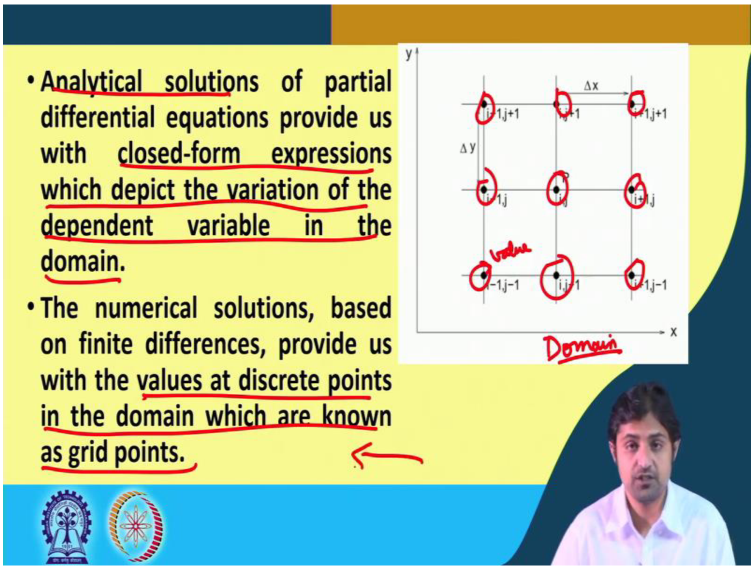

Say like this so, analytical solution of the partial differential equation provide us with closed form expressions which depicts the variation of the independent variable in the domain, so, this is the domain here and the numerical solution based on finite differences provide us with the values at discrete points in domain which are known at grid points. So, the difference between the analytical or the mathematical solution.

What does analytical solution of the partial differential equation provide us they provide us with closed form expressions we depict the variation of the dependent variable in the entire domain. Whereas, using the numerical simulation based on finite difference it provides us with the values at discrete points in the domain. So, it will provide us the value here. Here, whereas mathematical solution will give us closed form expression which is valid everywhere not only at some points.

So, of course, if we are able to obtain the analytical solution that is the best case scenario, but in majority most of the cases actually majority of the cases we are not able to do that therefore, we resort to the technique of finite difference method.

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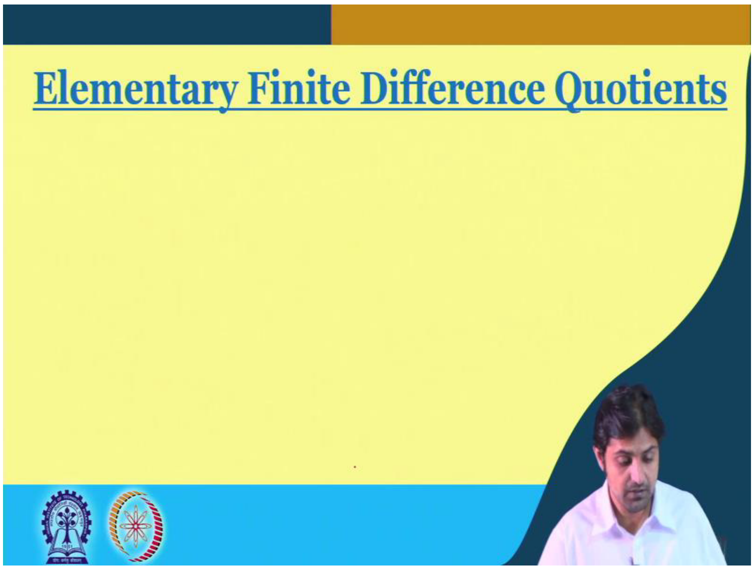

So, I think this is a nice point to stop and in the next lecture we will start with elementary finite difference quotients. So, thank you so much for listening and I will see you in the next lecture.