Hydraulic Engineering
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Lecture # 58
Computational Fluid Dynamics (Contd.,)

Welcome back to the last lecture of this module computational fluid dynamics and the last lecture we ended at a point where we saw the Reynolds shear stress equation. We discussed about the different terms in it you do not need to worry about the mathematical equation of those terms, but some common terms like Reynolds shear stress, those things are recommended that you remember them, but not the complex terms. So, the up in proceeding in the direction we are going to study the closure problem. So, the effect of the Reynolds shear stress rho in tau i j as you can see on the slide now the effect of rho in tau i j on the mean flow is like that of a stress term.

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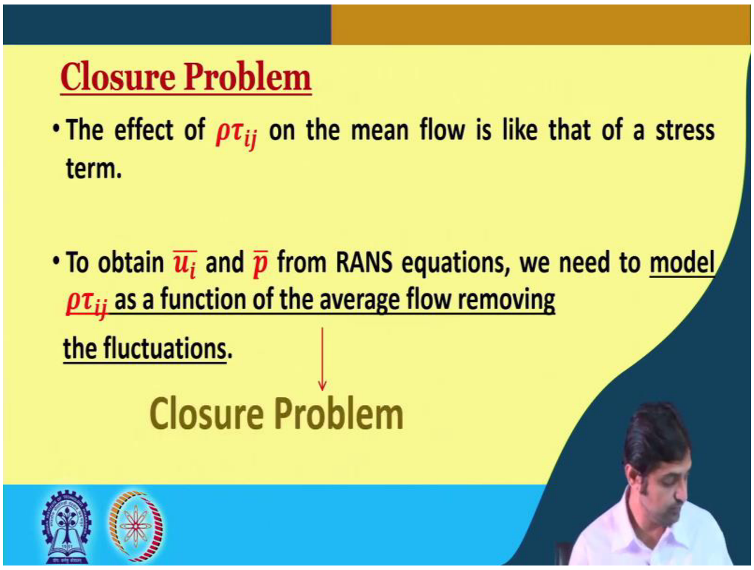

So I am going to write here rho in tau i j is actually Reynolds shear stress. So it is like shear stress to obtain u i and pressure. So, u i is the average flow velocities and pressure from RANS equation, we need to model this shear stress rho in tau i j is a function of the average flow so if you look at the equations. So take you back to this.

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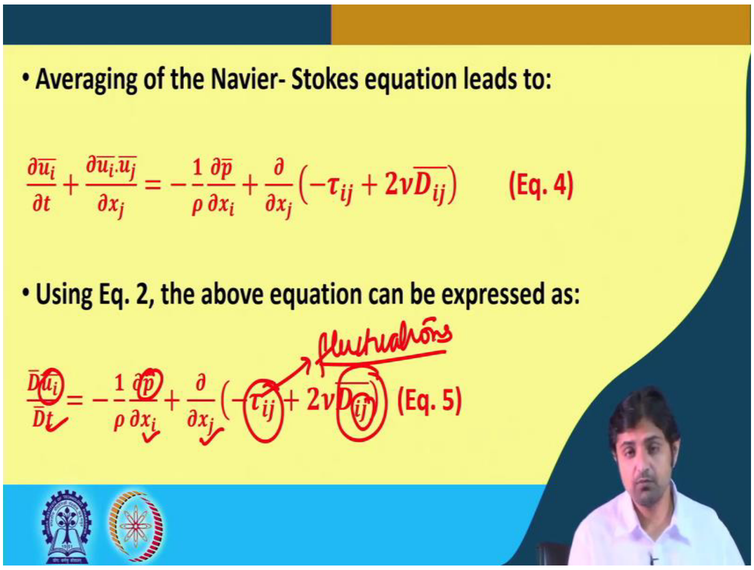

Where we have written the Reynolds average here you see this particular equation this is average quantity that we need to find out this we need to find out these are the independent variables. D i j we also know because this is the average value and it comprises of only this is the only term that has fluctuations and this increases our complexity. So, we need to be able to equate tau ij to something that is known known or some unknown which we are actually calculating.

So, the best ways to put it in some form of an average value and to do that, that particular problem is called a closure problem. So, we will go back to the closure problem so as I said, to obtain u i and pressure from Reynolds equation, we need to model this Reynolds's shear stress given by rho into tau i j as a function of average flow, this will remove the fluctuations and this process is called the closure problem.

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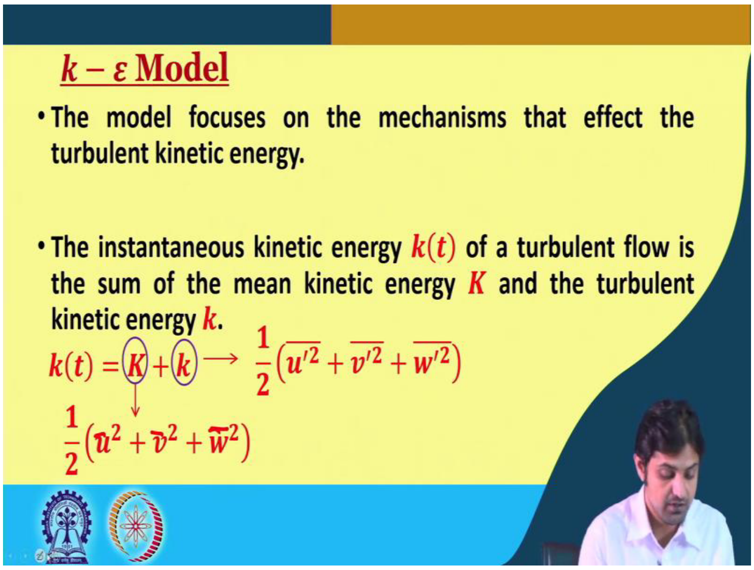

There are elegant ways the first of the methods is called the k epsilon model. So, this in this technique the model focuses on the mechanism that effects the turbulent kinetic energy, k stands for kinetic energy. So, the instantaneous kinetic energy k as a function of time k t of a turbulent flow is the sum of the mean kinetic energy and the turbulent kinetic energy k. So, k of t can be written as capital K + small k.

So, this is small k and capital K is written is simply half u square + v squared + w squared actually u bar v bar and w bar and small k is written is half of u dash squared + v dash squared + w dash squared.

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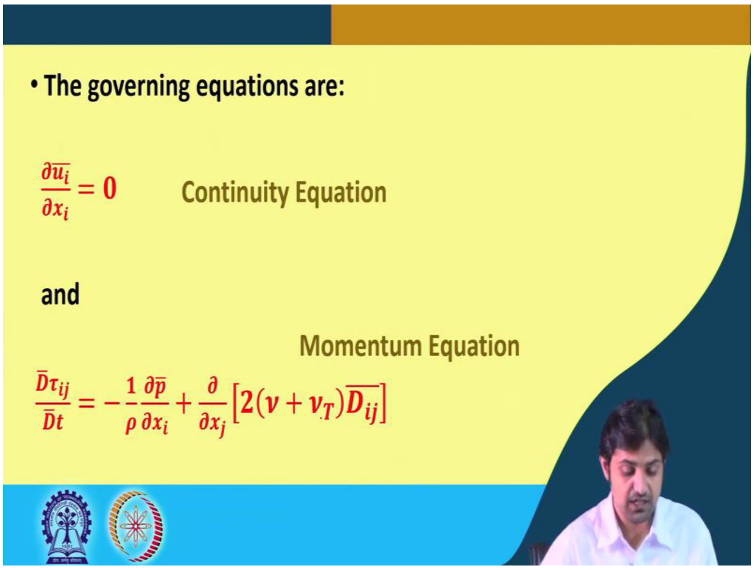

The governing equations for this are one we have continuity equation and the other is so, the way we write is So, D D t of tau i j that is what we are modeling is -1 by rho del p del x i + del x j into 2 nu + nu t into del i j. How does equations have been derived we are not going to discuss but these are the 2 equations that we use for momentum. See, it looks like a momentum equation itself so modeling of the Reynolds shear stress in as the momentum equation.

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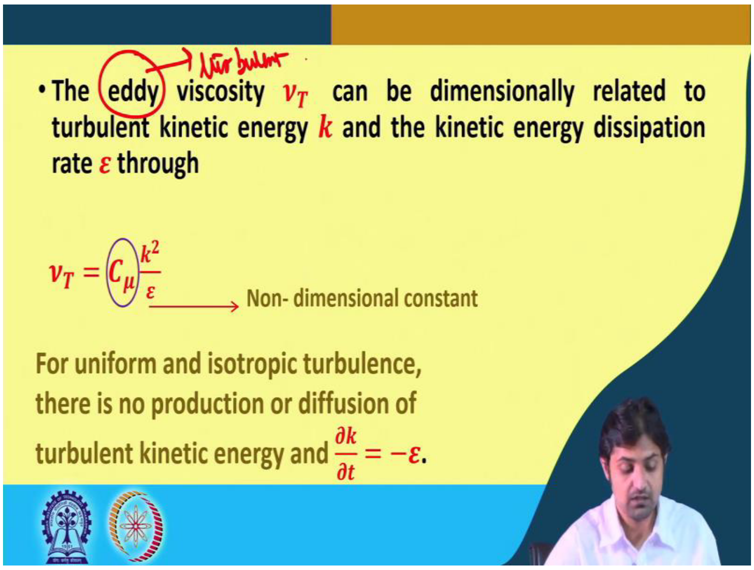

So, if you see there is a term called nu T. So, this nu t is the eddy viscosity. So, this actually is not should better be called as turbulent eddy viscosity, nu T, it can be actually dimensionally related to kinetic energy k kinetic energy dissipation rate epsilon through nu T is written as c mu into k square by epsilon. So, nu T we got also able to find on infinite terms of k and epsilon and we substitute this into this equation and C mu is a non-dimensional constant. Which values we know from experiments for uniform and isotropic turbulence there is no production or diffusion of turbulent kinetic energy, therefore, del k delta t will be equal to - epsilon E.

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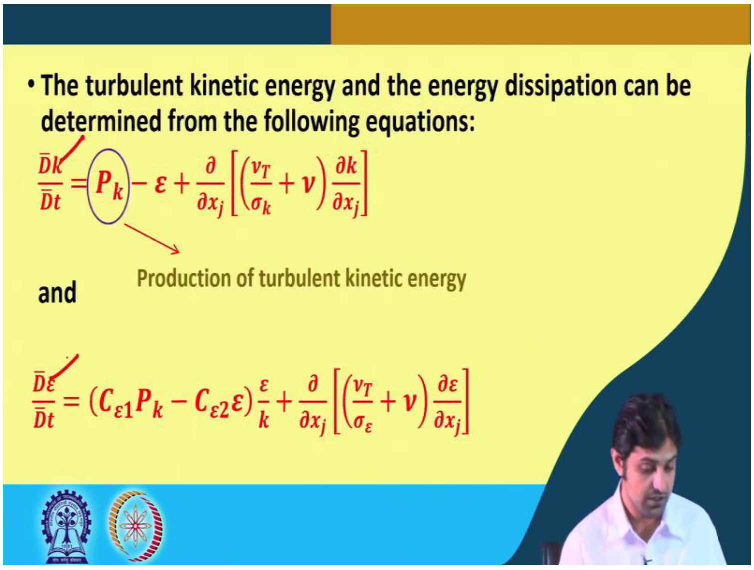

The turbulent kinetic energy and the energy dissipation can be determined from the following equation. So, now, we said that this is the equation which we are going to use for determining tau i j and that we can use in our average equation Reynolds average Navier stokes equation, we saw that there are some terms one is nu t that we do not know again others are again like del p bar, D i j bar. Those are known means do quantity that are in terms of average quantities.

So nu T is something that we yet do not know. And we write it nu to in terms of C mu into k square by epsilon. So, again, we have 2 things to again find out k and epsilon we still we do not know so, the next step is finding out this turbulent kinetic energy k and epsilon that that is how this equation gets in the model gets its name. And to be able to do that we have 2 another equation 1 is in terms of kinetic energy 1 is in terms of epsilon. And this is production of turbulent kinetic energy this term here. So, we use these 2 equation to solve for k and epsilon.

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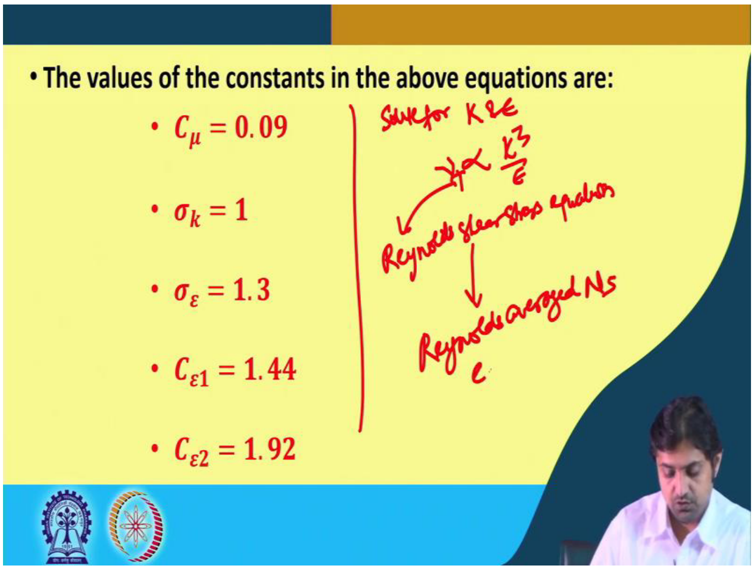

And the values of the constant in above equation are C mu = 0.09, sigma k = 1 if you see, C mu was there first, and then there is sigma k here then there is sigma E here and also there are some terms there is sigma there is the C epsilon 1 there is C epsilon 2. So, those things are already known these are the values that we use so, what we do is we solve the k and epsilon equations use those equations to find out the turbulent kinetic turbulent eddy viscosity nu T put it into their an Reynolds shear stress equation.

And use it in the average equation of Reynolds Navier Stokes equation. So, this for just writing it down solve for k and epsilon then nu T is related to k square by epsilon. Nu T, put this in Reynolds shear stress equation and use that in Reynolds average Navier stokes equations. It is a quite a complex way, but it gives good results.

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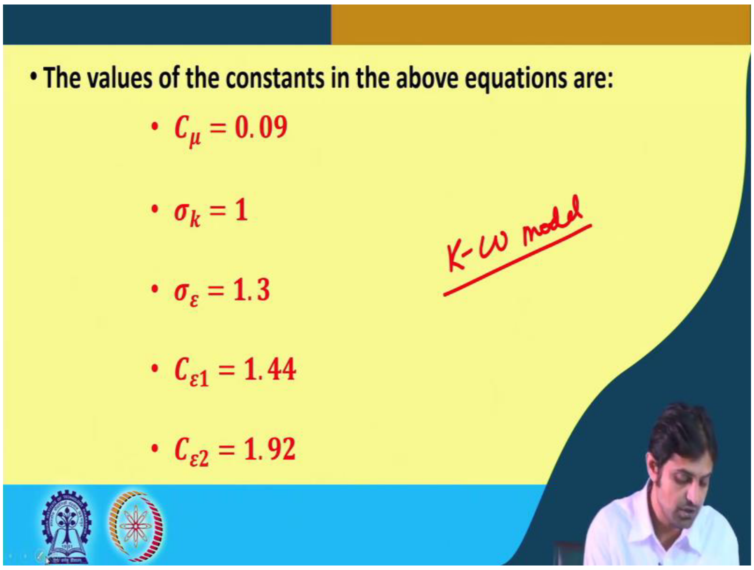

Most of the turbulence model in the world follow k and epsilon, there is one other turbulence model for k omega. So, k is the same kinetic energy omega is somehow related to dissipation epsilon, but that is also the outside the scope, but it is better to remember the name. So, the other model is k omega model like epsilon, these are the two world's most widely used turbulence models, and some of each of them have their advantages and disadvantages.

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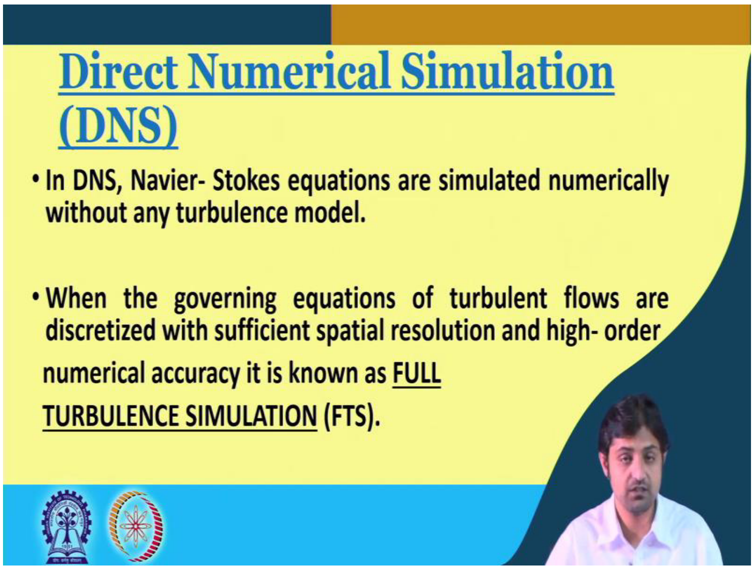

So, another method apart from Reynolds average navier stokes equation for solving the turbulence, for solving the computational fluid dynamics navier stokes equation is called direct numerical simulation. So, in direct numerical simulation DNS navier stokes equation or simulated numerically without any turbulence model. So, they solve for the exact solution when the governing equations of turbulent flows are discretized with sufficient spatial resolution and high order numerical accuracy, it is known as full turbulence simulation FTS.

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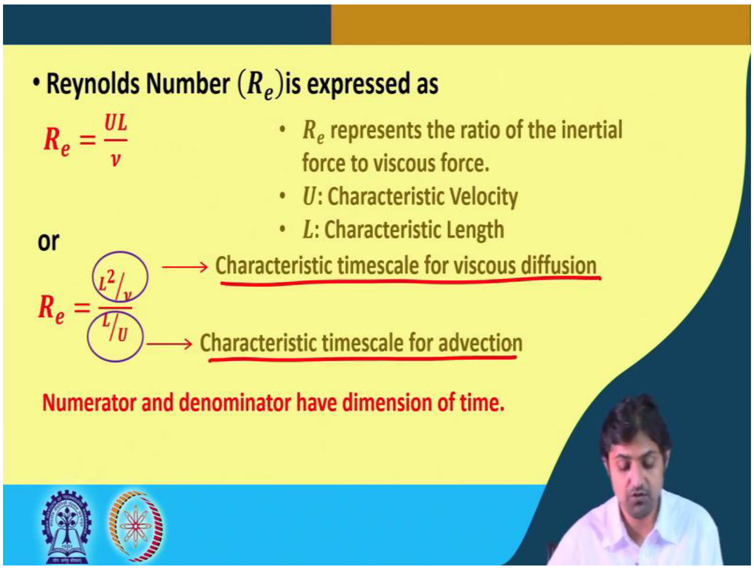

So, in this modeling Reynolds number is expressed as UL by nu, where R e represents the ratio of the inertial forces to viscous forces or Reynolds number, use the characteristic velocity L is the characteristic length or Reynolds number can also be written as square by nu divided by L by U. Numerator and denominators both have dimensions of time so, this L square by nu is the characteristic timescale for viscous diffusion whereas L by U is the characteristics timescale for advection.

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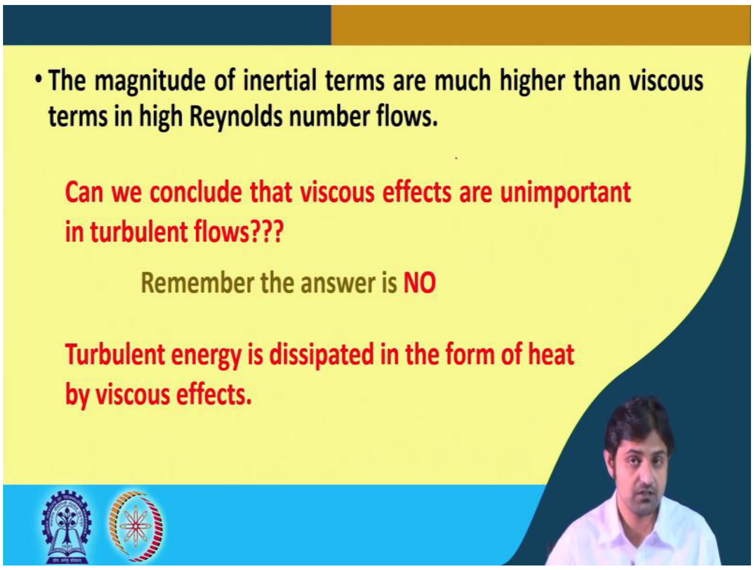

The magnitude of inertial terms are much higher than viscous term in high Reynolds number flows that is true, because it is the ratio of viscous diffusion because the ratio of the inertial forces to viscous forces. So, can we conclude that viscous effects are unimportant in turbulent flows no it is very important because turbulent energy is dissipated in form of heat by viscous effects? Therefore, the viscous effects are very important in high energy turbulent flows.

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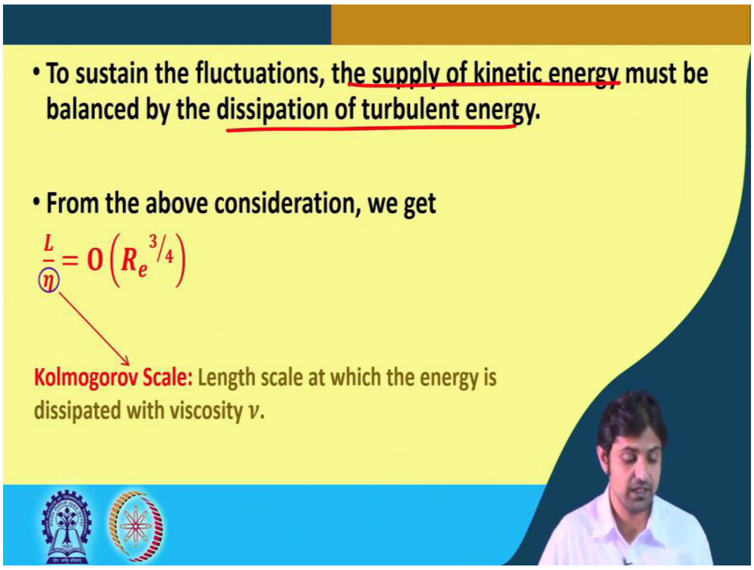

To sustain the fluctuations the supply of kinetic energy must be balanced by these dissipation of turbulent energy this is important thing supply of kinetic energy must be balanced by the dissipation of turbulent energy. So, from the above consideration what we get is all definitely outside the scope the derivation of this that L by eta is of the order of Reynolds number to the power 3 by 4 this this has been obtained and this eta is a Kolmogorov length scale. So, it is the length scale at which the energy is dissipated with viscosity nu you should remember the order and everything but the derivation is not required.

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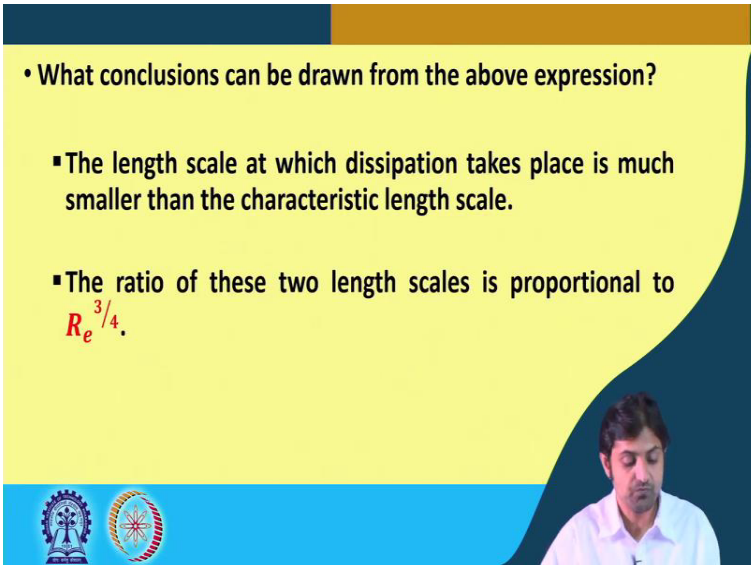

So, what can we convert conclusions can be drawn from the above expect expression this is important so, L by eta so length of the flow divided by the Kolmogorov length scale at which energy is dissipated is the function of Reynolds number to the power 3 by 4. So, if this is the equation what are the conclusions from it? This means that the length scale at which the dissipation takes place is much smaller than the characteristic length scale. L by eta is proportional to R e to the power so in turbulent flow and Reynolds number is very high.

So that means L by eta is very high. This means numerator is much, much larger than the denominator. That is implying the length scale at which dissipation takes place is much eta is much smaller than the characteristic length scale L. Valid conclusion from the equation. The ratio of these 2 length scale is proportional to R e to the power 3 by 4 as we have seen in the last slide.

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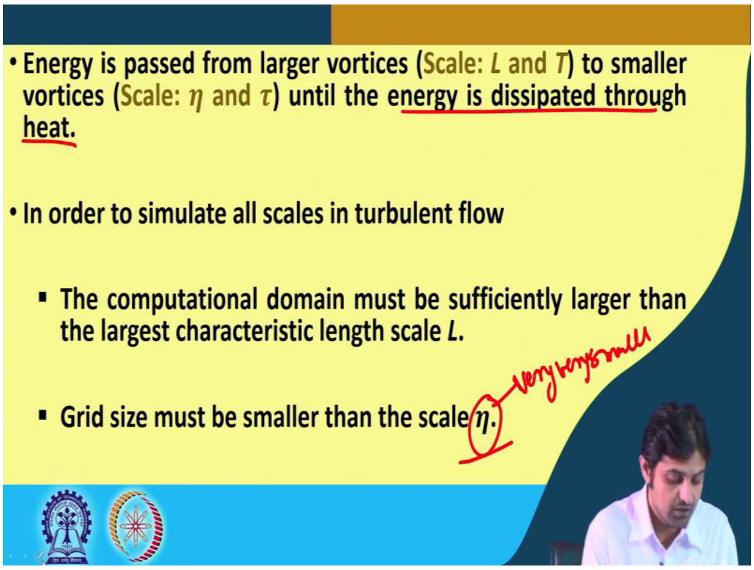

Second thing is the energy is passed from large vortices, that means, the flow is of the scale of length of L and T to smaller vortices that is, so, because see the dissipation happens at the Kolmogorov length scale. So, the eddies become transfer energy to each other they keep on losing the energy you know and until they become they come into range of this Kolmogorov length scale and at this point this energy is dissipated through the heat.

So, in order to simulate all the scales in turbulent flow So, now you see we have a length of scale from capital L to eta it is Kolmogorov length scale which is of the order of several order of magnitude less than millimeters the computational domain must be sufficiently large than the characteristic length scale L . So, to see there will be what is this which will be of the length L as well so, the computational domain must be sufficiently large than the character length L.

But also it is important that the grid size must be smaller than this may then this then this it is very small of the order of 10 to the power -5, 6 meters and length the scale of the typically of the order of let us say 1 or 2 meters or 10 to the power 1 meter, let us say so to be able to model all the type of energies energy in direct numerical simulation, the two things that are important is that the total domain means the study area should be larger than the length L which is of the order of meters several time 100 of meters but the grid size must be smaller than this Kolmogorov length scale which is very, small.

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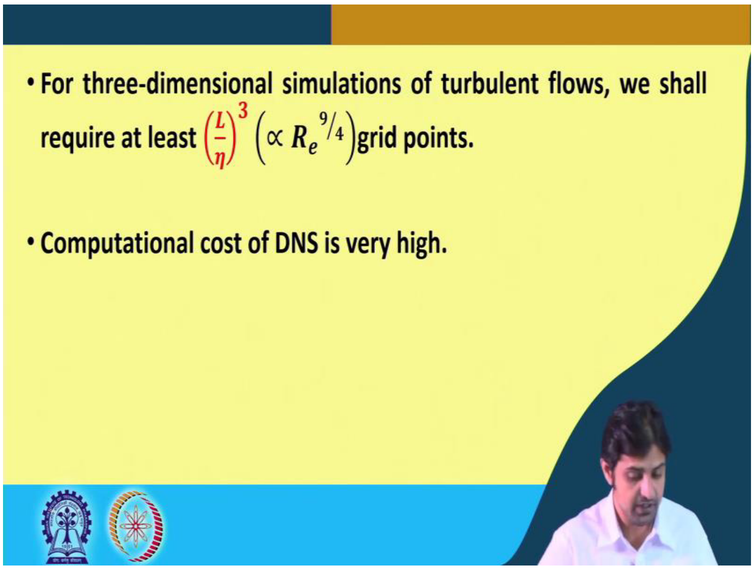

The implication of this is that for 3 dimensional simulation of turbulent flow we will require at least L by eta to the power 3 that means, Reynolds number 2 the power 9 by 4 grid points let us say R e is very, not very high Reynolds number let us see Reynolds number is 10 to the power 4. So, how many grids is needed? R e that is 10 to the power 4 raise to the power 9 by 4. So, we need 10 to the power nine grids almost so much high more than billions.

You know around billion grids, which currently looking at the capacity of our computational facilities it is not possible. So, these things like R e to the power 9 by 4, R e to the power 3 by 4 those are the term which you must the values which you must remember for the computational cost of DNS is very high.

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