Hydraulic Engineering
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Lecture - 38
Pipe flow

Welcome students. This week we are going to study a module that is called pipe flow, which will go on for another week. This is a very long and important chapters of hydraulic engineering, same as open channel flow. So, this comprises of almost one sixth of the portion of the entire course.

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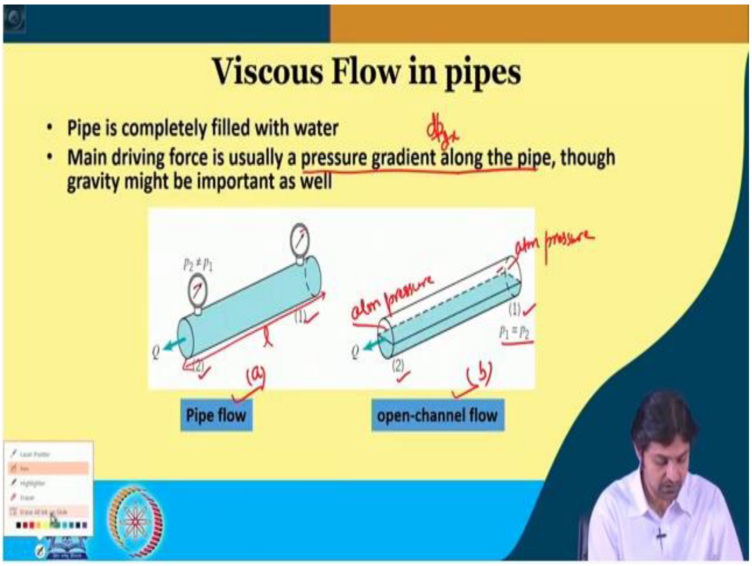

So, to get started, one of the important things that you must know that the flow in the pipes are viscous in nature. Therefore, we call it viscous flow in pipes. So, an important property of a pipe flow is that the pipe is completely filled with water or any other fluid, whichever it can be; it can be with oil or anything, but the pipe should be completely filled with it. Here, the main driving force is usually a pressure gradient along the pipe.

So, if you remember, in open channel flow the main driving gradient was gravity. But here, it is pressure gradient along the pipe. If you remember, from dimensional analysis, we derived equation for the pressure drop per unit length in the pipe. So, you would imagine that the pressure gradient along this pipe delta p/delta x, let us say, if x is in this direction, this is very important. Here, gravity may or may not be that important. But to define the pipe flow, we said pressure gradient must be there.

So, these are the 2 figures, there is a flow. So, this is figure, a and this is b. These are section 1 and 2 in this pipe. This is section 1 and 2 in this pipe. If, there is a flow occurring in both the flows, if you put the pressure transducers or something that can measure pressure here and here, this will say that p2 is not equal to p1. That means, there is a pressure gradient, along this length.

And in the second case, the pipe is not completely filled with water and if you see, if it is open, you know, here also the atmospheric pressure will be there and here also atmospheric pressure will be there and p1 will be equal to p2. So, this is classified as pipe flow, whereas here there is no pressure gradient, the flow is occurring and it has a free surface. That means, it is an open-channel flow.

So, this is pipe flow, as I told you, explained you and the figure on the right hand side is open-channel flow because it is exposed. It has an exposed free surface to atmosphere and the main driving force here was the gravity. So, I will take these slides away.

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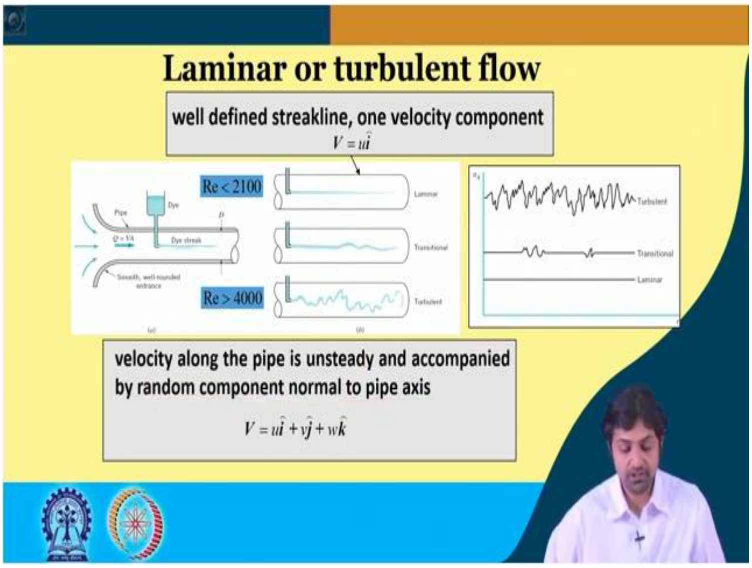

So, now, when the flow in the pipes occur, the important question is, whether it is laminar or turbulent flow because that is one of the classification of the flows that we also saw in the open-channel flow. Therefore, what is laminar and turbulent flow? That I am going to explain. So, the figure a, represents a pipe in which the water is flowing, water or any liquid for that purpose and we have a set for a dye, this is a dye.

What we do is, we drop a little bit of dye here, using this apparatus. You know what dye is? It is a coloured thing, that takes the color of the liquid in which it is. And with the velocity, this dye will also start moving d, y, e, dye. So, this is called a dye streak. The line, which the dye follows is called the dye streak. As you would see, so this, the figure number b is the detailed figure of how this dye streak looks like.

You see, this a, this b and this c. So, if the flow velocity is less or if the flow is laminar, in case of less velocity there is likelihood, more likelihood that the flow is going to be laminar. It will be almost like a streamline, you see, like this. As you keep on increasing the velocity, you will see, there is going to be starting of some disturbance, like this. This is a transitional flow and if the velocity increases very high, the flow becomes fully turbulent.

So, there will be a lot of fluctuations, like this. So, this is one of the experimental setups in a pipe flow, where you can actually observe the differences between, the physical and the visual difference between the laminar, transitional and the turbulent flow. So, it has been found out that for laminar flow, the Reynolds number should be less than 2100. This is an important Reynolds number that I expect you to remember.

So, for the flow in pipes, the Reynolds numbers should be less than 2100 to be laminar. Whereas, for the turbulent flow, if the Reynolds number is greater than 4000, that flow is definitely laminar and for the range in between 2100 and 4000 Reynolds number, the flow is transitional. You see, this is the u, the velocity in x direction have been plotted, with respect to time. So, for laminar flow, it is going to be a straight line, very straight line, here.

For the transitional flow, you see, there are some disturbance, at some point then it becomes straight and then it becomes, so this is transitional. The Reynold number has risen, but not that highest so that it becomes fully turbulent. And when we plot for turbulent, you see, there are fluctuations right from the beginning, at every point it is like this, you see, here. So, this is a completely turbulent flow.

So, for laminar flow, it is a well defined streakline and there is only 1 velocity component that is u of i, i means, in x direction. A unit vector in the x direction is i cap. So, velocity is u i cap. Whereas, in the turbulent flow, the velocity along the pipe is unsteady and it is accompanied by random component, normal to the pipe axis. So, first of all it is unsteady and it is accompanied by random component, normal to pipe axis.

And it the velocity will be the sum in velocities, in all the 3 directions and it is going to be

$$V = u\hat{i} + v\hat{j} + w\hat{k}$$

. That is an important property of the turbulent flow.

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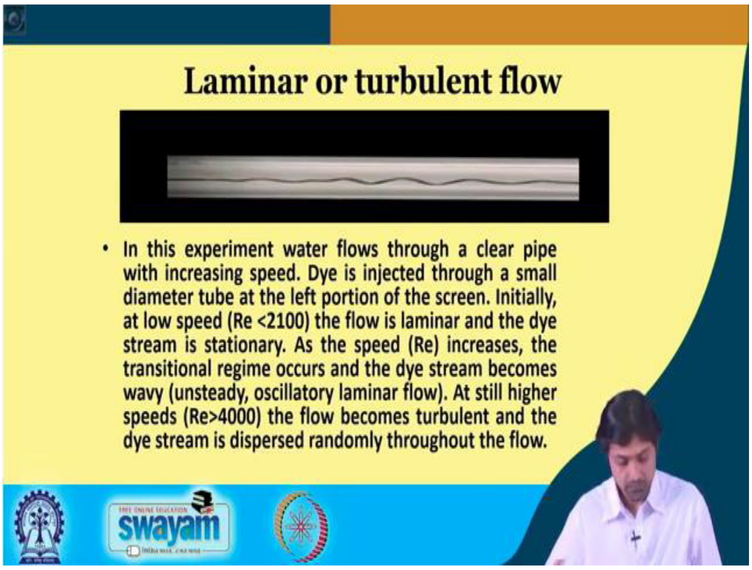

So, this is a figure of an experiment. The link I have written, I have not attached the video because of the copyright issues. But this is an image of the experiment, which was done in the pipe, as I have shown you. And this has been done with the help of a dye. So, what happens is, in this experiment, the water flows through a clear pipe. So, there was a clear pipe and the speed has been increased in steps.

Initially, the dye is injected through a small diameter tube at the left portion of the screen. So, from here, the dye has been injected. Initially, when the speed is low or we can also say when the Reynolds number was less than 2,100 the flow is laminar. If, you watch the full video following the link given, you can see, the flow is laminar and the dye stream is stationary. It is like a pure streamline, one single line.

Now, as the speed and consecutively the Reynold lumber increases, the transitional regime occurs and the dye stream becomes wavy. It becomes unsteady and oscillatory laminar flow. So, in transition, you see, there is, you would see in the video that it becomes wavy, a little bit of, you know, something like this, you know. Whereas, if you keep the Reynolds number even higher, that is, greater than 4,000 or what you can do, you cannot control the Reynolds number. just like that. What you do is, you increase the speed.

So, if you increase the speed, the flow becomes fully turbulent and the dye stream is dispersed randomly throughout the flow. So, you see, we have read in the chapter of laminar and turbulent flow, one important property of turbulence is dispersion and mixing. So, you see, in this particular video, you will see, that the dye stream is dispersed, in all the directions.

And clearly indicating what we had learned in the laminar and turbulent fluid flow chapter, in this particular course of hydraulic engineering was correct. We can verify that experimentally and by visual investigation.

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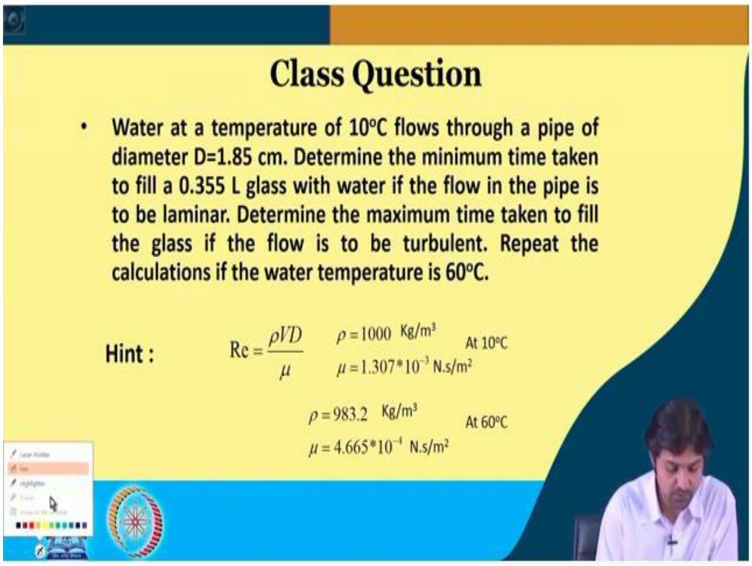

Now, there is a question. So, water at a temperature of 10 degree centigrade flows through a pipe of diameter 1.85 centimeters. Determine the minimum time taken to fill a 0.355 litre glass with water if the flow in the pipe is to be laminar. The second part is. determine the maximum time taken to fill the glass if the flow is to be turbulent. And we will have to repeat the calculation for the same thing if the water temperature is 60 degree centigrade.

What we have been given? We know that the Reynolds number is rho VD/Mu, we also have been given the rho and mu at 10 degree centigrade and we also have been given the rho and Mu at 60 degree centigrade. So, this is all the information that we have for now. So, this is the simplest most basic question, but this will help you clarify the idea of laminar and turbulent flow in pipes. So, I am going to go to the white screen.

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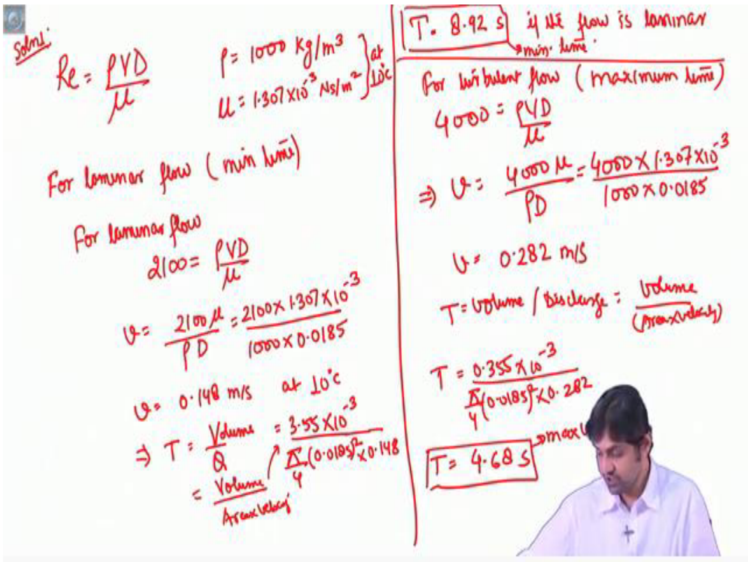

And we are going to solve this problem. So, let us start with the 10 degrees' case. So, we know that Reynolds number is rho VD/Mu. D is the diameter of the pipe, rho is 1000 kilogram per meter cube and Mu is 1.307 into 10 to the power - 3 Newton second per meter square, at 10 degree centigrade. So, for laminar flow, what was the Reynolds number which was critical? So, for any flow, which was less than 2100 Reynolds number was laminar.

So, the question is, determine the maximum time taken to, sorry, determine the minimum time taken to fill a 0.355 liter glass. So, when will the minimum time be taken? Minimum time will be taken when we have the maximum velocity and maximum velocity that can be taken, in case of laminar flow would correspond to Reynolds number of 2100, because that is the maximum Reynold number.

So, for laminar flow, Reynolds number we have taken 2100 because the velocity corresponding to this Reynolds number of 2100 will take the minimum time to fill the glass is equal to rho VD/Mu. So, the velocity is going to be, 2100 Mu/rho into D or we can write, 2100 into 1.307 into 10 to the power - 3/1000 into the diameter of the pipe, which is given as, 0.0185, 1.85 centimeter is 0.0185 meters.

And this will give us, 0.148 meters per second, at 10 degree centigrade. Therefore, the time taken to fill is velocity, so, sorry, not velocity, volume by and which is 3.55 liter into 10 to the power -

3 meter cube and Q is area into velocity. What is the area? Pi/4 into D square into velocity, which we have got, 0.148. So, I think I should write this step again, volume/area into velocity.

Therefore, if you do this calculation, time that will be taken to, minimum time taken to fill will be 8.92 second, if the flow is laminar. Now, the second part, it says is what is the maximum time taken to fill the glass, if the flow is to be turbulent? So, turbulent regime starts from Reynolds number of 4000. If the velocity is high that means, time taken will be very less.

But here, the question is what is the maximum time taken if the flow is to be turbulent? So, the maximum time taken will be, when the velocity is minimum. So, minimum velocity required for a turbulent flow will correspond to a Reynolds number of 4000. So, for turbulent flow, minimum time here, maximum time.

So, for turbulent flow, the maximum time, we write, 4000 = rho VD/Mu or velocity is going to be 4000 Mu/rho into D or 4000, Mu is 1.307, into 10 to the power -3/rho is 1000, diameter is 0.0185 and this will give us, a velocity of 0.282 meters per second. So, the time taken will again be volume/discharge or volume/area into velocity. So, this will come out to be 0.355 into 10 to the power -3/pi/4 0.0185 and the velocity is 0.282.

So, the time required, maximum time required is going to be 4.68 seconds, max time. This was minimum time. So, you have seen how this is calculated. Now, the second part is, we have to repeat the calculations if the temperature is 60 degree centigrade. So, I will quickly go through the calculations again. So, first I will,

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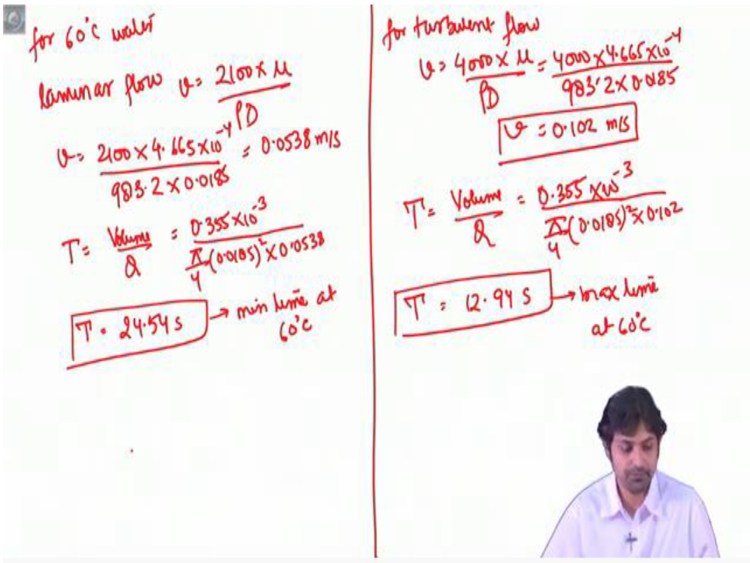

So, for 60 degree centigrade water, so I will just take the, so for laminar flow velocity is 2100 into mu/rho D and therefore, the velocity is 2100, mu in case of 60 degree is 4.665 into 10 to the power -4 divided by, in density was 983.2 and the diameter remains same. So, this will give us 0.0538 meters per second. Therefore, time required would be volume/Q and this is 0.355 into 10 to the power -3/pi/4 0.0185 whole square divided by 0.0538.

So, the time required in this, comes to be 24.54 seconds, so minimum time at 60 degree centigrade. So, for turbulent flow, V will be similar thing, 4000 into Mu/rho D, Mu is 4.665 into 10 to the power -4. So, 983.2 into 0.0185 and this will come, the velocity is going to come 0.102 meters per second. Therefore, the time taken would be volume/discharge. So, volume is 0.355 into 10 to the power -3 divided by, maximum time at 60 degree centigrade.

So, this was one simple question that we solved with the application of laminar and turbulent boundary values. But rest of the concepts of the Reynolds number, what is the formula and other things remain the same. So, after this we will go back to our lectures, the slides.

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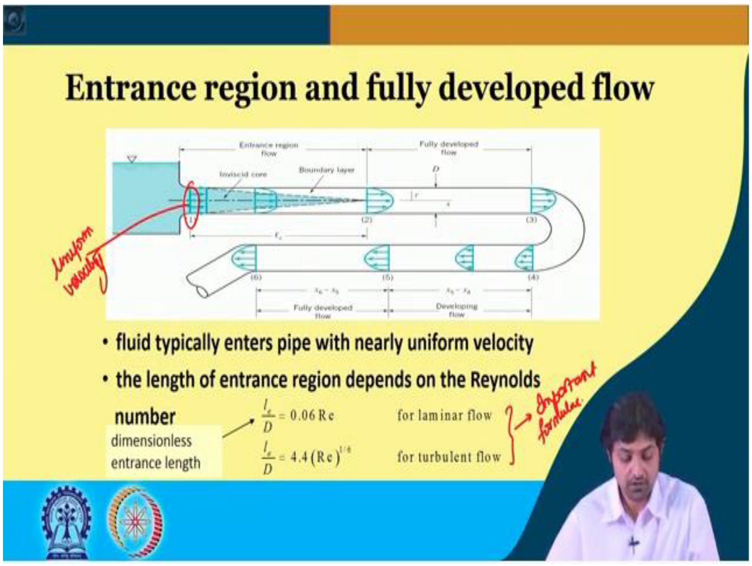

Now, there is an entrance region and fully developed flow. We should talk about this what entrance region is and what a fully developed flow in a pipe flow is. So, you see, there is a figure here, which I will explain it to you later. So, there is a pipe coming out from the reservoir and the, so this is the and this is pipe. So, as soon as the water comes out from the reservoir to the pipe, earlier there was no water.

So, as soon as this water comes out, the water will take some time to become fully developed. We are going to go into that in a little more time but I will explain you. So, this region, this length le is called the entrance region. And as soon as the water starts moving the boundary layer formation will start because near the wall the velocity is going to be almost to the 0 and the maximum velocity will occur along the centre line, as we have already seen in the laminar and turbulent flow analysis.

Inviscid core here means that there is no viscosity. The viscosity is at the boundary. And after it has gone through this entrance region le, the velocity becomes fully developed. And you see, this is the velocity profile in the pipe flow that we get, parabolic in nature. So, now just going into a little, you know, so what we see is fluid typically enters pipe with nearly uniform velocity.

So, the velocity with which it enters at section 1 is uniform velocity. The length of this entrance region depends on the Reynolds number. So, how long this entrance length will depend upon how fast the water is flowing or in other word, the length of the or in the other words, Reynolds number. So, the dimensionless entrance length is given by le/D = 0.06 Re. Whereas, for turbulent flow, it is given as le/D = 4.4 into Reynolds number to the power 1/6.

These are the important formulas. So, you should remember them. So, if the flow is laminar we will have a shorter entrance length. Whereas if the, I mean, Reynolds number is high or the flow is turbulent, we will have a longer entrance length region.

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Now, as the fluid moves through the pipe, viscous effects cause it to stick to the pipe wall because of the no slip boundary condition. As I told you here, as it starts moving because of the no-slip boundary condition, the viscous effect will cause it to stick to the pipe wall and the boundary layer will grow in thickness to completely fill the pipe. Now, the viscous effects are of considerable importance within this boundary layer, as we had discussed.

For the fluid outside the boundary layer within the inviscid core surrounding the center line from 1 to 2, viscous effects are negligible. As I said here, this is inviscid core that means there is no viscosity in this inviscid core. But after the end of the entrance region, the boundary layer shall occupy the entire pipe and the inviscid core will cease to exist.

So, the for fluid outside the boundary layer, that was within the inviscid core surrounding the center line from 1 to 2, viscous effects are negligible. The calculation of velocity profile and pressure distribution within the entrance region is very complex and therefore outside the scope of this course. So, we are not going to calculate the velocity and pressure distribution within the entrance region.

The only thing that you should remember is what is going to be the entrance length, depending on the Reynolds number.

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So, as soon as the flow reaches the end of the entrance region, the flow becomes simple, much more simple and the velocity will be dependent upon the radial distance r and the velocity will also be independent of x. What is r? So, this is a pipe., so this is x and because it is a pipe, it is circular, so this is r, radial distance from the center line. The flow between section 2 and 3 is called fully developed flow.

So, I will just take you there again. So, flow between this section 2 and 3 is called fully developed after this entrance region has ended. And why only until 3? Because at 3 there is again a bend. So, this is the complete description of that figure. And I think, this is a fine point to stop the lecture here.

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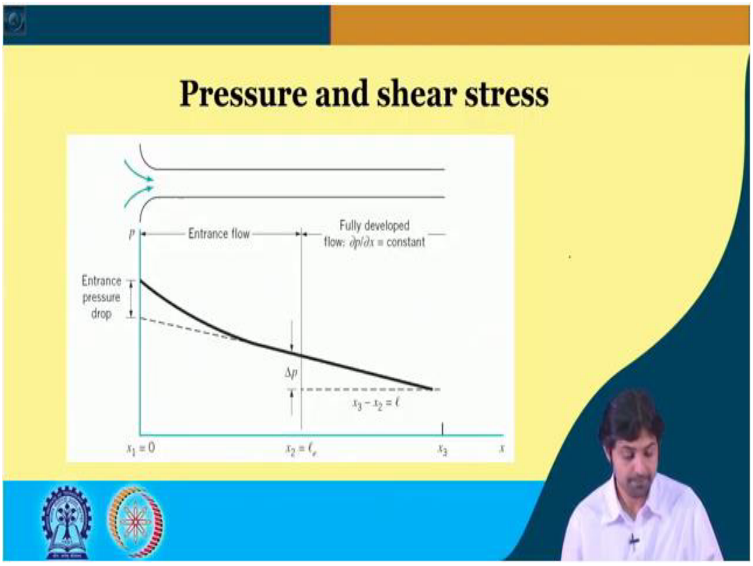

And we will start with the pressure and the shear stress distribution, in our next lecture of this module. Thank you so much for listening. See you in the next lecture.