Chapter (3): Truss Analysis

3.1 Introduction:

Truss is an assemblage of straight members connected at their ends by flexible connections to form a rigid configuration. Because of their light weight and high strength, trusses are widely used, and their applications range from supporting bridges and roofs of buildings to being support structures in space stations. Modern trusses are constructed by connecting members, which usually consist of structural steel or aluminum shapes or wood struts, to gusset plates by bolted or welded connections.

If all the members of a truss and the applied loads lie in a single plane, the truss is called a plane truss. Plane trusses are commonly used for supporting decks of bridges and roofs of buildings.

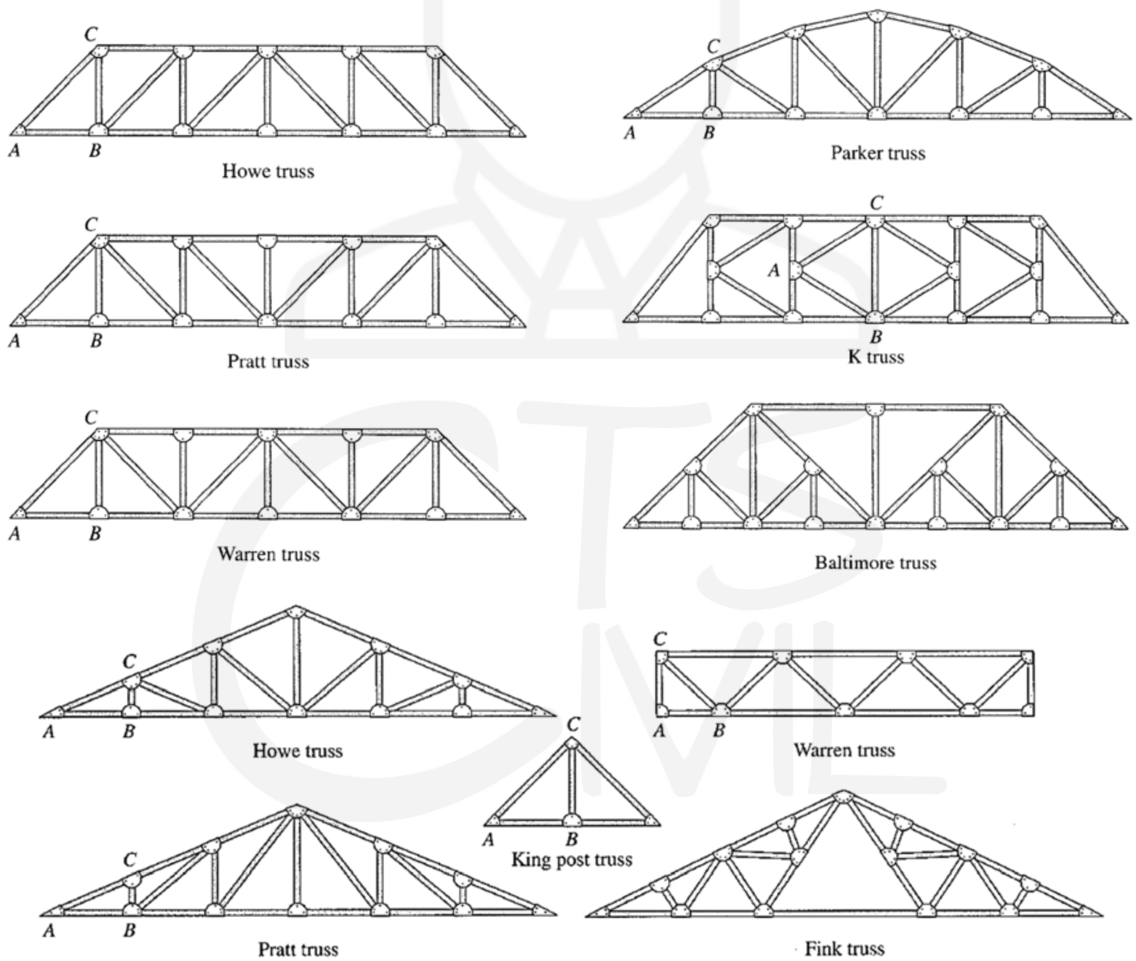


Figure 3-1: Common roof trusses

3.2 Assumptions for Analysis of Trusses:

The analysis of trusses is usually based on the following simplifying assumptions:

1- All members are connected only at their ends by frictionless hinges in plane trusses and by frictionless ball-and-socket joints in space trusses.
2- All loads and support reactions are applied only at the joints.
3- The centroidal axis of each member coincides with the line connecting the centers of the adjacent joints.

3.3 Method of Joints:

3.3.1 Procedure for Analysis

The following step-by-step procedure can be used for the analysis of statically determinate simple plane trusses by the method of joints.

1- Check the truss for static determinacy. If the truss is found to be statically determinate and stable, proceed to step 2. Otherwise, end the analysis at this stage.
2- Determine the slopes of the inclined members (except the zero-force members) of the truss.
3- Draw a free-body diagram of the whole truss, showing all external loads and reactions.
4- Examine the free-body diagram of the truss to select a joint that has no more than **two unknown** forces (which must not be collinear) acting on it. If such a joint is found, then go directly to the next step. Otherwise, determine reactions by applying the three equations of equilibrium and the equations of condition (if any) to the free body of the whole truss; then select a joint with two or fewer unknowns, and go to the next step.
5- a. Draw a free-body diagram of the selected joint, showing tensile forces by arrows pulling away from the joint and compressive forces by arrows pushing into the joint. It is usually convenient to assume the unknown member forces to be tensile.
   b. Determine the unknown forces by applying the two equilibrium equations (x and y direction). A positive answer for a member force means that the member is in tension, as initially assumed, whereas a negative answer indicates that the member is in compression.

   If at least one of the unknown forces acting at the selected joint is in the horizontal or vertical direction, the unknowns can be conveniently determined by satisfying the two equilibrium equations by inspection of the joint on the free-body diagram of the truss.

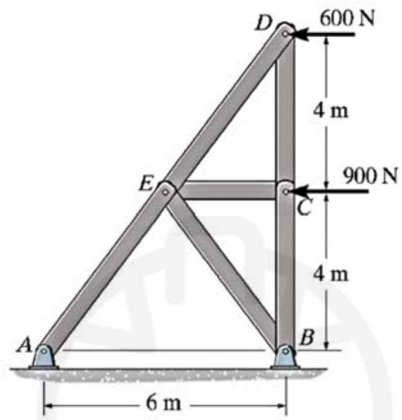
6- If all the desired member forces and reactions have been determined, then go to the next step. Otherwise, select another joint with no more than two unknowns, and return to step 5.

7- If the reactions were determined in step 4 by using the equations of equilibrium and condition of the whole truss, then apply the remaining joint equilibrium equations that have not been utilized so far to check the calculations. If the reactions were computed by applying the joint equilibrium equations, then use the equilibrium equations of the entire truss to check the calculations. If the analysis has been performed correctly, then these extra equilibrium equations must be satisfied.

For the following examples, find the forces in the members of the truss and indicate if the member is in tension or compression.

3.3.2 Examples:

Example (1):



Solution:

Method of Joints: We will begin by analyzing the equilibrium of joint D, and then proceed to analyze joints C and E.

Joint D: From the free-body diagram in Fig. *a*,

$\overset{+}{\rightarrow} \Sigma F_x = 0;$ $\quad F_{DE}\left(\frac{3}{5}\right) - 600 = 0$

$F_{DE} = 1000 \text{ N} = 1.00 \text{ kN (C)}$ **Ans.**

$+\uparrow \Sigma F_y = 0;$ $\quad 1000\left(\frac{4}{5}\right) - F_{DC} = 0$

$F_{DC} = 800 \text{ N (T)}$ **Ans.**

Joint C: From the free-body diagram in Fig. *b*,

$\overset{+}{\rightarrow} \Sigma F_x = 0;$ $\quad F_{CE} - 900 = 0$

$F_{CE} = 900 \text{ N (C)}$ **Ans.**

$+\uparrow \Sigma F_y = 0;$ $\quad 800 - F_{CB} = 0$

$F_{CB} = 800 \text{ N (T)}$ **Ans.**

Joint E: From the free-body diagram in Fig. *c*,

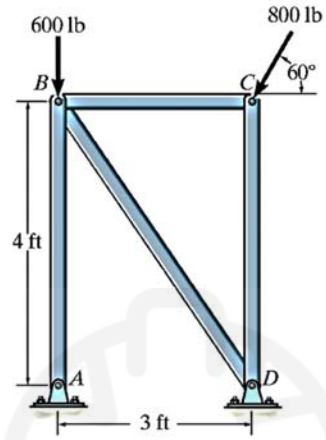
$\searrow + \Sigma F_x' = 0;$ $\quad -900 \cos 36.87° + F_{EB} \sin 73.74° = 0$

$F_{EB} = 750 \text{ N (T)}$ **Ans.**

$\nearrow + \Sigma F_y' = 0;$ $\quad F_{EA} - 1000 - 900 \sin 36.87° - 750 \cos 73.74° = 0$

$F_{EA} = 1750 \text{ N} = 1.75 \text{ kN (C)}$ **Ans.**

Example (2):



Solution:

Joint C:

$\xrightarrow{+} \Sigma F_x = 0;$ $\quad F_{CB} - 800 \cos 60° = 0$

$F_{CB} = 400 \text{ lb (C)}$ **Ans.**

$+\uparrow \Sigma F_y = 0;$ $\quad F_{CD} - 800 \sin 60° = 0$

$F_{CD} = 693 \text{ lb (C)}$ **Ans.**

Joint B:

$\xrightarrow{+} \Sigma F_x = 0;$ $\quad \frac{3}{5} F_{BD} - 400 = 0$

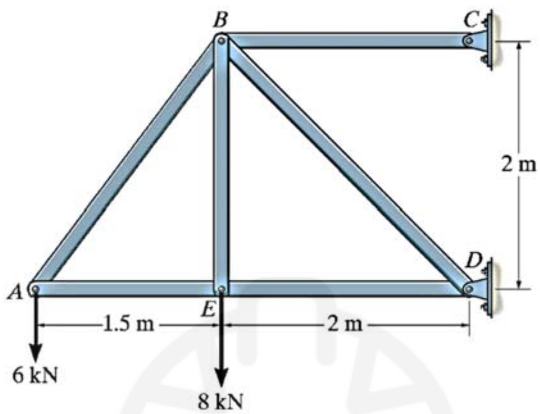
$F_{BD} = 666.7 = 667 \text{ lb (T)}$ **Ans.**

$+\uparrow \Sigma F_y = 0;$ $\quad F_{BA} - \frac{4}{5}(666.7) - 600 = 0$

$F_{BA} = 1133 \text{ lb} = 1.13 \text{ kip (C)}$ **Ans.**

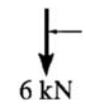
Member AB is a two-force member and exerts only a vertical force along AB at A.

Example (3):



Solution:

Joint A:



$+\uparrow \Sigma F_y = 0;\qquad \frac{4}{5} F_{AB} - 6 = 0$

$F_{AB} = 7.5 \text{ kN (T)}$ **Ans.**

$\overset{+}{\rightarrow} \Sigma F_x = 0;\qquad -F_{AE} + 7.5\left(\frac{3}{5}\right) = 0$

$F_{AE} = 4.5 \text{ kN (C)}$ **Ans.**

Joint E:

$\overset{+}{\rightarrow} \Sigma F_x = 0;\qquad F_{ED} = 4.5 \text{ kN(C)}$ **Ans.**

$+\uparrow \Sigma F_y = 0;\qquad F_{EB} = 8 \text{ kN (T)}$ **Ans.**

Joint B:

$+\uparrow \Sigma F_y = 0;\qquad \frac{1}{\sqrt{2}}(F_{BD}) - 8 - \frac{4}{5}(7.5) = 0$

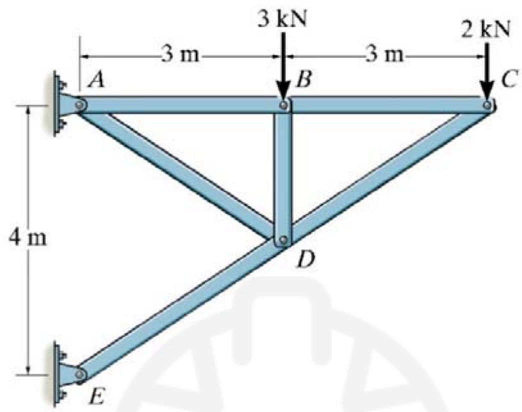
$F_{BD} = 19.8 \text{ kN (C)}$ **Ans.**

$\overset{+}{\rightarrow} \Sigma F_x = 0;\qquad F_{BC} - \frac{3}{5}(7.5) - \frac{1}{\sqrt{2}}(19.8) = 0$

$F_{BC} = 18.5 \text{ kN (T)}$ **Ans.**

C_y is zero because BC is a two-force member .

Example (4):



Solution:

Joint C:

$+\uparrow \Sigma F_y = 0; \quad \frac{2}{\sqrt{13}} F_{CD} - 2 = 0$

$F_{CD} = 3.606 = 3.61 \text{ kN (C)}$ **Ans.**

$\overset{+}{\rightarrow} \Sigma F_x = 0; \quad -F_{CD} + 3.606\left(\frac{3}{\sqrt{13}}\right) = 0$

$F_{CB} = 3 \text{ kN (T)}$ **Ans.**

Joint B:

$\overset{+}{\rightarrow} \Sigma F_x = 0; \quad F_{BA} = 3 \text{ kN (T)}$ **Ans.**

$+\uparrow \Sigma F_y = 0; \quad F_{BD} = 3 \text{ kN (C)}$ **Ans.**

Joint D:

$\overset{+}{\rightarrow} \Sigma F_x = 0; \quad \frac{3}{\sqrt{13}} F_{DE} - \frac{3}{\sqrt{13}}(3.606) + \frac{3}{\sqrt{13}} F_{DA} = 0$

$+\uparrow \Sigma F_y = 0; \quad \frac{2}{\sqrt{13}}(F_{DE}) - \frac{2}{\sqrt{13}}(F_{DA}) - \frac{2}{\sqrt{13}}(3.606) - 3 = 0$

$F_{DA} = 2.70 \text{ kN (T)}$ **Ans.**

$F_{DE} = 6.31 \text{ kN (C)}$ **Ans.**

3.4 Problems:

Determine the force in each member of the trusses shown in the figures below and indicate if the members are in tension or compression. Summarize your answers in a table format showing member's name, force value, and force type.

Question № 1:
Let $P_1 = 800$ lb and $P_2 = 400$ lb.

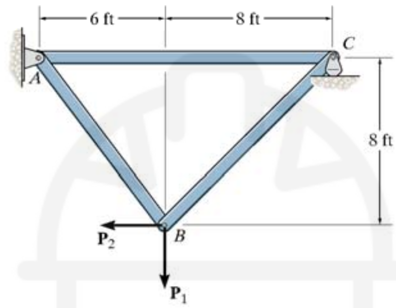


Figure 1

Question № 2:

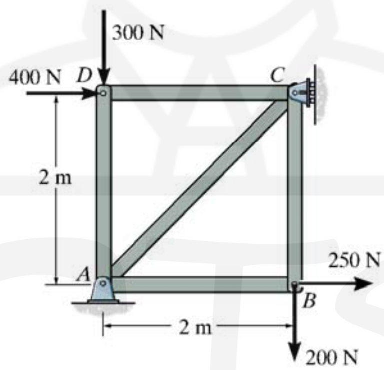


Figure 2

Question № 3:

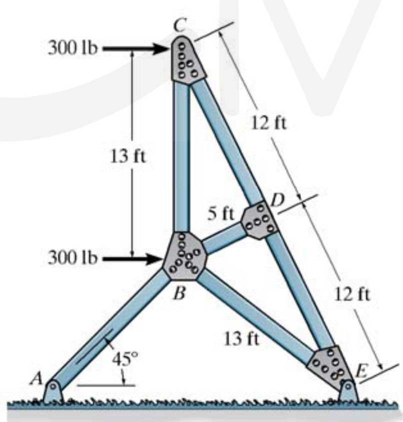


Figure 3

Question № 4:

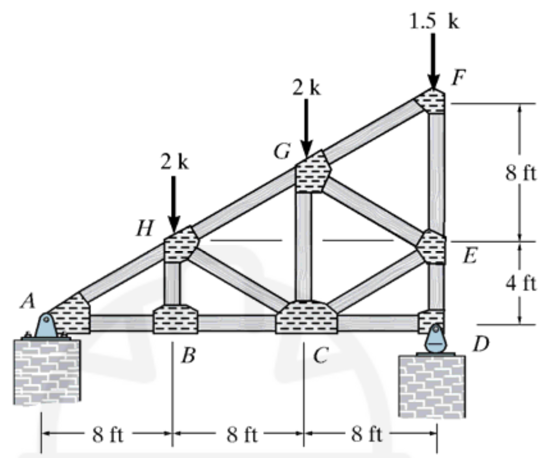


Figure 4

Question № 5:

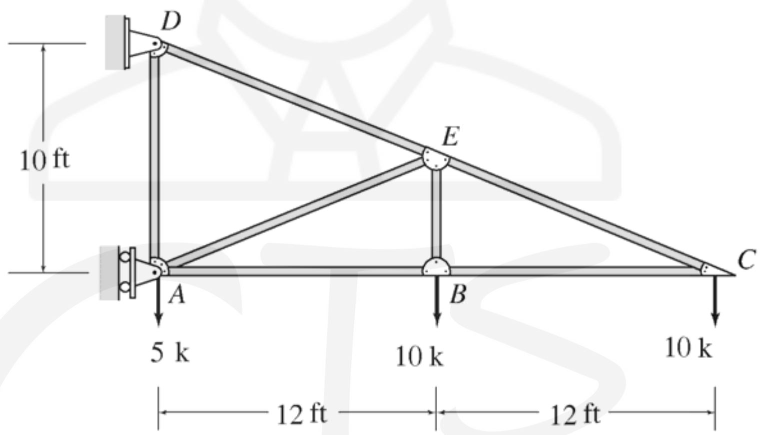


Figure 5

Question № 6:

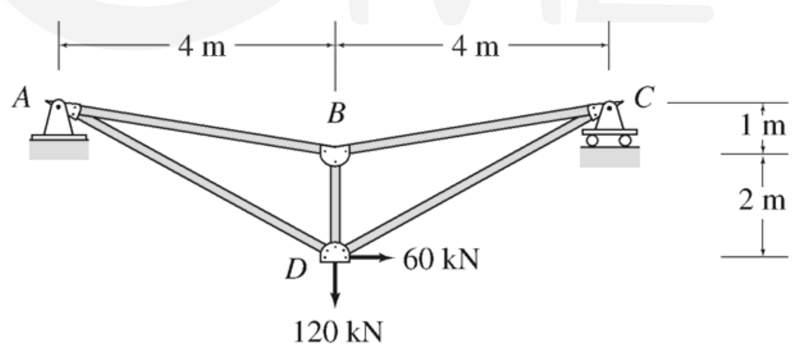


Figure 6

Question № 7:

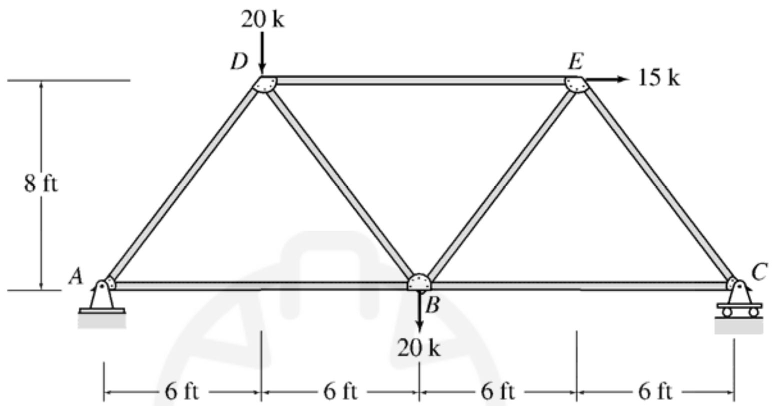


Figure 7

3.5 Method of Sections:

3.5.1 Procedure for Analysis:

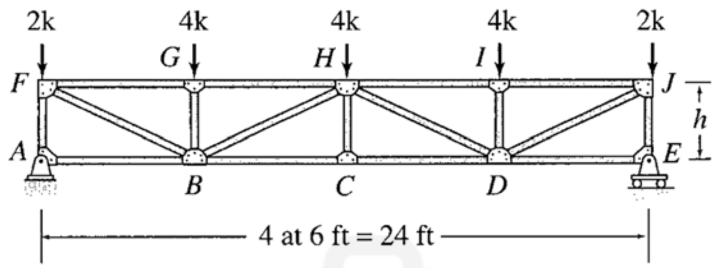
The following step-by-step procedure can be used for determining the member forces of statically determinate plane trusses by the method of sections.

1. Select a section that passes through as many members as possible whose forces are desired, but not more than three members with unknown forces. The section should cut the truss into two parts.
2. Although either of the two portions of the truss can be used for computing the member forces, we should select the portion that will require the least amount of computational effort in determining the unknown forces. To avoid the necessity for the calculation of reactions, if one of the two portions of the truss does not have any reactions acting on it, then select this portion for the analysis of member forces and go to the next step. If both portions of the truss are attached to external supports, then calculate reactions by applying the equations of equilibrium and condition (if any) to the free body of the entire truss. Next, select the portion of the truss for analysis of member forces that has the least number of external loads and reactions applied to it.
3. Draw the free-body diagram of the portion of the truss selected, showing all external loads and reactions applied to it and the forces in the members that have been cut by the section. The unknown member forces are usually assumed to be tensile and are, therefore, shown on the free-body diagram by arrows pulling away from the joints.
4. Determine the unknown forces by applying the three equations of equilibrium. To avoid solving simultaneous equations, try to apply the equilibrium equations in such a manner that each equation involves only one unknown. This can sometimes be achieved by using the alternative systems of equilibrium equations (Sum of moment equations) instead of the usual two-force summations and a moment summation system of equations.
5. Apply an alternative equilibrium equation, which was not used to compute member forces, to check the calculations. This alternative equation should preferably involve all three-member forces determined by the analysis. If the analysis has been performed correctly, then this alternative equilibrium equation must be satisfied.

For the following examples, use the method of sections to solve for the required members (indicated by x) and state whether the members are in tension or compression.

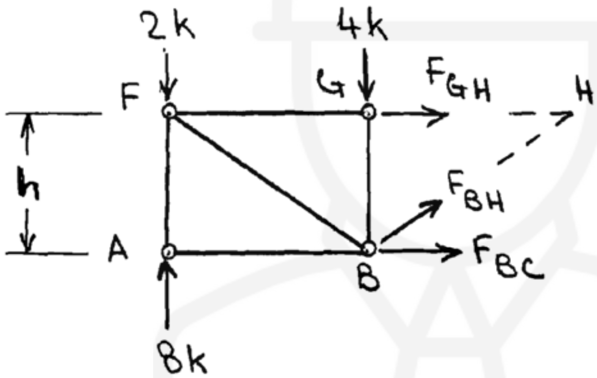
3.5.2 Examples:

Example (1):



Solution:

<u>Section through members BC, BH and GH:</u>



$+\circlearrowright \Sigma M_B = 0 \qquad -8(6) + 2(6) - F_{GH}(h) = 0$

$$F_{GH} = -\frac{36}{h} \qquad (1)$$

$+\circlearrowright \Sigma M_H = 0 \qquad -8(12) + 2(12) + 4(6) + F_{BC}(h) = 0$

$$F_{BC} = \frac{48}{h} \qquad (2)$$

Equations (1) and (2) indicate that the magnitudes of F_{GH} and F_{BC} are inversely proportional to the truss height h.

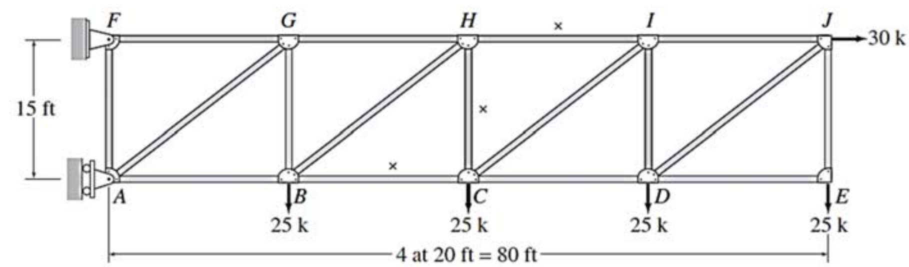
<u>For h = 3 ft:</u> $F_{GH} = -\frac{36}{3} = -12\,k = \underline{12\,k\,(C)}$

$F_{BC} = \frac{48}{3} = \underline{16\,k\,(T)}$

<u>For h = 6 ft:</u> $F_{GH} = -\frac{36}{6} = -6\,k = \underline{6\,k\,(C)}$

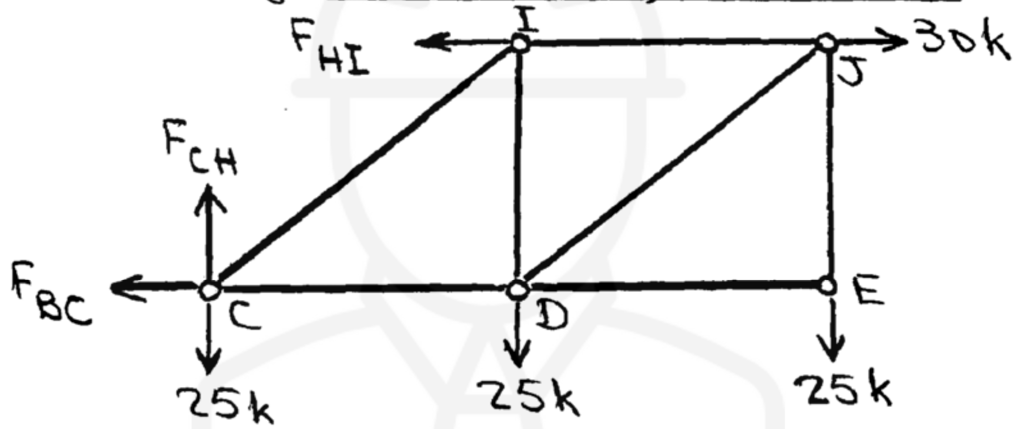
$F_{BC} = \frac{48}{6} = \underline{8\,k\,(T)}$

Example (2):



Solution:

Section through members BC, CH and HI:



$+\uparrow \Sigma F_y = 0 \qquad F_{CH} - 3(25) = 0$

$$F_{CH} = 75\text{ k (T)}$$

$+\circlearrowleft \Sigma M_C = 0$

$$F_{HI}(15) - 30(15) - 25(20) - 25(40) = 0$$

$$F_{HI} = 130\text{ k (T)}$$

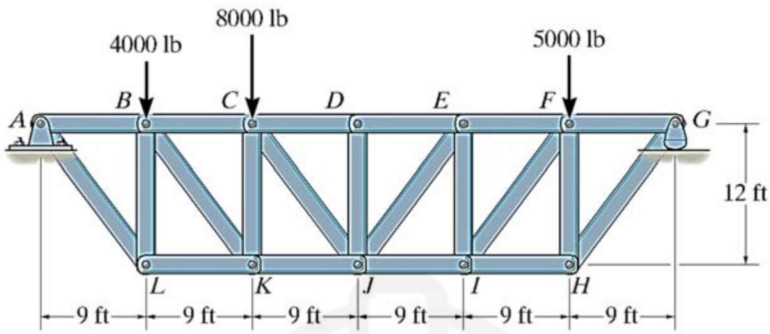
$\overset{+}{\rightarrow} \Sigma F_x = 0$

$$-F_{BC} - 130 + 30 = 0$$

$$F_{BC} = -100\text{ k}$$

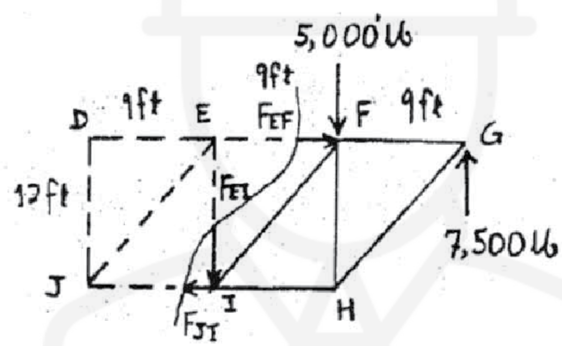
$$F_{BC} = 100\text{ k (C)}$$

Example (3):



Members: *EI, JI*

Solution:



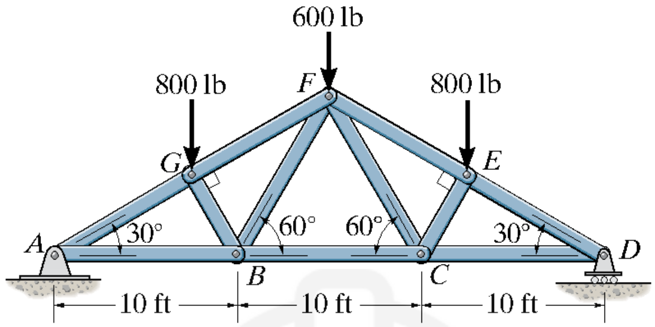
$\circlearrowleft + \Sigma M_E = 0;$ $\quad -5000(9) + 7500(18) - F_{JI}(12) = 0$

$F_{JI} = 7500\text{ lb} = 7.50\text{ kip (T)}$ **Ans.**

$+\uparrow \Sigma F_y = 0;$ $\quad 7500 - 5000 - F_{EI} = 0$

$F_{EI} = 2500\text{ lb} = 2.50\text{ kip (C)}$ **Ans.**

Example (4):



Members: *FE, EC*

Solution:

***Support Reactions*:** Due to symmetry,

$+\uparrow \Sigma F_y = 0;$ $\quad 2B_y - 800 - 600 - 800 = 0; B_y = 1100 \text{ lb}$

***Method of Sections*:**

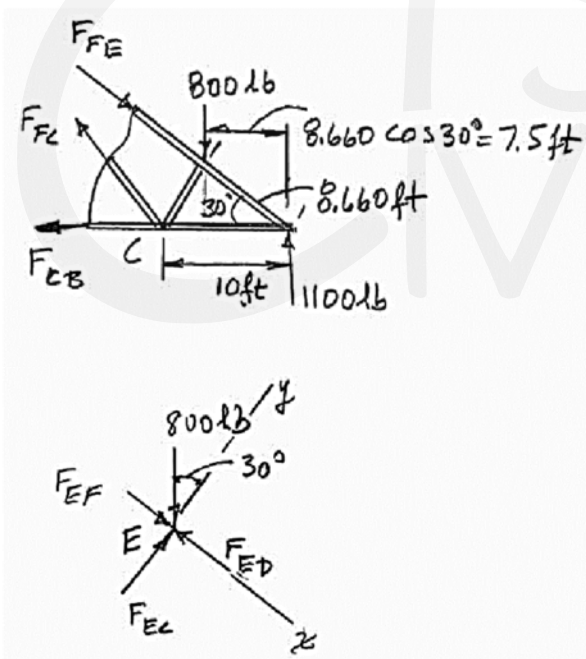
$\circlearrowleft + \Sigma M_C = 0;$ $\quad 1100(10) - 800(10 - 7.5) - (F_{FE} \sin 30°)(10) = 0$

$F_{FE} = 1.80 \text{ kip (C)}$ **Ans.**

Joint E:

$+\uparrow \Sigma F_y = 0;$ $\quad F_{EC} - 800 \cos 30° = 0$

$F_{EC} = 693 \text{ lb (C)}$ **Ans.**



3.6 Problems:

Determine the forces in the members identified by "x" of the trusses shown by the method of sections. Indicate if the members are in tension or compression. Summarize your answers in a table format showing member's name, force value, and force type.

Question № 1:

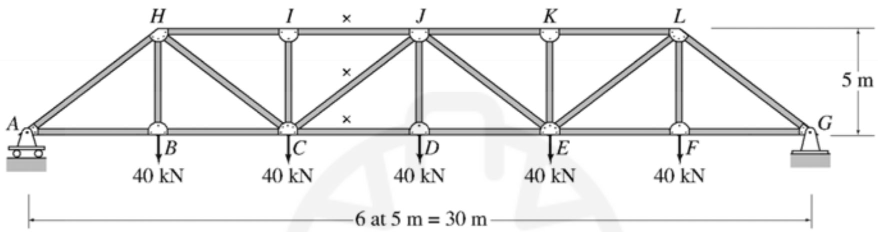


Figure 1

Question № 2:

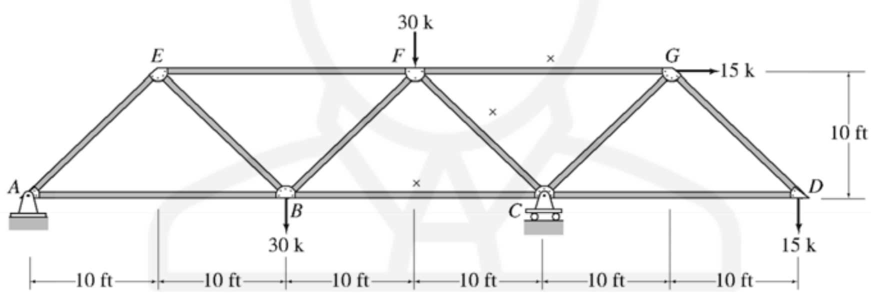


Figure 2

Question № 3:

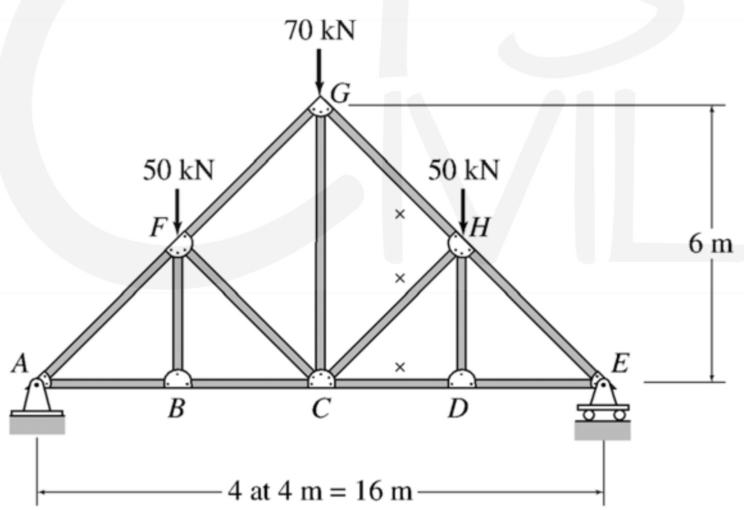


Figure 3

Question № 4:

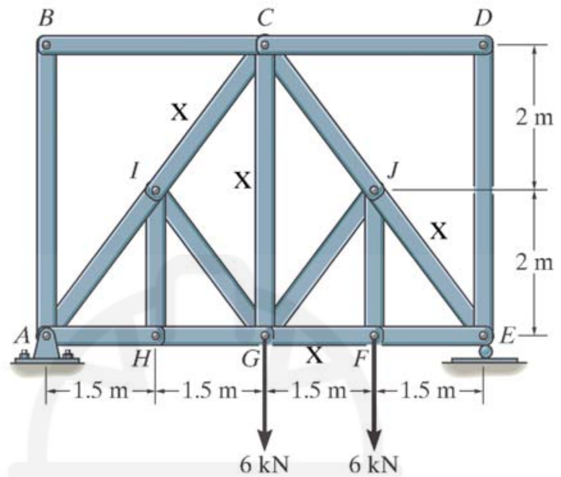


Figure 4

Question № 5:

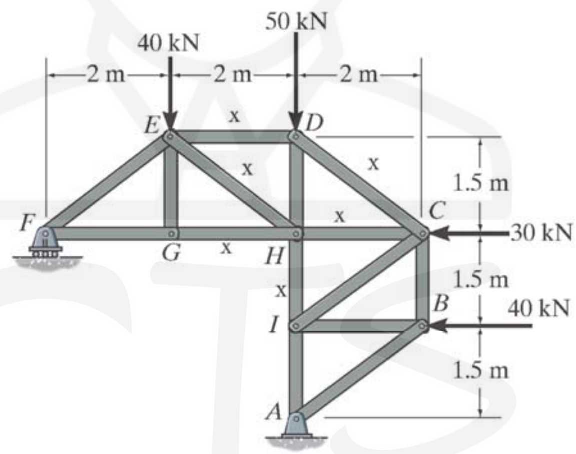


Figure 5

Question № 6:

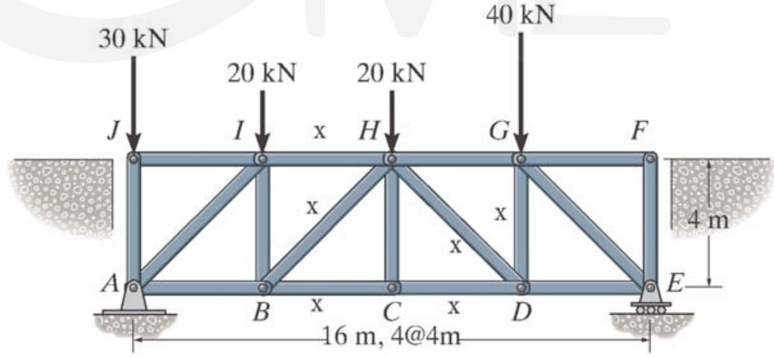


Figure 6