Chapter 14: Natural Frequencies

Introduction

In earthquake engineering, understanding the concept of *natural frequency* is fundamental to the dynamic analysis of structures. Every structure, whether a building, bridge, or tower, has a certain frequency at which it naturally vibrates when subjected to a dynamic force. When the frequency of an external force, such as an earthquake, matches the structure's natural frequency, resonance occurs—leading to amplified vibrations that can cause severe damage or failure.

Natural frequencies are influenced by the structure's mass and stiffness. Determining these frequencies is critical for designing earthquake-resistant structures, analyzing their response to seismic excitation, and applying concepts such as damping and modal analysis.

14.1 Basic Concepts of Vibrations

14.1.1 Types of Vibrations

- **Free Vibration**: Occurs when a structure is displaced and allowed to vibrate on its own.
- **Forced Vibration**: When a structure is subjected to an external periodic or random force.
- **Undamped and Damped Systems**: Ideal vs. real-world systems that account for energy dissipation.

14.1.2 Natural Frequency Definition

Natural frequency is the rate at which a system oscillates in the absence of any driving or damping force. Mathematically, for a single-degree-of-freedom (SDOF) system:

$$f_n = \frac{1}{2\pi}\sqrt{\frac{k}{m}}$$

Where:

- f_n = natural frequency (Hz)
- k = stiffness (N/m)
- m = mass (kg)

14.2 Single Degree of Freedom Systems (SDOF)

14.2.1 Mathematical Modeling

- **Mass-Spring-Damper System**: Simplest representation of structural systems.
- **Differential Equation of Motion**:

$$m\ddot{x} + c\dot{x} + kx = 0$$

14.2.2 Undamped Natural Frequency

For an undamped system:

$$\omega_n = \sqrt{\frac{k}{m}} \quad \text{(rad/s)}$$

14.2.3 Units and Interpretation

- Natural frequency is expressed in Hz or rad/s.
- A higher stiffness or lower mass leads to a higher natural frequency.

14.3 Multi-Degree of Freedom Systems (MDOF)

14.3.1 Introduction to MDOF Systems

Structures like multi-storey buildings have more than one natural frequency due to their multiple degrees of freedom.

14.3.2 Eigenvalue Problem

To find the natural frequencies:

$$[K - \omega^2 M]\phi = 0$$

Where:

- K = stiffness matrix
- M = mass matrix
- ϕ = mode shape
- ω = natural frequency

Solving this gives multiple eigenvalues (frequencies) and eigenvectors (mode shapes).

14.4 Modal Analysis

14.4.1 Principle of Modal Superposition

The response of an MDOF system can be considered as a combination of several SDOF systems vibrating independently.

14.4.2 Mode Shapes

- Represent the shape that a structure assumes at each natural frequency.
- Modes are orthogonal and each has a distinct frequency.

14.4.3 Orthogonality Conditions

Two different mode shapes ϕ_i and ϕ_j are orthogonal with respect to mass and stiffness matrices:

$$\phi_i^T M \phi_j = 0 \quad (i \neq j)$$

14.5 Numerical Methods for Frequency Calculation

14.5.1 Rayleigh's Method

An approximate technique using assumed mode shapes:

$$\omega^2 = \frac{\int_0^L EI \left(\frac{d^2y}{dx^2}\right)^2 dx}{\int_0^L \rho A y^2 dx}$$

14.5.2 Finite Element Method (FEM)

- FEM is widely used in practice to compute natural frequencies and mode shapes.
- Discretizes the structure into elements and solves the eigenvalue problem numerically.

14.6 Factors Affecting Natural Frequency

14.6.1 Mass Distribution

- Adding mass lowers the natural frequency.
- Uneven mass distribution leads to irregular modal responses.

14.6.2 Stiffness Variation

- Increased stiffness raises the natural frequency.
- Weak or flexible stories (soft storey effect) reduce stiffness and can cause vulnerability.

14.6.3 Boundary Conditions

- Natural frequency is affected significantly by how the structure is supported.
- Fixed vs. pinned vs. free conditions change stiffness characteristics.

14.7 Resonance and Structural Response

14.7.1 Resonance Phenomenon

Occurs when the frequency of external seismic excitation matches the natural frequency of the structure:

$$f_{earthquake} \approx f_{structure}$$

Leads to dangerously amplified vibrations.

14.7.2 Avoiding Resonance in Design

- Altering mass or stiffness.
- Using base isolators or dampers to shift natural frequency.
- Designing to avoid common seismic frequency ranges (1–10 Hz).

14.8 Experimental Determination of Natural Frequency

14.8.1 Ambient Vibration Testing

- Measures response due to wind, traffic, or other minor vibrations.
- Uses accelerometers and FFT (Fast Fourier Transform) to find dominant frequencies.

14.8.2 Forced Vibration Test

- A known force is applied to the structure and response is measured.
- More accurate than ambient methods but intrusive.

14.8.3 Free Vibration Method

- Structure is displaced and allowed to vibrate freely.
- Frequency is extracted from time-history response.

14.9 Importance in Earthquake Engineering

- Matching structural frequency with site frequency can be fatal.
- Helps in seismic zoning and design of dynamic dampers.
- Essential input for response spectrum and time-history analysis.
- Critical for retrofitting and performance-based design.

14.10 Frequency Content of Ground Motion

14.10.1 Fourier Spectrum

- Earthquake ground motion can be decomposed into frequency components using **Fourier Transform**.
- The **Fourier amplitude spectrum** shows how different frequencies contribute to the total motion.
- Helps in identifying dominant frequencies of ground motion that may affect specific structures.

14.10.2 Power Spectral Density (PSD)

- PSD represents how power (or energy) is distributed over frequency.
- Used in stochastic earthquake response analysis and for designing structures subjected to random vibrations.

14.10.3 Bandwidth of Earthquake Motions

- **Narrow-band** vs **broad-band** motions:
  - Narrow-band ground motion: high energy in limited frequency range; dangerous if matches structural frequency.
  - Broad-band motion: energy spread across many frequencies; affects multiple modes.

14.11 Site Effects on Natural Frequency

14.11.1 Local Soil Conditions

- Soft soils lower the natural frequency of the ground, which may match the frequency of certain buildings and lead to resonance.
- Example: Mexico City 1985 earthquake – soft lake-bed soils amplified motion.

14.11.2 Site Amplification

- Seismic waves get amplified when passing through low-stiffness materials.
- This can modify the effective ground motion and impact structures with matching frequencies.

14.12 Tuning of Structures

14.12.1 Structural Tuning Concept

- Adjusting mass and stiffness so that natural frequencies **do not coincide** with dominant ground motion frequencies.

14.12.2 Tuned Mass Dampers (TMDs)

- Devices added to structures to absorb energy at specific frequencies.
- Widely used in high-rise buildings, towers, and bridges to control vibrations.

14.13 Frequency Matching and Base Isolation

14.13.1 Base Isolation Systems

- These systems **decouple** the building from ground motion by introducing flexibility at the base.
- This shifts the natural frequency of the structure **below** the dominant earthquake frequencies.

14.13.2 Frequency Shift Strategy

- Making the structure flexible (lower frequency) or stiffer (higher frequency) to avoid resonance.
- Key part of performance-based earthquake design.

14.14 Frequency Ratios and Modal Participation

14.14.1 Frequency Ratio

- Ratio between the frequency of ground excitation and the structure's natural frequency:

$$r = \frac{\omega}{\omega_n}$$

- Where:

- ω: forcing frequency
- ω_n: natural frequency

- Resonance occurs when $r \approx 1$.

14.14.2 Modal Participation Factor

- Measures how much a particular mode contributes to total structural response.
- Important in **response spectrum analysis** and **mode superposition method**.

14.15 Frequency Considerations in Design Codes

14.15.1 IS Code Provisions (IS 1893:2016)

- Specifies natural period estimation formulas:
  - For RC moment-resisting frame buildings:

$$T = 0.075h^{0.75}$$

- For steel frame buildings:

$$T = 0.085h^{0.75}$$

- h = height of building in meters
- These formulas help approximate natural periods for code-based seismic design.

14.15.2 Design Response Spectra

- Natural frequency (or period) of the structure is used to extract spectral acceleration from design response spectra.
- Determines seismic forces to be considered.

14.16 Practical Case Studies and Failures Due to Frequency Matching

14.16.1 Case Study: Mexico City Earthquake (1985)

- Soft soil layers had natural frequencies around 0.5–1 Hz.
- Mid-rise buildings (6–15 stories) had similar natural frequencies → severe damage due to resonance.

14.16.2 Case Study: Kobe Earthquake (1995)

- Short-period structures (low natural frequency) suffered from stiff soil-ground interaction and pounding effects.

14.16.3 Lessons Learned

- Matching structural frequency with site frequency or ground excitation is dangerous.
- Frequency-based analysis must be integrated early in the design stage.
