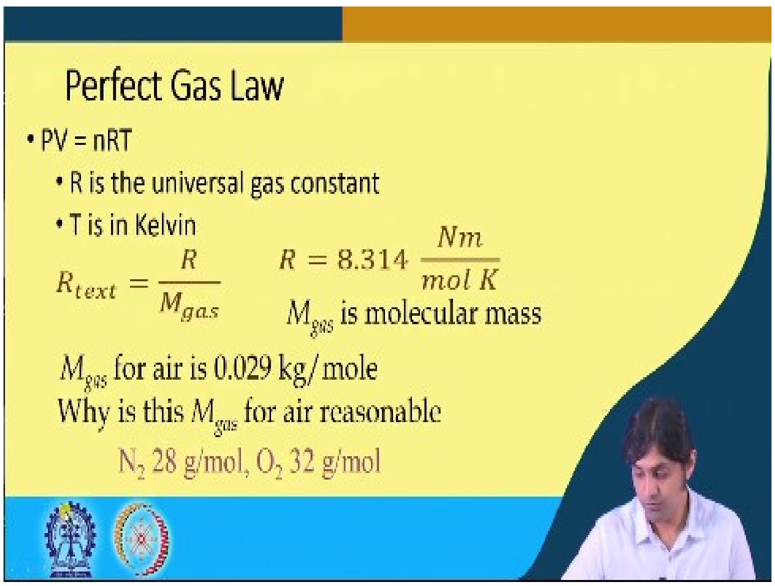
Hydraulic Engineering

Prof. Mohammad Saud Afzal
Department of Civil Engineering
Indian Institute of Technology Kharagpur

Lecture – 02
Basics of Fluid Mechanics- 1 (Contnd.)

Welcome back, this is the second lecture and we are going to study fluid properties again. So, we in the last class we studied mainly the shear stress and fluid viscosities So, today we will proceed a little further, this is for course called hydraulic engineering.

(Refer Slide Time 00: 45)

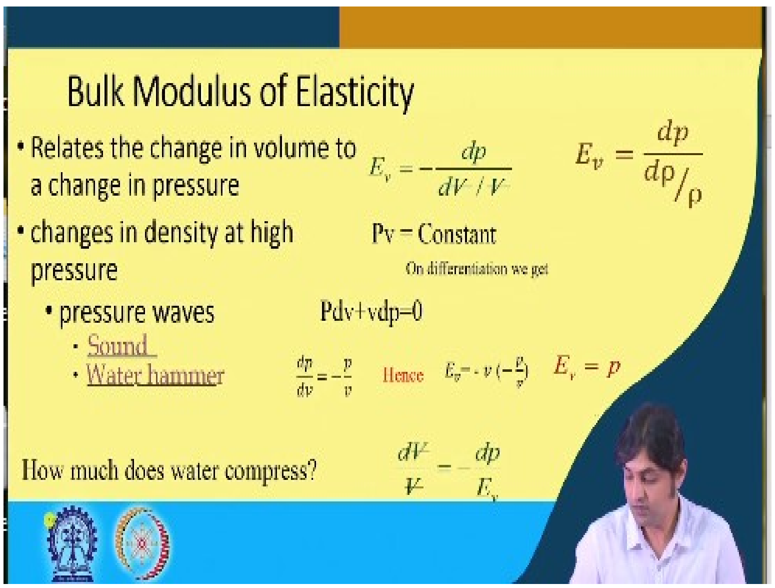


So, as we said that viscosity and other concepts are for all fluids and gas is also a fluid. So, we will also see what a perfect gas law is you have studied that already in your class 10th and 12th but this is these first 10 lectures are going to be a revision of your basics fluid mechanics course. So, I will go a little fast because I know you have already done it in your previous course called fluid mechanics, but to make the course more complete these topics are going to be covered again.

So, what does it say? If you remember it says $PV = nRT$. Okay? Where P is pressure, V is volume, R is gas constant, T is temperature and n is the number of moles. R is a universal gas constant. Temperature is taken in *Kelvin*. The value of R is given as $8.314 \frac{N.m}{mol.K}$. In the text, one of the textbooks that we are referring Munson, Okiishi and Young or Cengel, the R in the text has been written as $R_{t\,ext} = \frac{R}{M_{gas}}$, where *M* gases molecular mass, *M* gas for air is 0.029 $\frac{kg}{mol}$.

Can you guess why? Why is this *M* gas for a reasonable? The reason lies in that air comprises of almost *80%* nitrogen and *20%* oxygen, the molecular mass of nitrogen is *28* $\frac{gm}{mol}$ and oxygen is *32* $\frac{gm}{mol}$. So, I am going to you know, so, if you see this 32 is 28. So, this is 80%. Okay? So, this is how this value will come. Okay? So, proceeding to the next slide.

(Refer Slide Time: 03:18)

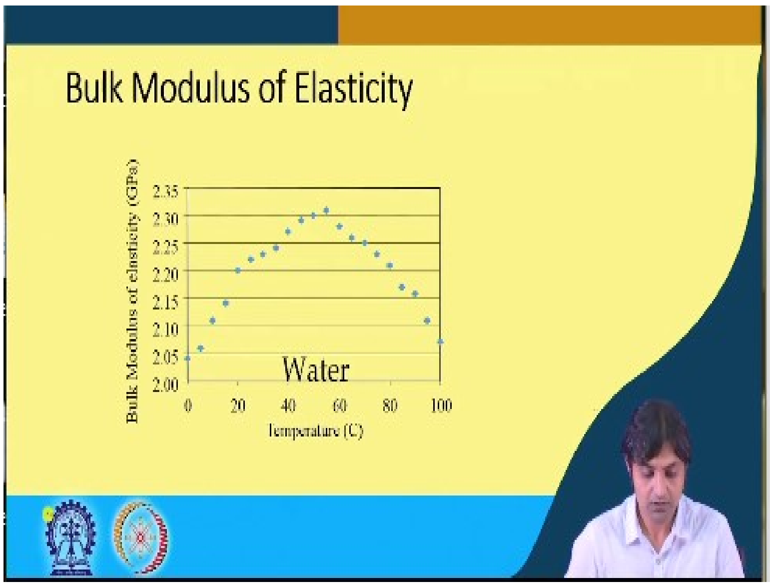


One of the important other property in terms of gases is bulk modulus of elasticity. So, what does bulk modulus elasticity do? It relates the change in volume to the change in pressure. So, bulk modulus of elasticity is defined as, so, you see $E_v = \frac{-dp}{dv\,/\,V}$ this is the definition of bulk modulus of elasticity. Yes, so we are going to the highlighter now. So, this is the equation that we are going to come later and how do we get it? This is shown here after some more description of bulk modulus elasticity.

The changes in density at high pressure for example, in terms of pressure waves is an example is the sound wave and other is a water hammer. So, now, I promise that I will show you how to come from this equation to this equation. So, we already know *PV=constant*. So, on differentiation, we will get $Pdv + vdp = 0$. And if you take *Pdv* to the other side and do $\frac{dp}{dv} = \frac{-p}{v}$.

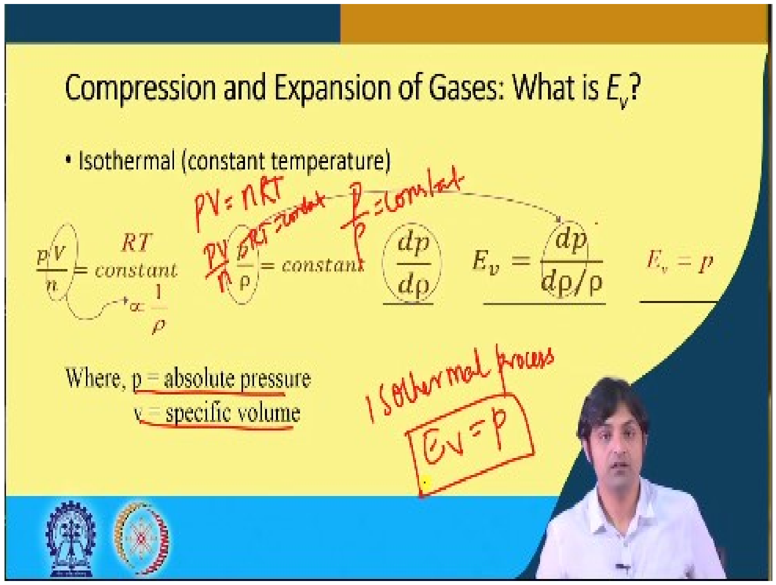
Hence, E_v can be written as if you use this equation and substitute the value of $\frac{dp}{dv}$, so, putting these 2 equations together, we will get E_v as pressure p in this case. Okay? We can also say that, why dp divided by dV / V can be written as dp / rho. So, I will just go and now, so, if you see this equation $\frac{dp}{dv} = \frac{-p}{v}$, so we can if you bring p on this side and dv so it is $\frac{dp}{dv} = -\frac{dv}{V}$ and this minus dv / v comes here and therefore E_v= $-dp$ and $\frac{dV}{V} = -\frac{dp}{p}$. Okay? So this becomes pressure, so now we proceed to the next slide. So how much does the water compress it is the same equation $\frac{dV}{V} = \frac{-dp}{E_v}$.

(Refer Slide Time: 07:01)



So, this is the figure, which shows the bulk modulus elasticity how it varies with the temperature. So, as you can see the bulk modulus elasticity increases as the temperature increases to a certain value and then starts to decreasing these are the normal ranges of temperature in air, 100 degrees is very rare but the most common ones are 0 to 30 degrees in India it can be 0 to 40 degrees. So, for water, so, I am sorry this bulk modulus elasticity is for water.

(Refer Slide Time: 07:43)

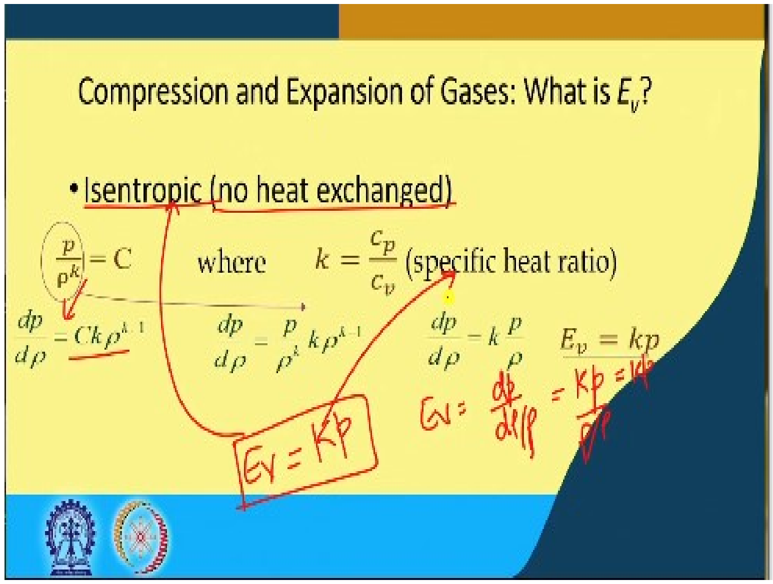


So, compression and expansion of gases. So, one of the phenomenon's is isothermal, which is constant temperature. So, what happens is $PV = nRT$, that equation we already know So,

$PV = nRT$. So $\frac{pV}{n}$ =RT which is constant. Okay? So, this is RT. So, $\frac{V}{n} \propto \frac{1}{P}$ or inversely proportional to p where p is absolute pressure and V is specific volume. And therefore, we can also write $\frac{P}{\rho} = const.$ So, we need to find out $\frac{dP}{d\rho}$ for to be able to find out E_v because E_v is given as $E_v = \frac{-dp}{d\rho\ /\ \rho}$.

So, $\frac{P}{\rho} = const.$. Right? So is $\frac{dp}{d\rho}$ and $\frac{dp}{d\rho}$ when substituted, it will give us E_v=P . So, more importantly for isothermal process what you must remember is that E_v=P . This is most important finding of this particular slide.

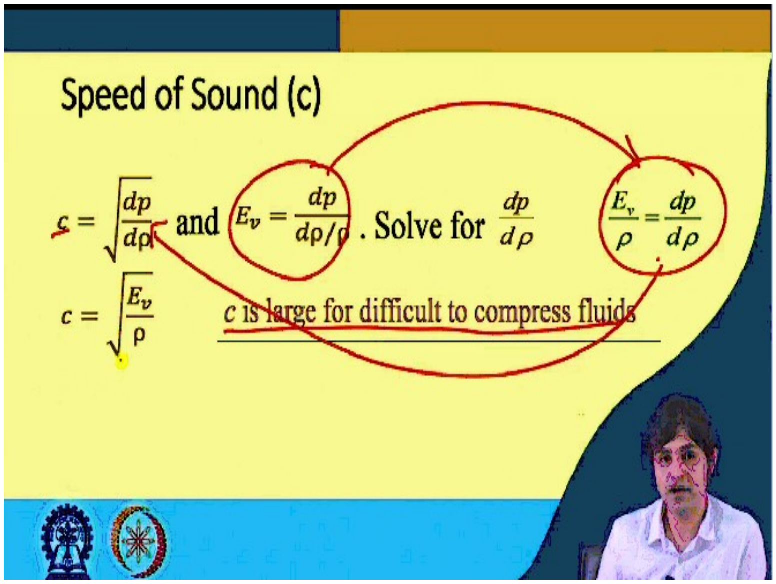
(Refer Slide Time: 09:31)



So, we are going to look at other phenomenon or process called Isentropic where no heat is exchanged. In this case, the equation for isentropic process is given by $\frac{p}{\rho^k} = C$ where $k = \frac{c_p}{c_p}$. This is a very standard terminology in thermal physics that you have already seen in your class 10th and 12th. So, this is also called a specific heat ratio. So, if you do $\frac{dp}{d\rho}$ from this equation, you will get $\frac{dp}{d\rho} = Ck\rho^{k-1}$ therefore $\frac{dP}{d\rho} = \frac{p}{\rho^k} k \rho^{k-1}$.

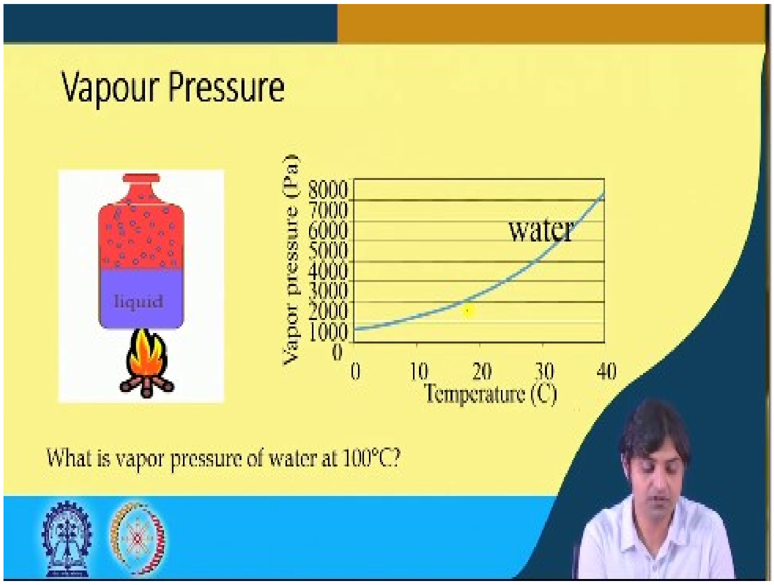
We are using the substituting the value of C in terms of $\frac{p}{\rho^k}$and therefore, $\frac{dP}{d\rho} = k\frac{p}{\rho}$, or we can also write E_v, because, E_v is given as $E_v = \frac{dp}{d\rho}$. Therefore, we have got $\frac{dP}{d\rho} = k\frac{p}{\rho}$, so we get kp more importantly E_v in case of Isentropic process is given by kp, k is specific heat ratio. So for isothermal process, this is going back $E_v = p$ and for Isentropic process $E_v = kp$, that is very important to note. Okay?

(Refer Slide Time: 11:45)



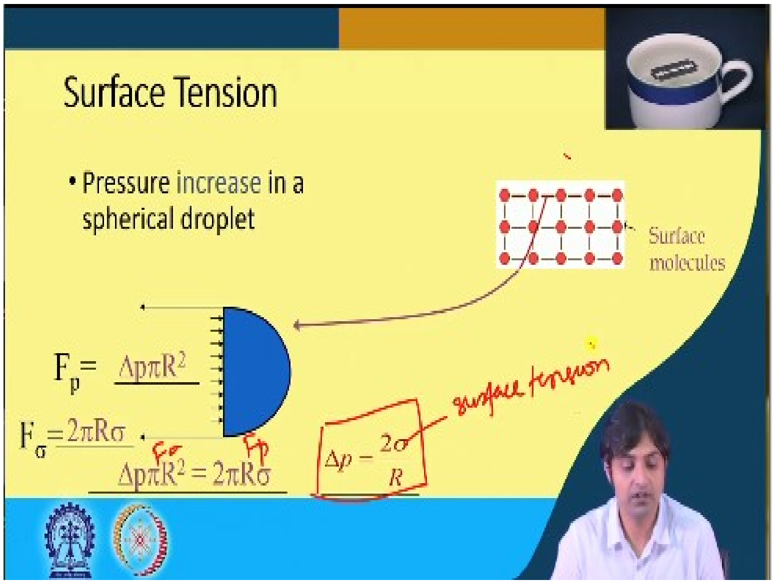
So another important thing that we should be aware of speed of sound is speed of '*c*' is given as this is the formula $c = \sqrt{\frac{dP}{d\rho}}$ and E_v we know that it is $E_v = \frac{dp}{d\rho/\rho}$. Now, we have to solve for $\frac{dP}{d\rho}$. We already know from this equation that $\frac{E_v}{\rho}$ can be written as $\frac{dp}{d\rho}$. Right? Therefore, if we put this equation in this equation, then we can get '*c*' as $c = \sqrt{\frac{E_v}{\rho}}$, where '*c*' is generally very large for compressible, I mean it is very difficult to compress fluids because E_v in case of fluid is too much the bulk modulus. So, we should be if we know this equation and we know what type of process is there we should be easily able to find out the speed of sound

(Refer Slide Time: 13:01)



So, another concept is vapour pressure. So, this is the variation of vapour pressure along with the temperature as you see as you keep on heating the temperature the vapour pressure increases to a great extent at *40 degrees* it is over *7000 Pascal*, you would have observed that on heating the water the molecules are easily able to escape. So, this is due to the vapour pressure and to be you know, some of the key values that we must be aware in these type of you know, courses is for example, what is the vapour pressure of water at 100 degrees centigrade it is *101 kPa*, so, this is an important value to remember. Okay?`

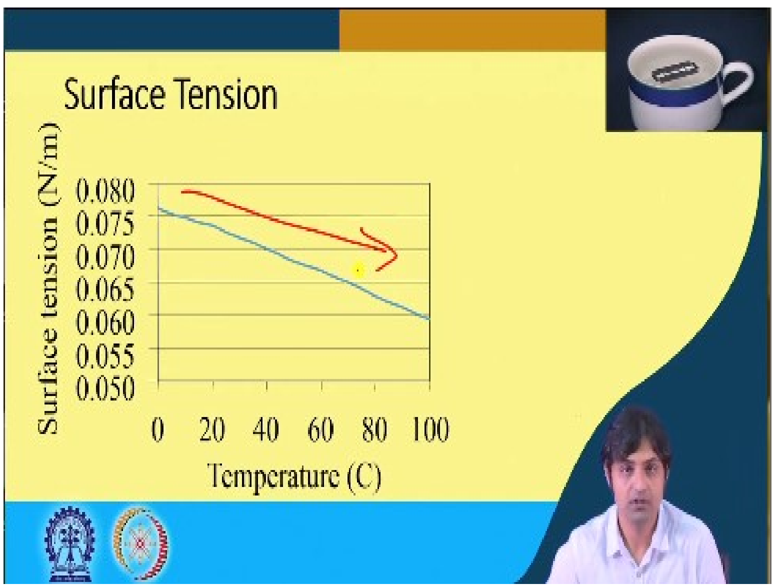
(Refer Slide Time: 13:49)



Another such properties, the surface tension. An example here is that the pressure increases in a spherical droplet, this is the surface of a droplet for example, and this is yeah, surface molecules that we are actually seeing here. And the phenomenon that is going to happen if you closely you know, area between you know, these 2 molecules that we are enlarging and zooming. So, what is happening is there are 2 type of forces. So, there will be force $F_p = \Delta P(\pi R^2)$ due to the pressure difference.

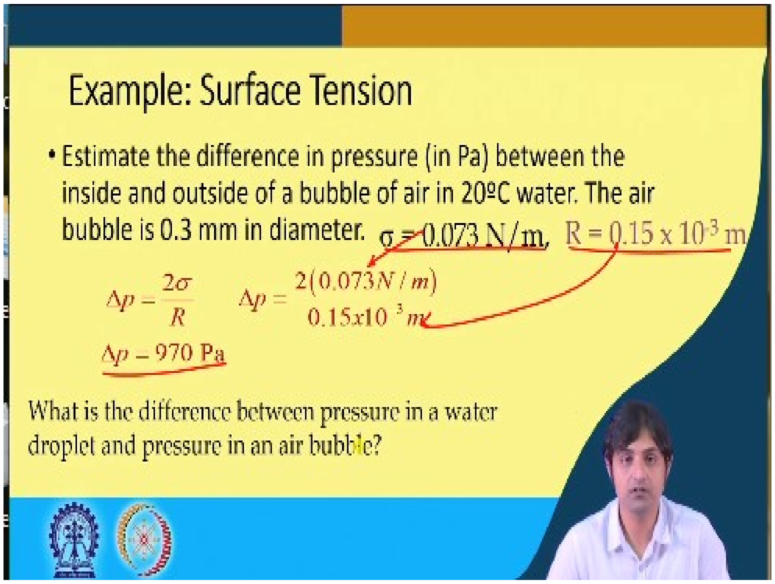
And the force of surface tension that will be resisting, it is going to be $F_\sigma = (2\pi R)\sigma$. This is the standard surface tension that you have read from before but I am repeating it again. So, in case of force balance, what is going to happen that this $F_\sigma = \Delta P(\pi R^2)$ should be equal to, so, this is $F_\sigma = F_p$ and what does it give? It gives the pressure difference in the droplet is $\Delta P = \frac{2\sigma}{R}$, σ is surface tension, R is radius of the molecule or the drop or the bubble whatever that is.

(Refer Slide Time: 15:29)



So, this as you can see from this curve, as soon as you increase the temperature of the fluid or water the surface tension is going down.

(Refer Slide Time: 15:40)

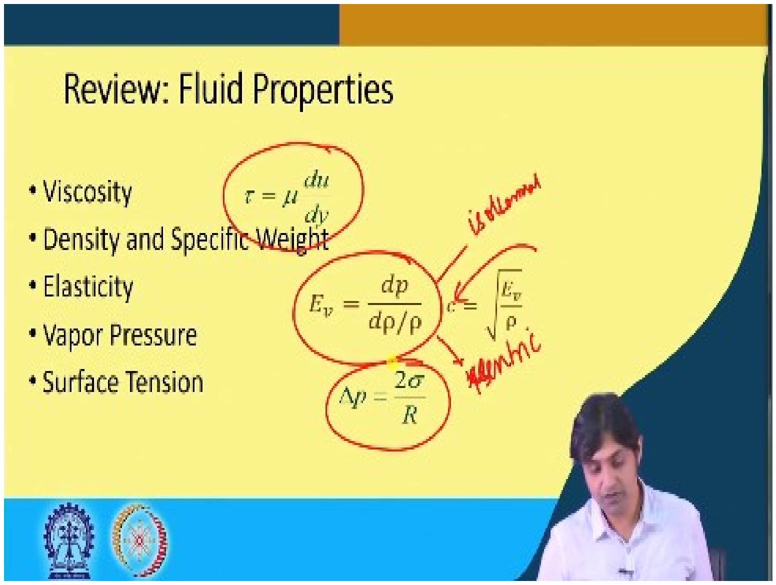


So, we have included an example of surface tension here, but, estimate the difference in pressure between the inside and outside of bubble of air in *20º* water *20º* of water. The air bubble is 0.3 *mm* in diameter. So, we already know the equation here σ is given as 0.073 *N/m* and *R* is the radius $0.15 * 10^{-3}$.

We have already seen the equation $\Delta P = \frac{2\sigma}{R}$. So, ΔP , 2σ sigma is the value here and *R* is 0.15 that we have come up with this.

So, the pressure difference comes out to be 970 *Pa*. I think it is very simple to calculate. So, you can use your calculator or just your. So, what is the difference? What is the difference between the pressure in water droplet and pressure in an air bubble? So, this is a question for you to think what just a hint it has something to do with the number of surfaces of air and water droplets.

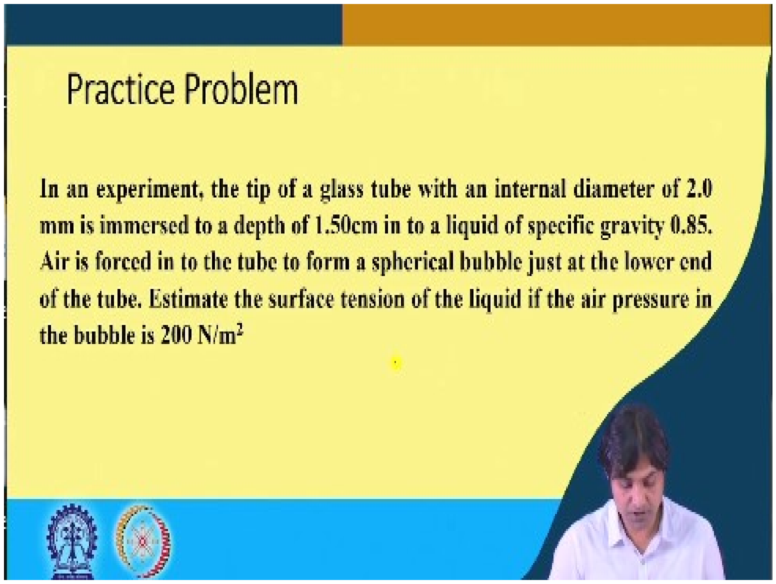
(Refer Slide Time: 17:03)



So, this normally, you know, the revision of our fluid properties is complete and the basics that we have seen. So, I try to have a review in such cases where we are revising something. So, what we have done we have read about viscosity, we have read that $\tau = \mu \frac{du}{dy}$ this is one of the most important equations actually, we have read about density and specific weight density is ρ specific weight is γ that is $\gamma = \rho g$ we have read about elasticity, bulk modulus of elasticity $E_v = \frac{dp}{d\rho/\rho}$.

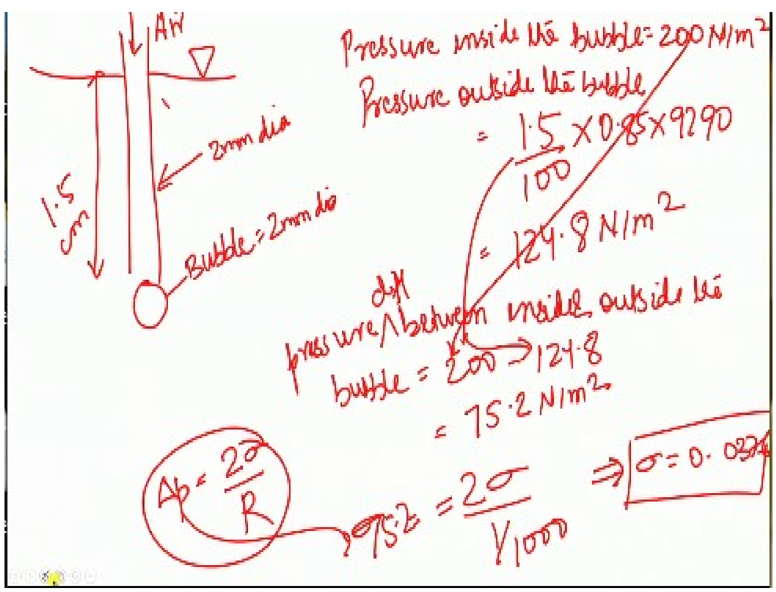
We have seen that for Isentropic, we have seen that for isothermal. And Isentropic and we have also related that to the speed of sound. We have seen about vapour pressure, we have seen about surface tension. And this was one of the important equation $\Delta P = \frac{2\sigma}{R}$. So to review the properties viscosity, density and specific weight, elasticity, vapour pressure and surface tension. These are the important properties that we have read. Now I have included for your convenience 4 different problems and encompassing all the concepts that we have done.

(Refer Slide Time: 18:41)



So we will go into these questions one by one. So the question says, in an experiment, that tip of the glass tube with an internal diameter of 2.0 *mm* is immersed to a depth of 1.5 *cm* into a liquid of specific gravity 0.85. Air is forced into the tube to form a spherical bubble just at the lower end of the tube, estimate the surface tension of the liquid if the air pressure in the bubble is $200\ N/m^2$. So, how do we approach this problem? Okay? So, first and the foremost thing that we should be doing is drawing the diagram of whatever piece of information that is there which need to know.

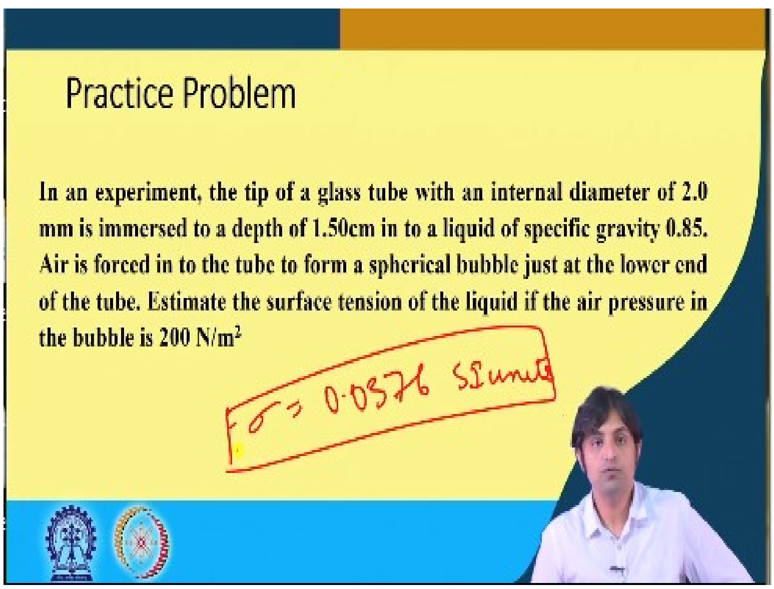
(Refer Slide Time: 19:35)



So, what it says is so, this is water and this is air is coming here. This is the bubble that is 2 *mm* diameter. This is 2 *mm* diameter and this is 1.5 *cm*. Okay. So, the pressure inside the bubble is given as 200 N/m^2 and it is also told, pressure outside the bubble is given by *1.5*100*0.85* 9790* that is the specific weight, you see this length 1.5 *cm* here so *1.5 /100* and 2.85 is this specific gravity and this is a specific weight. This comes out to be 124.8 N/m^2.

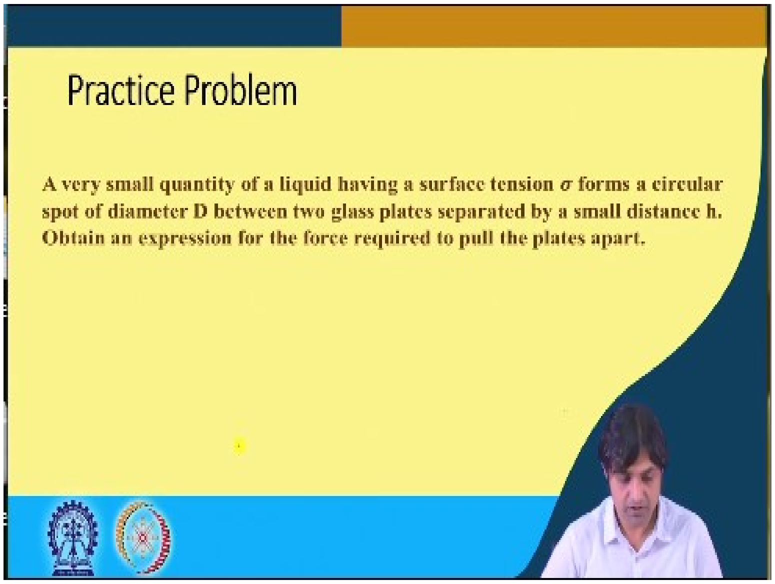
So, what is the pressure difference now, between inside and outside, between inside and outside the bubble is 200-124.8. So, this is here and this is here and this pressure difference comes out to be 75.2 N/m^2. So, what was the formula for the pressure difference that we have seen in couple of slides before, that $\Delta P = \frac{2\sigma}{R}$. So correct? So, using this equation, σ can be written as, or we just substitute the value this value is 75.2, this is equal to 2σ divided by R, R is 1/1000. This will gives σ = 0.0376 SI units. So, this is one of the examples of the surface tension forces. We can now

(Refer Slide Time: 23: 32)



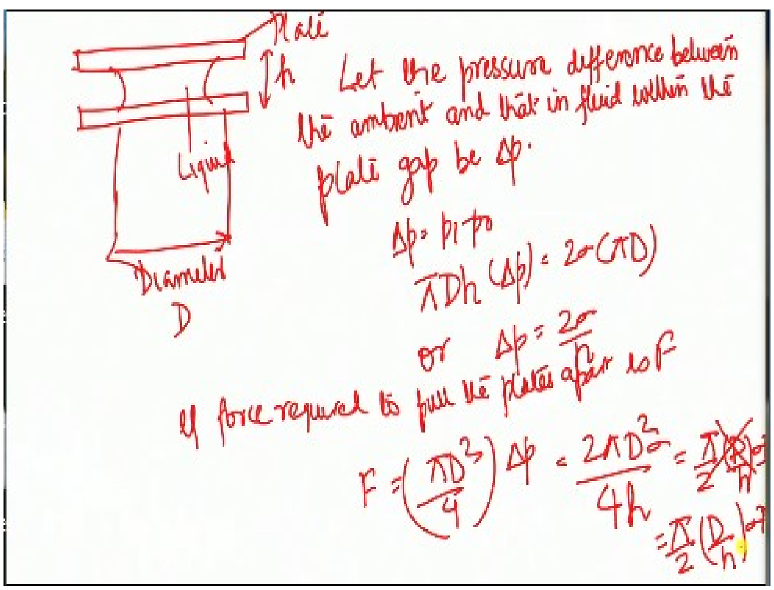
See that in this, we found out we will go, we will select the pen and then this σ here came out to be 0.0376 SI units.

(Refer Slide Time: 24:03)



Now we go to the next problem. It is also very much related to the surface tension. So, the problem says a very small quantity of liquid having a surface tension sigma forms a circular spot of diameter D, between 2 glass plates separated by a small distance h. Obtain the expression for the force required to pull the plates apart.

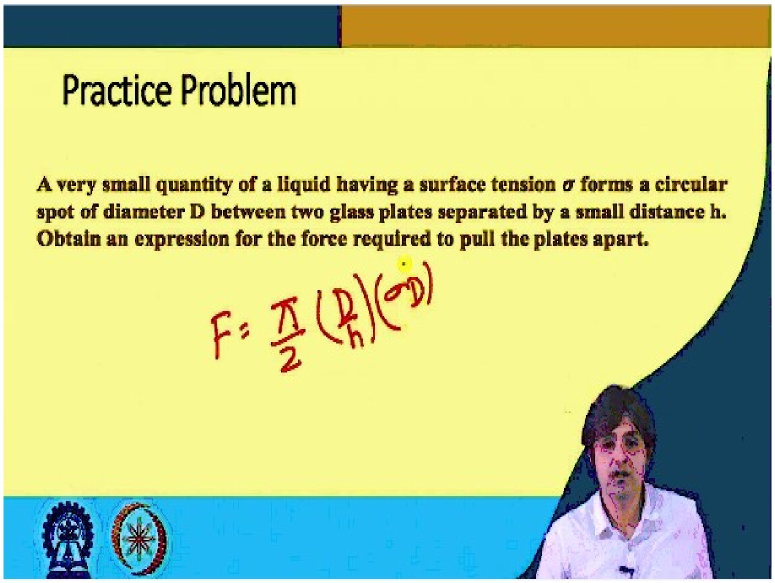
(Refer Slide Time: 24:37)



So, as I said the most important thing to do is to be able to draw whatever is given. So, let us draw, 1 plate, 2 plates. So, this is the plate, this is liquid, this is given by h and this distance diameter D. So, these are the some of the figures that we already know. So now what we say is let the pressure difference between the ambient and that in fluid within the plate gap be ΔP. Then

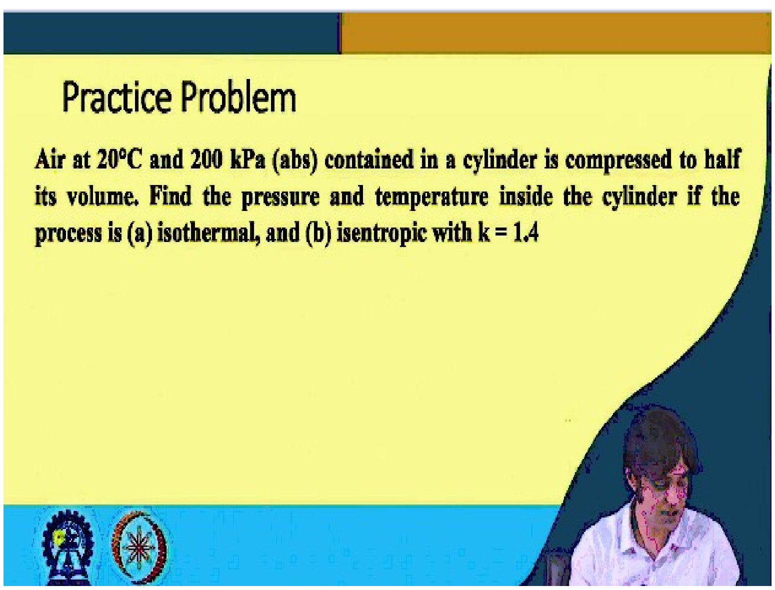
ΔP can be given as $\Delta P = P_1 - P_0$ or $(\pi D h)\Delta P = 2\sigma(\pi D)$ or $\Delta P = \frac{2\sigma}{h}$ in our case. So, if force required to pull the plates apart is F, F can be written as $F = \frac{\pi D^2}{4} \Delta P$ and $2\sigma(\pi D)$ by, we substitute the value of delta p or it can also be written as $\frac{\pi D}{2h} \sigma D$. Sorry, I will write it again $\frac{\pi D}{2h} \sigma D$.

(Refer Slide Time: 27:50)



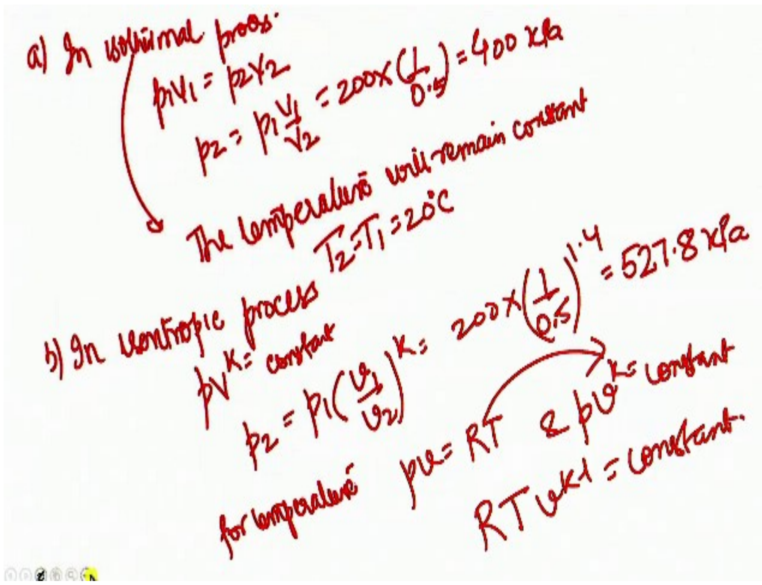
So writing the final equation here F. F that is required will be $F = \frac{\pi D}{2h} \sigma D$,.

(Refer Slide Time: 28:21)



Now we go to our next problem that is related to the air and the compression so, mostly related on the gas laws. So, air at 20º and 200 *kPa* contained in a cylinder is compressed to half its volume. Find the pressure and the temperature inside the cylinder of the process (a) isothermal and (b) if it is isentropic with k =1.4. So, okay, the best way to go is to have the whiteboard. So, whiteboard is very good.

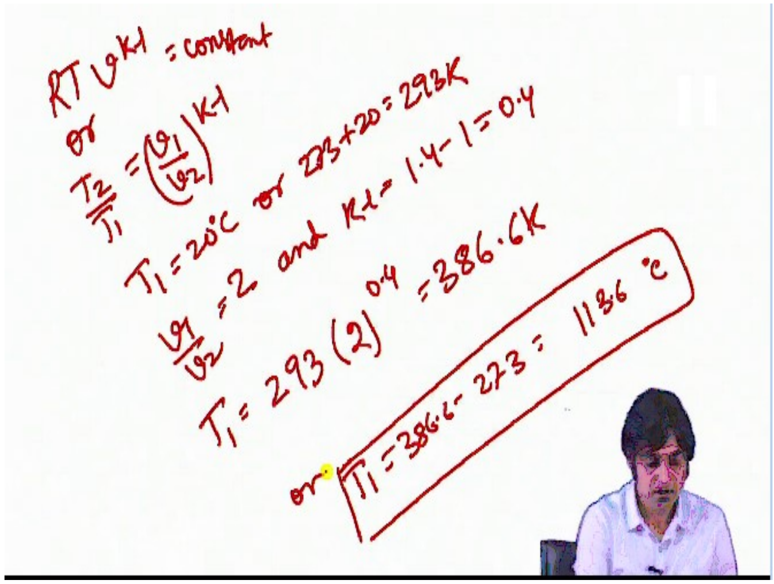
(Refer Slide Time: 29:02)



So, (a) Isothermal process $P_1V_1 = P_2V_2$ therefore $P_2 = (P_1V_1)/V_2$ or $200\frac{1}{0.5}$ is very simple to calculate and this is 400 *kPa* absolute temperature, because everything is given here. Because this is isothermal process temperature will remain constant and therefore, T_2 is equal to T_1 is equal to 20º. Right? The second part in isentropic process, what happens in this isentropic process $(pv)^k$ =*constant*. So, in this case P_2 will come to be $P_2 = P_1(V_1/V_2)^k$.

So, 200 * (1/0.5), so it is compressed to half, so volume 200 * $(1/0.5)^{1.4}$ and if you calculate this, it will come to out to be 527.8 *kPa.* Now for temperature *PV=RT* and $(pv)^k$ =*constant* . So we can use this equation here and then we can write RTv^{k-1} = *constant*. Well I do get the next page.

(Refer Slide Time: 30:02)



So if we use so what we found out in the last screen was RT v to the power k - 1 is equal to constant or we are also able to write T_2 / T_1 because R will be canceled on both sides, $(V_1 / V_2)^{k-1}$. We know that T_1 was 20°or in Kelvin, it is going to be 273 + 20 is equal to 293 *K* and (V_1 / V_2)= 2. And *k - 1* will give out *1.4 - 1* as 0.4.

So on substituting these values, we can get T_1 is equal to 293 * $2^{0.4}$ or 386.6 *K* or T_1 is *386.6 - 273* that is 113.6°. So, this is the temperature that we have got in case of a isentropic process. So, what I would like to do is I would like to end our fluid properties lecture here if I get more time we can solve one more problem at the end of this week's lecture, okay, thank you so much. See you in the next week lecture that will be on fluid statics, and that will be spread over 3 different lectures. Thank you so much.