


Large Eddy Simulations (LES)

- There is a big difference in the behaviors of large and small eddies in a turbulent flow field.

<p><u>Large Eddies</u></p> <ul style="list-style-type: none"> • More anisotropic. • Behavior is dictated by <ul style="list-style-type: none"> ▪ Geometry of the problem domain. ▪ The boundary conditions. ▪ Body forces acting. 	<p><u>Small Eddies</u></p> <ul style="list-style-type: none"> • Nearly isotropic. • Have a universal behavior.
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Remember: Large eddies extract energy from the mean flow.



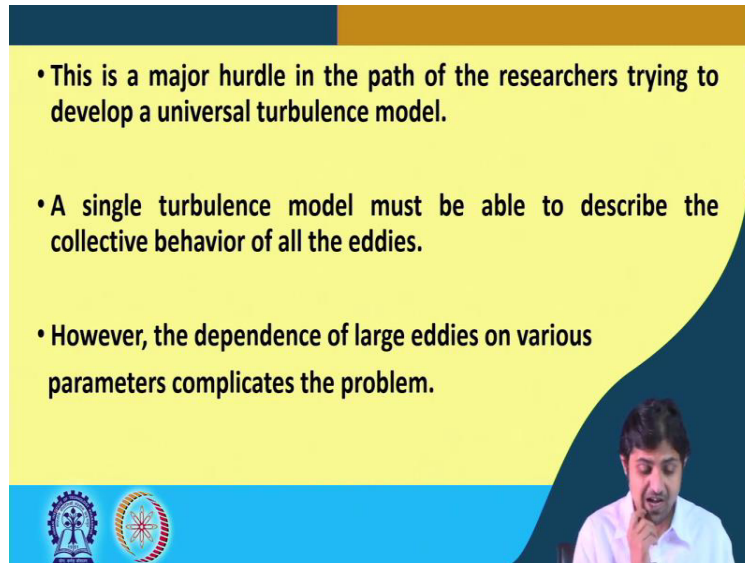

Another such technique is called Large Eddy simulation. See in the DNS one important thing to note was that we had the best accuracy but lot of computational time is required. LES is sort of a tradeoff between the Reynolds average in Reynolds average we do many approximations so the results are not that accurate compare to DNS, but LES is something which is a tradeoff between DNS and Reynolds average Navier Stokes equation. So there is a big difference in the behaviors of large and small eddies in turbulent flow field.

We were talking in DNS about the length scales, we said that there will be vortices or a LES that are as big as the length of the flow, there will be LES at the time of dissipation if heat will be very, very small, let us say order of 10^{-5} to 10^{-6} which also means that there is actually big difference in the behavior of these Eddies large Eddies will have a different behavior and smaller these will have a small bit, I mean, a different behavior.

So what are large Eddies they are large, Eddies are more, anisotropic. And their behavior is dictated by the geometry of the problem domain. And the boundary conditions the larger Eddies, they also must depend upon the they will also depend upon the body forces acting whereas small Eddies they are nearly isotropic they are very small they have not generally a universal behavior as demonstrated by Kolmogorov which we have not read in this course.

But just take it for granted that these have a universal behavior important thing to remember is that the large eddies extract energy from the mean flow, so, larger that is more than energy will be there, and they take energy from the mean flow that the flow real flow.

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- This is a major hurdle in the path of the researchers trying to develop a universal turbulence model.
- A single turbulence model must be able to describe the collective behavior of all the eddies.
- However, the dependence of large eddies on various parameters complicates the problem.

Whereas as small eddies take energy from the a little bit larger eddies which takes more energy from the larger Eddies than them. So energy is in form of a cascade. So, this this term is called Kolmogorov hypothesis. Again, I am telling it is outside the scope, but it is better to mention that this is actually a major hurdle in the path of research just trying to develop a universal turbulence model, which is what is the hurdle the variation of eddies from large scale to small scale?

And the fact that the largest scale eddies are not universal in nature only smaller eddies are a single turbulence model must be able to describe the collective behavior of all the eddies. However, in the dependence of large eddies on various parameter complicates the problem smaller eddies no problem because it is universal.

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- In LES, the larger eddies are computed with a time- dependent simulation whereas the influence of the small eddies are incorporated through a turbulence model.

LES

Large eddies → time dependent simulation

Small eddies → taken through a turbulence model

we can account for all large eddies

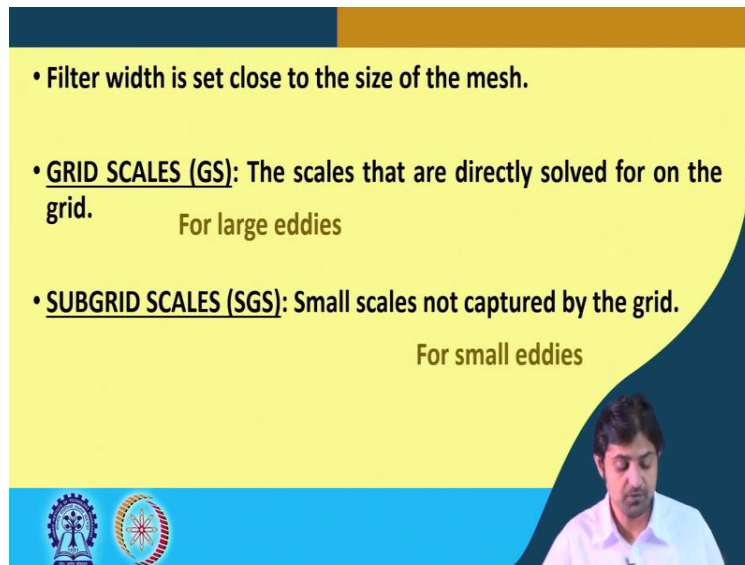
So, in LES the larger eddies are computed with a time dependent simulation where the influence of this small eddies are incorporated through turbulence model. So, we say in LES 2 parts large Eddies and there are small eddies. So they are solved through time dependent simulation and we account for everything for all large eddies but small eddies are so much the, the effects of the small eddies is taken through a turbulence model.

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- In LES, the larger eddies are computed with a time- dependent simulation whereas the influence of the small eddies are incorporated through a turbulence model.
- LES uses spatial filtering operation to separate the large and the small eddies.
- The filtered N- S equations are used as the governing equations for LES.

So our domain size does not is not mean we do not need to be, you know, accounting for the smaller eddies. We make our grid size that it captures the largest of the eddies. So, LES uses spatial filtering operation to separate the large and the small eddies as I have told you and the filtered navier stokes equation are used as the governing equation for large Eddy simulation.

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- Filter width is set close to the size of the mesh.
- GRID SCALES (GS): The scales that are directly solved for on the grid.
For large eddies
- SUBGRID SCALES (SGS): Small scales not captured by the grid.
For small eddies

Now, what would be the filter means up till what time what lens should we cut it the filter width is set to be close to the size of the mesh. There are some terms that when we LES when we use the size of the mesh Δ is the Grid Scale GS. The scales that are directly solved for on the grid are called the grid scales for large eddies they are they are the grid scales are used for the large eddies and for the smaller one the Subgrid Scales SGS.

So, small scales are not captured by those grid correct because they are larger in size to capture those eddies. The grid should be at least more I mean smaller than those so sub grid scale modeling is used for smaller eddies.

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Filtering

- The filtering operation is defined by a filter function $G(\mathbf{x}, \mathbf{x}', \Delta)$ as :

$$\bar{\phi}(\mathbf{x}, t) = \int_{-\infty}^{\infty} \int_{-\infty}^{\infty} \int_{-\infty}^{\infty} G(\mathbf{x}, \mathbf{x}', \Delta) \phi(\mathbf{x}', t) d\mathbf{x}'_1 d\mathbf{x}'_2 d\mathbf{x}'_3$$

Filtered function
Filter width
Unfiltered function

The overbar in $\bar{\phi}(\mathbf{x}, t)$ denotes spatial filtering and not time- averaging.

Now the filtering operation that we have talked about is defined by a filter function G of \mathbf{x} , \mathbf{x}' , Δ as it is a very complex but there is a filtering operation that you must know that in LES we use a filtering operation to cut off the smaller eddies and solve in reality for the larger eddies and then approximate this smaller eddies using the turbulent model. So this is a filtered function Δ is a filter with which is mostly said to the size of the mesh.

And this is the unfiltered function, $\phi(\mathbf{x}', t)$. The over bar in $\bar{\phi}(\mathbf{x}, t)$ denotes is spatial filtering and not time averaging. This is very important. So until now, when it there was bar we used to do average in time, but this is spatial filtering spatial filtering. So, average in space.

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- Examples of filtering functions are:

Top- Hat or Box- filter

$$G(\mathbf{x}, \mathbf{x}', \Delta) = \begin{cases} \frac{1}{\Delta^3}, & |\mathbf{x} - \mathbf{x}'| \leq \frac{\Delta}{2} \\ 0, & |\mathbf{x} - \mathbf{x}'| > \frac{\Delta}{2} \end{cases}$$

For a 3-D computation, the filter width is given by $\Delta = \sqrt[3]{\Delta x \Delta y \Delta z}$, where Δx , Δy and Δz are the length, width and height of the grid cells, respectively.

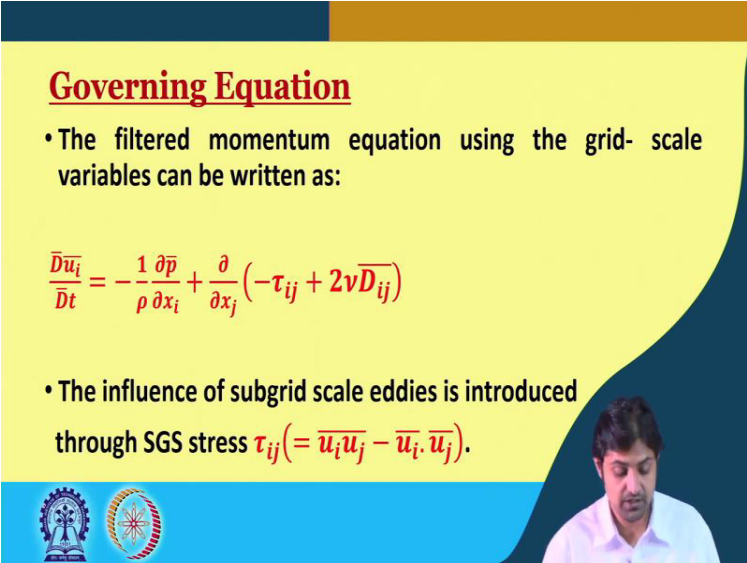
Gaussian filter

$$G(\mathbf{x}, \mathbf{x}', \Delta) = \left(\frac{6}{\pi \Delta^2} \right)^{3/2} \exp \left(-6 \frac{|\mathbf{x} - \mathbf{x}'|^2}{\Delta^2} \right)$$

So examples of filtering functions are there is a top hat or box filter, where $g(x)$ see, you see there is a G here, this is a filtering function and this therefore this is a filtered function. So $g(x)$ can be given by different box filter, it is a different study in its own is a Gaussian filter. So remembering the name of the filter is good enough you do not need to remember those equations. So for 3d computation, the filter with this is important.

The filter of it is given by this you must remember that Δ is given by cube root of Δx , Δy and Δz where Δx , Δy and Δz or the length width and height of the grid cells respectively. So, you saw what G is and what Δ is this Δ is here.

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Governing Equation

- The filtered momentum equation using the grid-scale variables can be written as:

$$\frac{D\bar{u}_i}{Dt} = -\frac{1}{\rho} \frac{\partial \bar{p}}{\partial x_i} + \frac{\partial}{\partial x_j} (-\tau_{ij} + 2\nu \overline{D_{ij}})$$

- The influence of subgrid scale eddies is introduced through SGS stress $\tau_{ij} (= \overline{u_i u_j} - \bar{u}_i \bar{u}_j)$.

So, now coming to the governing equations of LES the filtered momentum equation using the grid scale variables can be written so, this is again it is a space averaging. The influence of sub grid scale eddies introduced through SGS stress, sub grid scale stress as SGS and that have written as τ_{ij} you see here is a term τ_{ij} just similar to the Navier Stokes equation and that is written as $\overline{u_i u_j} - \bar{u}_i \bar{u}_j$ these are against space average in LES.




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• $\tau_{ij} = L_{ij} + C_{ij} + R_{ij}$

Leonard term, $L_{ij} = \overline{\overline{u_i} \cdot \overline{u_j}} - \overline{u_i} \cdot \overline{u_j}$

Cross term, $C_{ij} = \overline{\overline{u_i} \cdot u_j'} + \overline{u_i'} \cdot \overline{u_j}$

SGS Reynolds stress, $R_{ij} = \overline{u_i' u_j'}$

Where τ_{ij} is $L_{ij} + C_{ij} + R_{ij}$. The L_{ij} is the Leonard term given by this there is a cross term called C_{ij} the SGS Reynolds Stress R_{ij} .

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References:

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References





So, this gives you an overall idea the differences between the Reynolds shear stress in a little bit more detail. The DNS the idea of DNS, why it is computationally so expensive has been reduced. We went through the term like Re to the power 3 by 4 number of grid points are equal to Re to the power 9 by 4. The basic idea of LES where we studied that we basically use the filtering function to model the larger these various sub grid scale modeling is used to model the smaller Eddies.

So, information like this what are the different terms Leonard term, Cross term and SGS Reynolds stresses and things like that have been covered in the turbulence modeling part. Many complex equations are not supposed to be remembered by you, but an overall idea about those equations or whatever required in this particular module. And this actually concludes our module on computational introduction to computational fluid dynamics.

I hope you enjoyed this particular session, this particular module of the course and as always, these are the references of then I mean the reference books that you can actually use. And thank you so much for listening and I will see you next week.