Chapter (5): Beam Deflection

5.1 Introduction:

The axis of a beam deflects from its initial position under action of applied forces. Accurate values for these beam deflections are sought in many practical cases: elements of machines must be sufficiently rigid to prevent misalignment and to maintain dimensional accuracy under load; in buildings, floor beams cannot deflect excessively to avoid the undesirable psychological effect of flexible floors on occupants and to minimize or prevent distress in brittle-finish materials; likewise, information on deformation characteristics of members is essential in the study of vibrations of machines as well as of stationary and flight structures.

5.2 Factors Affecting Beam Deflections

Factor	Symbol	Type
Span length	l	Directly proportional
Applied load	w	Directly proportional
Modulus of Elasticity	E	Inversely proportional
Moment of Inertia	I	Inversely proportional

5.3 Calculating Beam Deflections:

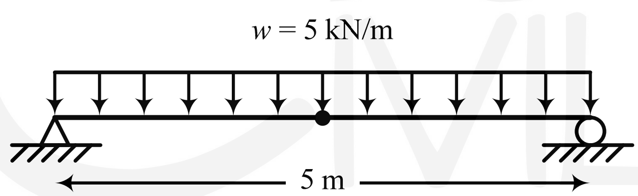
Calculations of beam deflections will depend on the formulae provided in the cases below.

5.4 Examples:

Example (1):

For the beam shown in the figure below, calculate the deflection of the beam at the mid-span.

Given: $E = 200\,\text{GPa}$, $I = 200 \times 10^6\ \text{mm}^4$



Solution:

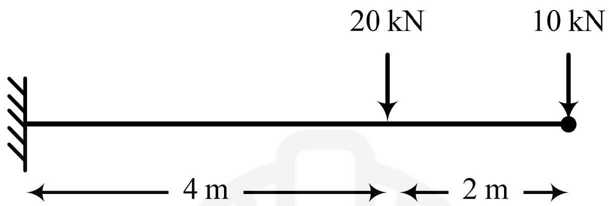
$$w = 5\,\text{kN/m} \times \frac{1\,\text{m}}{1000\,\text{mm}} = 0.005\,\text{kN/mm},\ L = 5\,\text{m} = 5000\,\text{mm}, \qquad E = 200\,\text{GPa} = 200\,\text{kN/mm}^2$$

$$\Delta = \frac{5}{384}\frac{wl^4}{EI} = \frac{5}{384}\frac{(0.005\,\text{kN/mm})(5000\,\text{mm})^4}{(200\,\text{kN/mm}^2)(200 \times 10^6\,\text{mm}^4)} = \boxed{1.017\ \text{mm}}$$

Example (2):

For the beam shown in the figure below, calculate the deflection of the beam at the free end.

Given: $E = 90\,\text{GPa}, \quad I = 100 \times 10^6\,\text{mm}^4$ $E = 90\,\text{GPa}, \quad I = 100 \times 10^6\,\text{mm}^4$



Solution:

$P_1 = 10\,\text{kN} \qquad P_2 = 20\,\text{kN} \qquad l = 6\,\text{m} = 6000\,\text{mm} \qquad I = 100 \times 10^6\,\text{mm}^4$

$x = 2\,\text{m} = 2000\text{ mm} \qquad b = 4\,\text{m} = 4000\text{ mm} \qquad E = 90\,\text{GPa} = 90\,\text{kN/mm}^2$

$$\Delta_1 = \frac{P_1}{6EI}\left(2l^3 - 3l^2 x + x^3\right)$$

$$= \frac{(10\,\text{kN})}{6\left(90\,\text{kN/mm}^2\right)\left(100 \times 10^6\text{ mm}^4\right)}\left(2(6000\,\text{mm})^3 - 3(6000\,\text{mm})^2(2000\text{ mm}) + (2000\text{ mm})^3\right)$$

$$= 41.48\,\text{mm}$$

$$\Delta_2 = \frac{P_2 b^2}{6EI}(3l - 3x - b)$$

$$= \frac{(20\,\text{kN})(4000\text{ mm})^2}{6\left(90\,\text{kN/mm}^2\right)\left(100 \times 10^6\text{ mm}^4\right)}\left(3(6000\,\text{mm}) - 3(2000\text{ mm}) - (4000\text{ mm})\right)$$

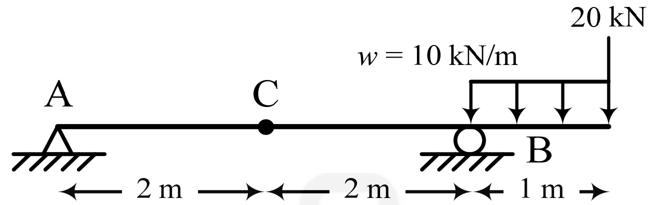
$$= 47.41\,\text{mm}$$

$$\Delta = \Delta_1 + \Delta_2 = 41.48\,\text{mm} + 47.41\,\text{mm} = \boxed{88.88\,\text{mm}}$$

Example (3):

For the beam shown in the figure below, calculate the deflection of the beam at point C.

Given: $E = 100\,\text{GPa}$, $I = 120 \times 10^6\ \text{mm}^4$



Solution:

$$w = 10\,\text{kN/m} \times \frac{1\,\text{m}}{1000\,\text{mm}} = 0.01\,\text{kN/mm} \qquad l = 5\,\text{m} = 5000\,\text{mm} \qquad E = 100\,\text{GPa} = 100\,\text{kN/mm}^2$$

$$P = 10\,\text{kN} \qquad x = 2\,\text{m} = 2000\ \text{mm} \qquad a = 2\,\text{m} = 2000\ \text{mm} \qquad I = 120 \times 10^6\ \text{mm}^4$$

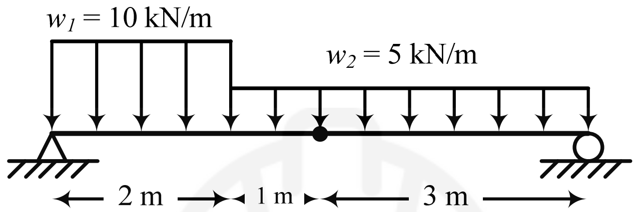
$$\Delta_1 = \frac{Pax}{6EIl}\left(l^2 - x^2\right)$$
$$= \frac{(10\,\text{kN})(2000\ \text{mm})^2(2000\ \text{mm})}{6\left(120\,\text{kN/mm}^2\right)\left(100 \times 10^6\ \text{mm}^4\right)}\left((5000\,\text{mm})^2 - (2000\ \text{mm})^2\right)$$
$$= 2.33\,\text{mm}$$

$$\Delta_2 = \frac{wax^2}{6EIl}\left(l^2 - x^2\right)$$
$$= \frac{(0.01\,\text{kN/mm})(2000\ \text{mm})(2000\ \text{mm})}{6\left(120\,\text{kN/mm}^2\right)\left(100 \times 10^6\ \text{mm}^4\right)}\left((5000\,\text{mm})^2 - (2000\ \text{mm})^2\right)$$
$$= 1.17\,\text{mm}$$

$$\Delta = \Delta_1 + \Delta_2 = 2.33\,\text{mm} + 1.17\,\text{mm} = \boxed{3.5\,\text{mm}}$$

Example (4):

For the beam shown in the figure below, calculate the deflection of the beam at the mid-span. Given: $E = 95\,\text{GPa}$, $I = 100\times10^6\ \text{mm}^4$



Solution:

$$w_1 = w_2 = 5\,\text{kN/m}\times\frac{1\,\text{m}}{1000\,\text{mm}} = 0.005\,\text{kN/mm}$$

$$l = 6\,\text{m} = 6000\,\text{mm} \qquad E = 95\,\text{GPa} = 95\,\text{kN/mm}^2$$

$$x = 3\,\text{m} = 3000\ \text{mm} \qquad a = 2\,\text{m} = 2000\ \text{mm} \qquad I = 100\times10^6\ \text{mm}^4$$

$$\Delta_1 = \frac{5w_1 l^4}{384EI} = \frac{5\left(0.005\,\text{kN/mm}\right)\left(6000\,\text{mm}\right)^4}{384\left(95\,\text{kN/mm}^2\right)\left(100\times10^6\ \text{mm}^4\right)} = 8.88\,\text{mm}$$

$$\Delta_2 = \frac{wa^2\left(l-x\right)}{24EIl}\left(4xl - 2x^2 - a^2\right)$$

$$= \frac{\left(0.005\,\text{kN/mm}\right)\left(2000\ \text{mm}\right)^2\left(6000\,\text{mm} - 3000\ \text{mm}\right)}{24\left(95\,\text{kN/mm}^2\right)\left(100\times10^6\ \text{mm}^4\right)\left(6000\,\text{mm}\right)}$$

$$\times\left(4\left(3000\ \text{mm}\right)\left(6000\,\text{mm}\right) - 2\left(3000\ \text{mm}\right)^2 - \left(2000\ \text{mm}\right)^2\right)$$

$$= 2.19\,\text{mm}$$

$$\Delta = \Delta_1 + \Delta_2 = 8.88\,\text{mm} + 2.19\,\text{mm} = \boxed{11\,\text{mm}}$$

5.5 Problems:

Question № 1:
For the beam shown in the figure (1) below, calculate the deflection of the beam at the mid-span. assuming $E = 80$ GPa and $I = 130 \times 10^6$ mm^4.

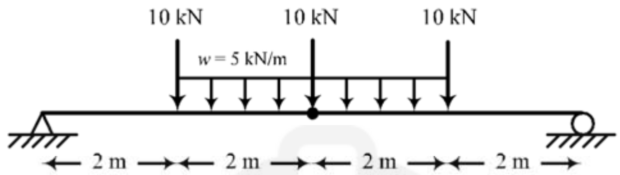


Figure 1

Question № 2:
Determine the displacement at point B for the cantilever beam shown in the figure assuming $E = 29000$ ksi.

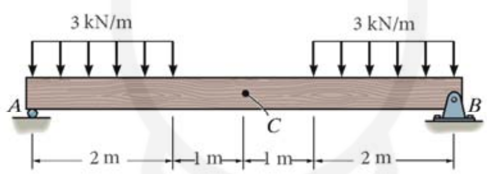


Figure 2

Question № 3:
Determine the deflection of the point located at mid–span between supports A and B for the beam shown in the figure. Assume $E = 200$ GPa and $I = 54 \times 10^6$ mm^4.

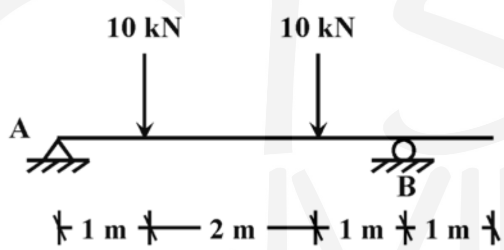
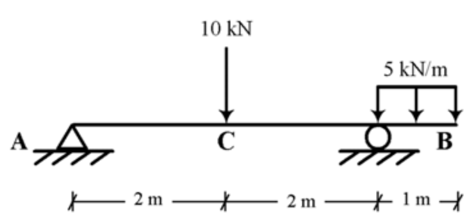


Figure 3

Question № 4:
Calculate the total displacement at point C of the beam shown in the figure given that and $I = 60 \times 10^6$ mm^4. and $E = 200$ GPa.



Question № 3