Hydraulic Engineering
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Lecture – 05
Basics of fluid mechanics-I(Cont.)

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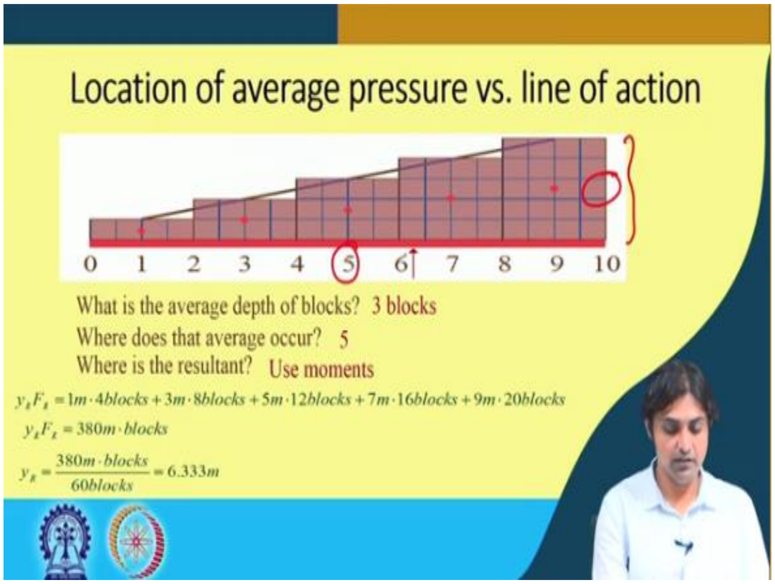

Okay. Welcome back. So, this is the again starting with the last slide from the last lecture and we figured out what was the average depth of the block and that came out to be, you know, the average depth was 3 blocks here, what where does that average occur? It was at block number 5 this average and we found out the resultant and the resultant came this. Okay.

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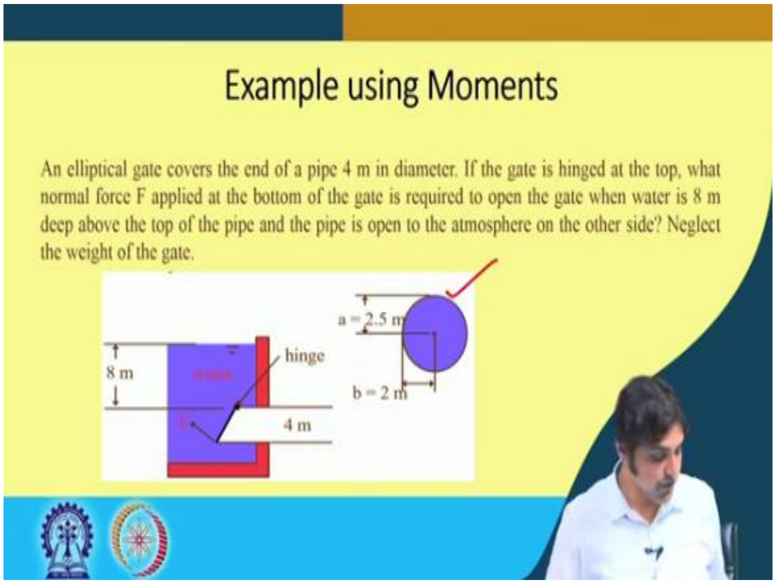

So, now we proceed with one of the real life examples of moment. So, the question is, an elliptical gate covers the end of a pipe 4 meters in diameter. So, this is the elliptical gate. I will delete this but I am trying to show where those different objects are. If the gate is hinged at the top here, okay, what normal force F applied at the bottom of the gate is required to open the gate when the water is 8 meters deep above the top of the pipe, okay, so this, and the pipe is open to atmosphere, okay, on the other side, here. Neglect the weight of the gate, okay. So, here we have seen, this is how this elliptical gate looks, this is the cross section, okay. So, once you have understood the question, we can proceed with these solutions now, okay.

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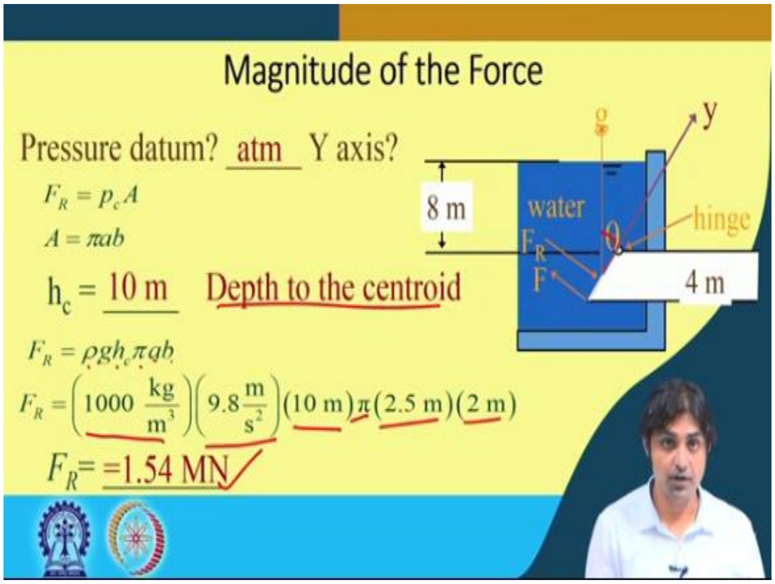

So, I have kept this figure on the right side. So, that you are able to follow, what we actually we are going to do. So, what is the pressure datum? This is the atmosphere, okay. So, the resultant force is going to be the pressure at the centroid into area that we have seen in the derivation in last lecture. What is the area? Area of the ellipse that we have seen is pi into a b, okay. What is the h c? So, just taking, you know, for h c first we need to understand, this is the y axis that we have assumed, okay. And this is the theta that we have assumed, okay. So, h c is 10 meter actually. Why? 8 meter plus 2 because of the ellipse, depth to this centroid, very clear. So, F R is going to be ρ g h c pi a b. So, ρ is 1000 kilogram per meter cube, g is 9.8 height of the centroid that we have seen is 10 meter, because 8 meter is this water depth and the depth location of the centroid is further 2 meters below this location so 10 meters. pi is pi, a is 2.5 and b is 2, as we have seen the values of a and b from the previous slide here is 2.5 and b is 2.

So, I am just going back. So, I mean, here. So, F R finally comes out to be 1.54 Mega Newton, very simple to calculate, okay. Mega Newton is 10 to the power 6, actualy. So, fine?

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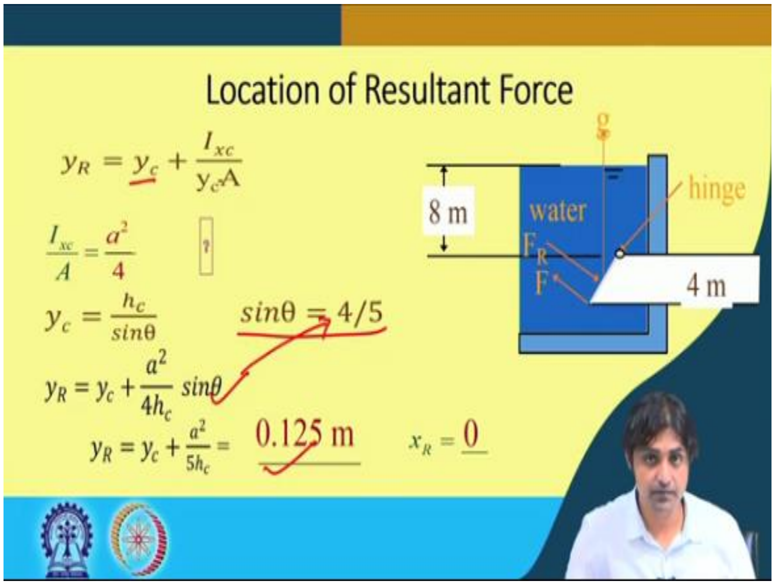

So, we proceed to the other part now. So, what is the location of the resultant? So, we need to find out y R and x R, y R is given by y c, correct. So, I x c / A from the set of the areas that we had seen in the shown in the last lecture is given as a square / 4. If you are confused, you can go and see it in the last lecture notes. y c can be written as h c / sin theta, this we have already seen in the derivation, okay, where sin theta is 4/5 for m, because minor axis and major axis is 2.5 and 2. So, y R can be written as y c + a square / 4 h c sin theta. Right?

If you just put the value of I x c and a, you will get this result this value. So, y R is y c + a square, if you have substitute the value of sin theta 4/5 you will get c + a square / 5 h c, and this comes out to be 0.125 meter. As ellipse we discussed these the it is symmetric about the centroid, so, we will not have any x R and x R was going to be 0. So, very simple calculations here, correct.

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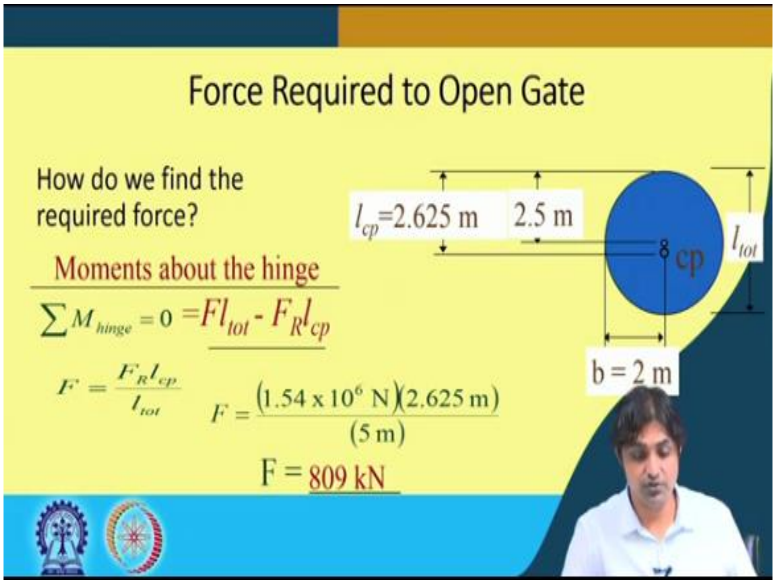

Now, the most important part is force that is required to open the gate. So, what all do we know? See, y R is 0.125, right. So, this will become 2.5 + 0.125 that will be 2.625, correct. And this is 2.5 meters, this is 2 and this is total length l total. So, how do we find the required force here? So, we have to take moments about the hinge, hinge was here. So, moment about hinge will be 0 and this means force into length of total will be the resultant force into length of it, we already know length of the, you know, center of pressure and the resultant force, we already know.

We need to find out this F, we already know l total. So, F total will be $\frac{F_R l_{cp}}{l_{tot}}$ and on substituting the value 1.54 into 10 to the power 6 for $F_R l_{cp}$ is 2.625 as can be seen here, and l total is very simple. It is 5 meters, right. So, yeah. So, this value will come out to be 809 kilo Newton. Okay, so, this makes sense. Correct. So, this makes our problem complete. What I am going to do again, I will erase all the ink on this slide. And now we can just proceed to the other part.

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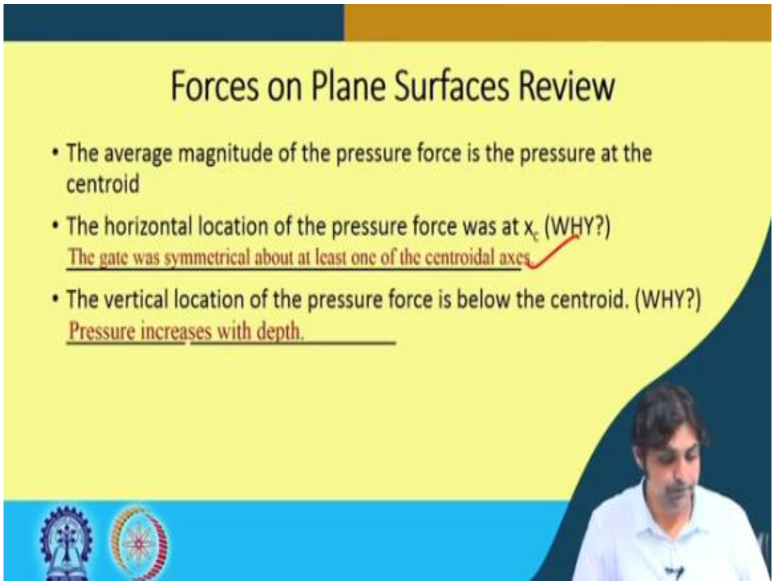

So, we also need to, you know, to wind up the, you know, the forces on the plane surfaces. Some of the important results, the average magnitude of the pressure force is the pressure at the centroid. This is what we have seen, correct. The horizontal location of the pressure force for that X c, why? In this particular problem, the gate was symmetrical about at least one of the centroid axis. In that case, it was 0 X c, the vertical location of the pressure forces below the centroid. This we have answered so many times that because the pressure increases with depth, correct.

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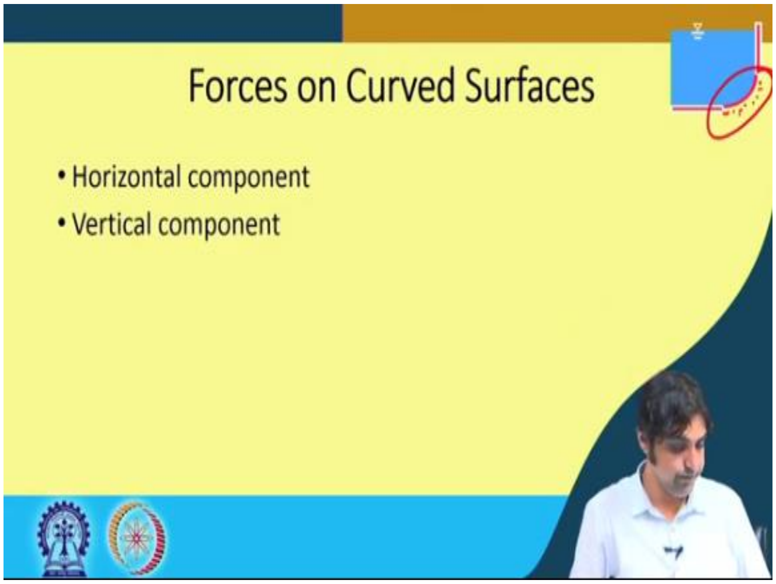

So, now, this is sorry, yeah. Now we need to calculate the pressure on the curved surface. So, this surface here, you see this, okay. So, how? We need to know the horizontal component, we need to find out the vertical component because it is curved, right.

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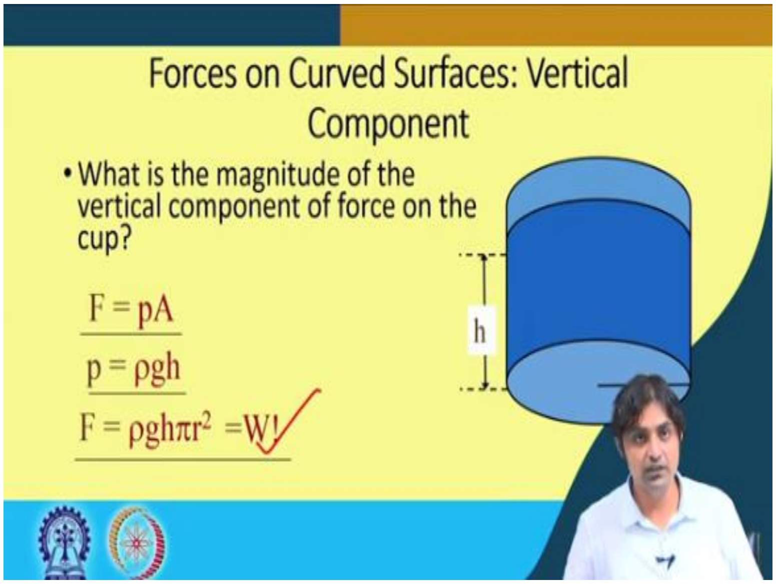

So, this is one of the examples. My question to you is what is the magnitude of the vertical component of force on the cup? Tell me? Force is pA, correct. Pressure is ρgh, putting it ρgh, area is πr^2, and this actually is the weight of the liquid supporting it. So that will be the force, vertical force.

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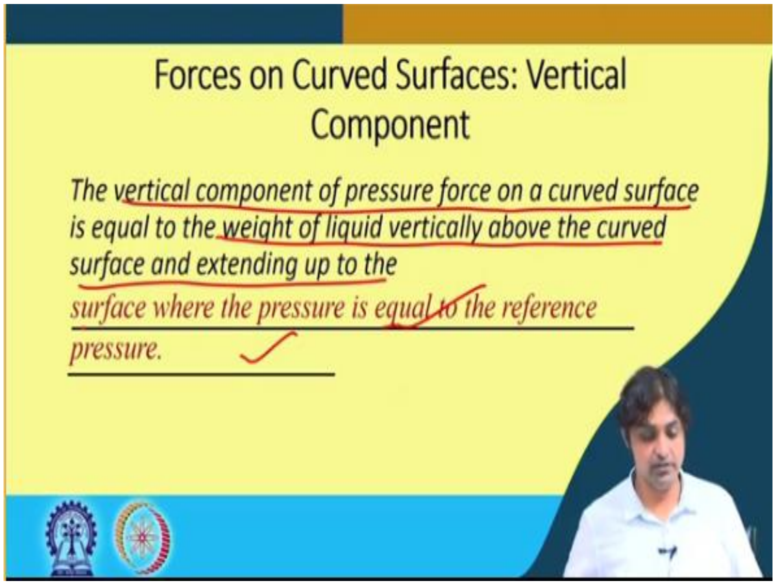

So, to sum the vertical component of the pressure force on a curved surface is equal to the weight of the liquid vertically above the curved surface and extending up to the surface where the pressure is equal to the reference pressure. This is the thumb rule. So, the vertical component of the pressure force on a curved surface is equal to the weight of the liquid vertically above the curved surface, okay. And extending up to the surface where pressure is equal to the reference surface.

So, that means up to the free surface, in our case. So, we will explain, we will understand this particular part using a solved example. This is pretty simple to understand.

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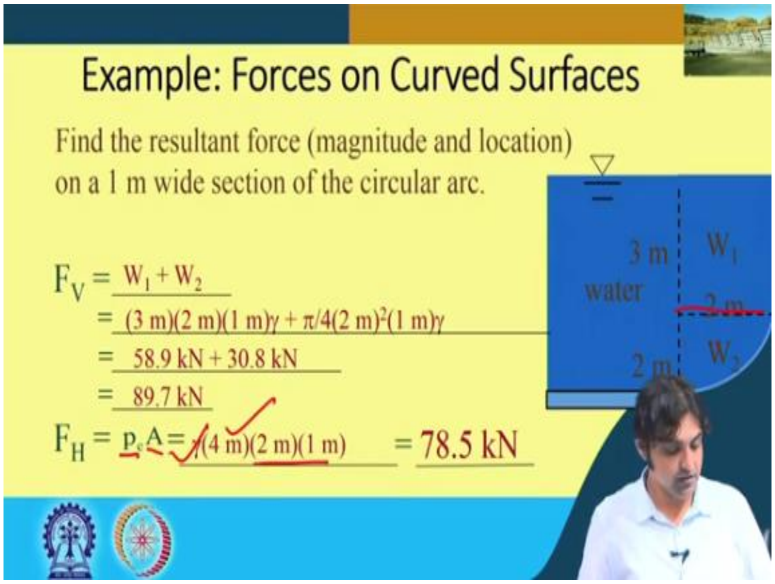

So, this is a gate, the all the components are indicated W1, the lengths are indicated 2 meters, W1, W2, 2 meters, 3 meters, water this is water, okay. So, let me erase this. We have to find the resultant force both the magnitude and location on a 1 meter wide section of the circular arc. So, actually this is 1 meter wide in that direction in the other direction not, shown here. So, what is going to be the vertical force? It is going to be what we according to the definition, it should be the sum of the liquid above this curved surface, extending from here up to the free surface.

So, basically what is that? W1 + W2, W1 is very simple, it is 3 meters into 2 meters into 1 meter, because that is the dimension that is told, 1 meter wide section into ϒ of water. This one it is pi / 4 r square. So, pi r square is a total area which is one fourth. So, pi / 4 r square into 1 meter into

ϒ, very simple calculation. This comes to be 58.9 kilo Newton + 30.8 kilo Newton and in total it comes to about 89.7 kilo Newton, okay, very simple.

Now, we also need to, want to know what the horizontal component is. So, the horizontal component is the pressure at the centroid multiplied by area. But what area that is? So, area is 2 into 1. So, pressure at the centroid is, this is centroid 1 meter below, so, 3 +1= 4 ϒ H is the pressure and this is the area 2 meter into 1 meter in that. So, this the horizontal force is going to be 78.5 kilo Newton.

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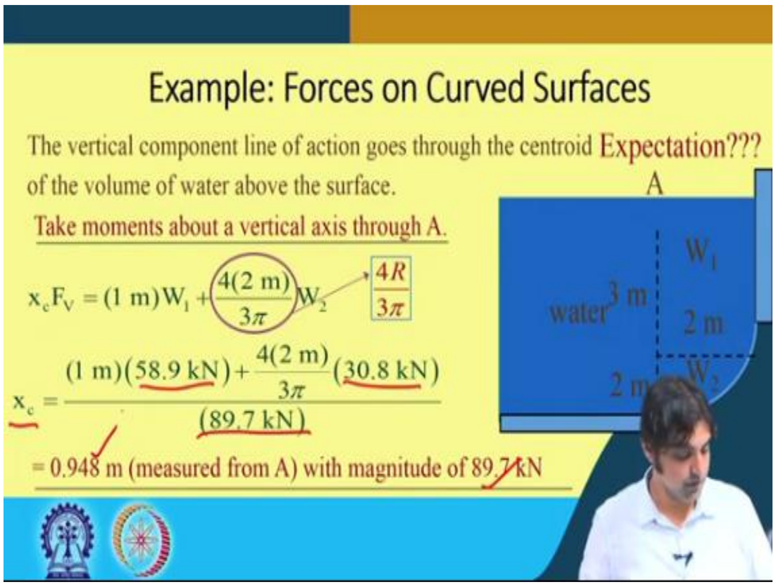

The vertical component line of action goes through the centroid of the volume of water above the surface and that is very understood, you know. So, this is A, now we have to take moments about the vertical axis through A here. So, if we want to find out the line of action, that is, x c, x c we know already, right? Sorry x c into F v will be equal to the weight is acting, 1 meter from there, W1. However, W2 I have told, if you remember, I told you to remember this equation 4 R / 3 pi in the lecture that we are going to use this and actually we are using this. This is 4 into 2 meter by the 4 into 2 / 3 pi and this, okay.

So, let me erase this, yeah. So, this is the $\frac{4R}{3\pi}$ and this will give us the location xc. So, it will be 1 meter weight was 50, w1 was 58.9 this we already know W2 was 30.8 and the total vertical force we found over 89.7 in the previous slide. So, this is actually going to give us 0.948 meter and it is measured from A with magnitude of 89.7 kilo Newton. So, this is the magnitude and this is the location.

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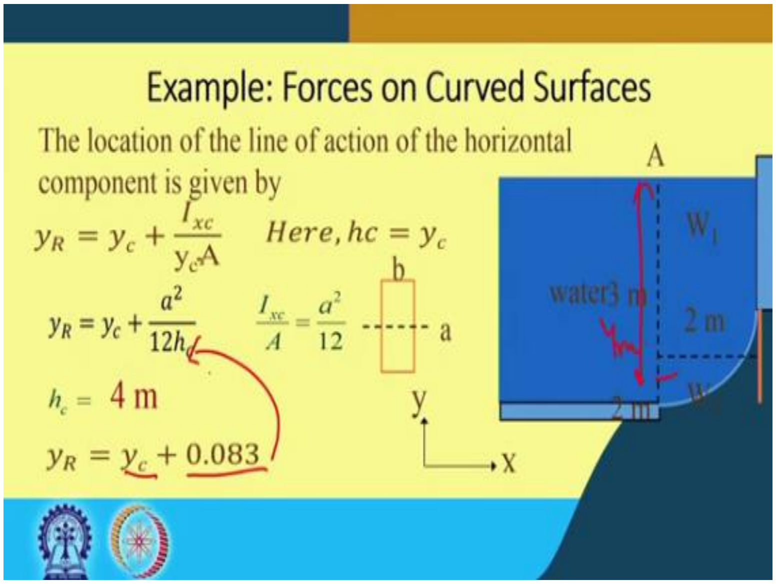

The location of the line of action of the horizontal component is given by $y_R = y_c + \frac{I_{xc}}{y_c A}$. This is what we have seen before here, h c is equal to y c, the height of the centroid is the actually also the y because we have assumed an axis in the vertical direction. So, y R is going to be y c + if you look at this, so, $\frac{I_{xc}}{y_c A}$ will come out to be, you know, because of a square / 12.

If you assume, b and a as this I xc / A is going to be a square / 12, as I have already indicated before, now, they have also the, you know, this one here. So, h c is going to be in this case, what h c is. So, 3 meter + 1 meter, so, that is 4 meter. So, y R is y c + 0.083 using this calculation, okay, because.

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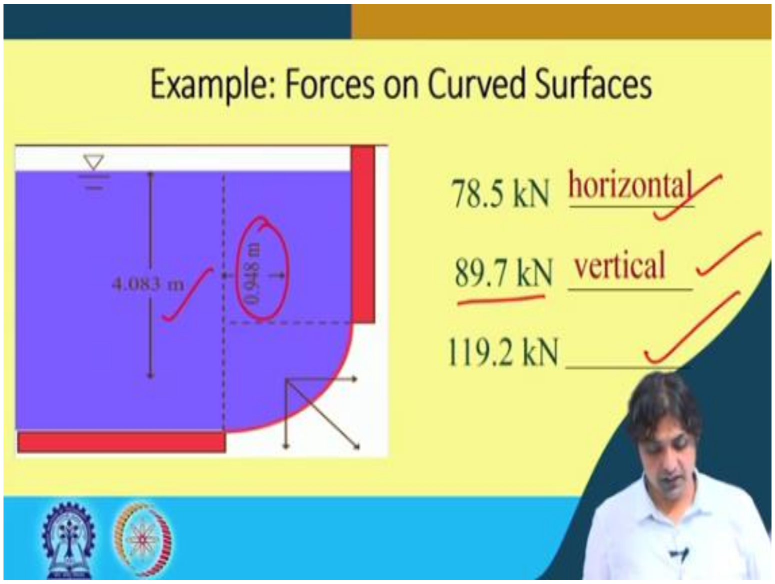

So, now to sum up, this is the horizontal force 78.5, this is 89.7 the vertical force and 119.2 is the resultant force and, yeah. The distances are shown, .948 meter that we have calculated you know 4.083 meter also we had calculated. So, this solves our this problem of forces on curved surface. So, this will, you know, give an overall picture of what is happening in this, you know, forces on curved surface.

So, the trick is, that you use always the force balance and the moment balance. Wherever, the curved surface is, the as I showed the vertical force is, the weight of the liquid above the curved surface. until the, you know, where the atmospheric pressure is or until the free surface. And the pressure the horizontal force will be the pressure at the centroid into the area, projected area, projected area, in the in because surface is 3 dimensional, but in the plane of 2d the projected area that is very simple to do that and so, yeah.

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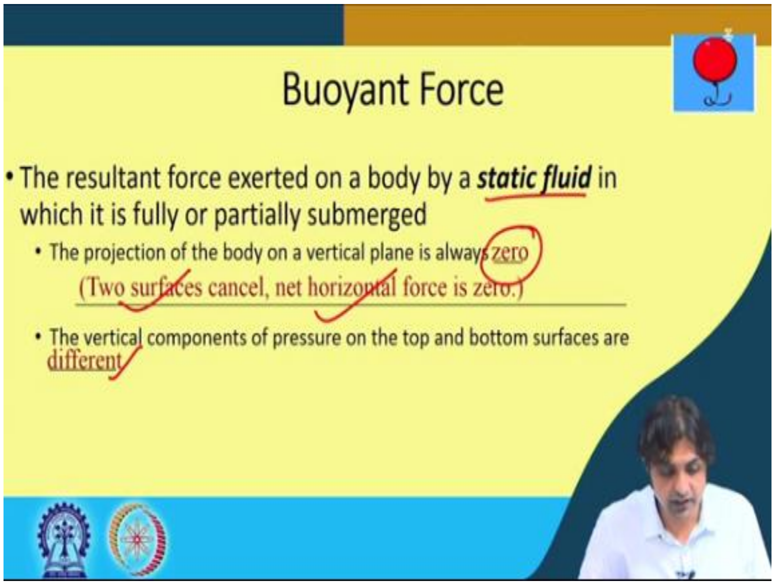

So, we will be proceeding now to the topic, last topic that is buoyant force after which we will also solve some problems. So, I would ask you the resultant force exerted on a body by a static fluid This is quite important term here in which it is fully or partially submerged is going to be the projection of the body on a vertical plane is always you have to fill this is going to be 0. This means, that 2 surfaces cancel the net horizontal force is 0. That is very important. The vertical components of pressure on the top and bottom surfaces are different however. So, in the horizontal plane the forces cancel to surface cancel therefore, there is no horizontal force, but in the vertical components of pressure on the top and bottom surfaces are different. The reason is very true, because at the top the pressure will be different and the bottom the pressure will be different.

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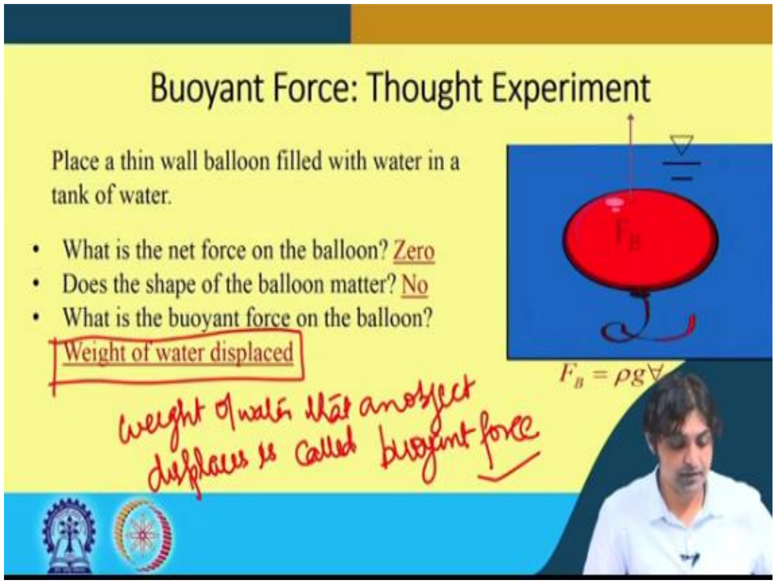

So, this is if you do try to do an experiment you place a thin water balloon filled with water in a tank of water, something like this. So, this is the balloon. And the, you know, this is filled with water and this is also filled this is placed in a tank of water. So, the buoyancy force, we call it ρ gv. I am asking what is the net force on the balloon? Should be 0. It is in if it is in equilibrium, net force should be 0. Here, does the shape of the balloon matters? No, because if you do that experiment, it does not really matter.

Now, the question is what is the buoyant force on the balloon? Actually, I have not derived it, but since you have already done it in your fluid mechanics class, the buoyant force on the balloon is actually the weight of the water displayed. So, this is the most important thing to note that the weight of water that an object displaces is called be buoyant force, this is very important. So, proceeding forward with these 2, you know, this particular concept, what is buoyant force? We are going to actually solve a problem.

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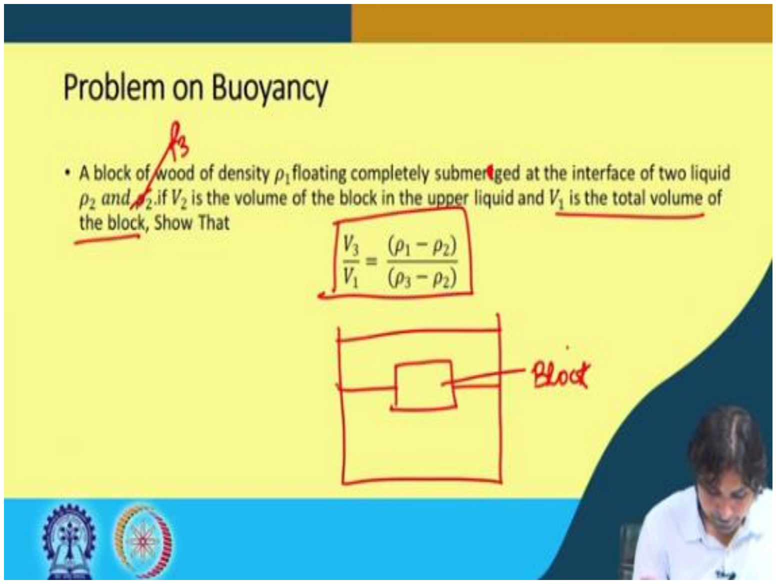

So, it says a block of wood of density ρ1 is floating completely submerged. I think the spelling is wrong but I delete it at the interface of 2 liquid ρ1 and ρ2, ρ2 and ρ3 yeah. So, it should be ρ 2 and this should be ρ 3, I am very sorry for this typing error. If V2, is volume of the block in the upper liquid and V1 is the total volume of the block we have to show this equation. First, of all when you have any questions like this, the first and the most important step here to do is draw the figure.

So, I am not very good at this, but anyways we will try. So, let us place the block here first. This is block; let us put a liquid layer here and another liquid layer here. So, block, this density is ρ 1, for example, this is liquid 3 because we have assuming density as ρ 3 and this is liquid 2 that is why this is ρ 2. So, I think this is the correct representation and now we also have to say this is here.

So, this is the, you know, this is the complete diagram. Now, I think, the question would be more clear to you. So, as always I am going to use, sorry, I am going to use white screen.

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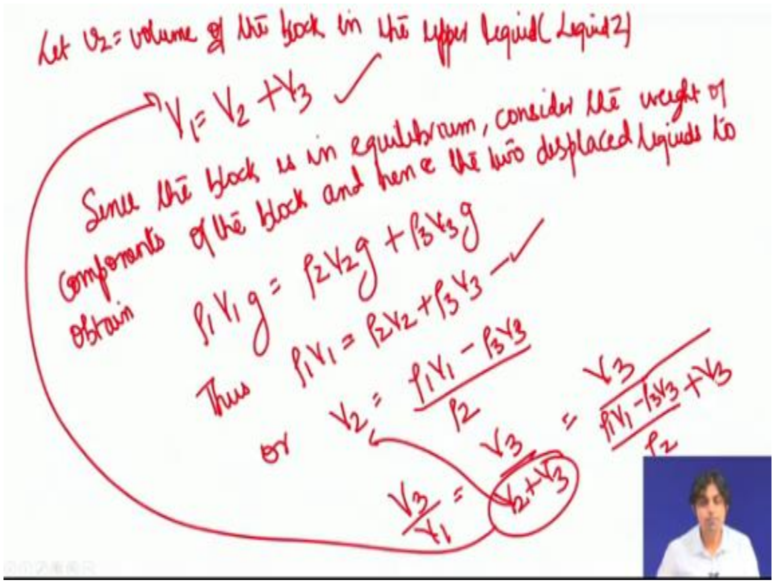

So, we say let v2 is equal to volume of the block in the upper liquid that is liquid 2. So, V1 will be equal to V2 + V3. Since, the block is in equilibrium, consider the weight of components of the block and hence the 2 displaced liquids to obtain. So, how do we get, that ρ1 V1 g is equal to ρ 2 V2 g Plus ρ3 V3 g. Thus, ρ 1 V1 is equal to ρ 2 V2 + ρ3 V3. That is correct or we can simply V2 is equal to using this equation here,ρ1 V1 -ρ3 V3 divided by ρ 2 or V 3 / V1 on the other hand can be written as V3 divided by V2 + V3 from here.

And this can be V3 now v2 + v3, v2 we can use from here. And this equation has actually come all the way from here. So, it will be v3 + and v2 we are going to use so it will be a ρ1 V1 - ρ3 V3 divided by ρ 2 + V3 fine. So, we will be expanding this in the next white screen. So, I am going screen on white, white screen.

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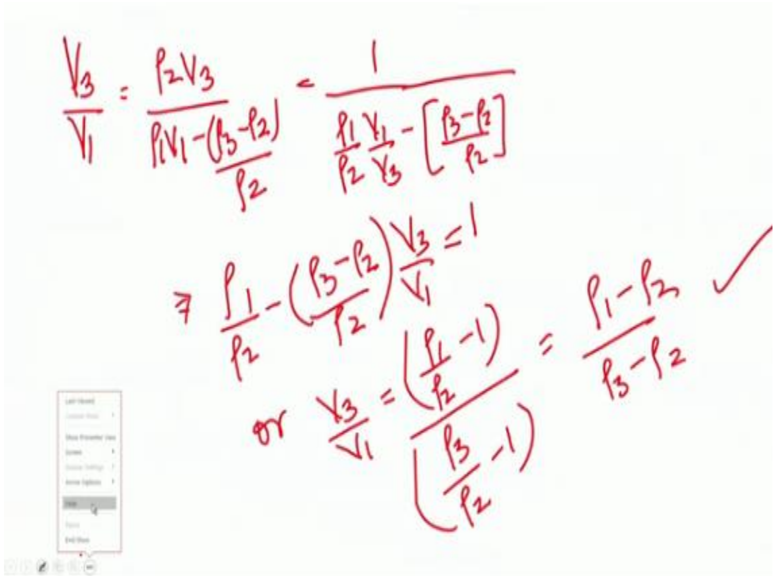

So, V3 / V1 is going to be ρ2 V3, we are just simply expanding this, ρ1 V1 - ρ3 - ρ2 divided by ρ2 or 1 / ρ1 / ρ2, V1 / V3 - ρ 3 - ρ2 divided by ρ 2, or this will we take the denominator upside, so, it will be ρ1 / ρ2 - ρ3 - ρ2 divided by ρ2 into V3 / V1 is equal to 1, or you simplify then V3 / V1 come out to be ρ1 / ρ2 - 1 are divided by ρ3 minus ρ2 - 1 or ρ1 - ρ 2 divided by ρ3 - ρ 2 and this was what was asked in the derivation.

So we have solved this problem, it is just nothing just simply forced balance in the vertical direction and some manipulation. I think, you should practice that more carefully in your home. So, now we are going to, you know, do the last.

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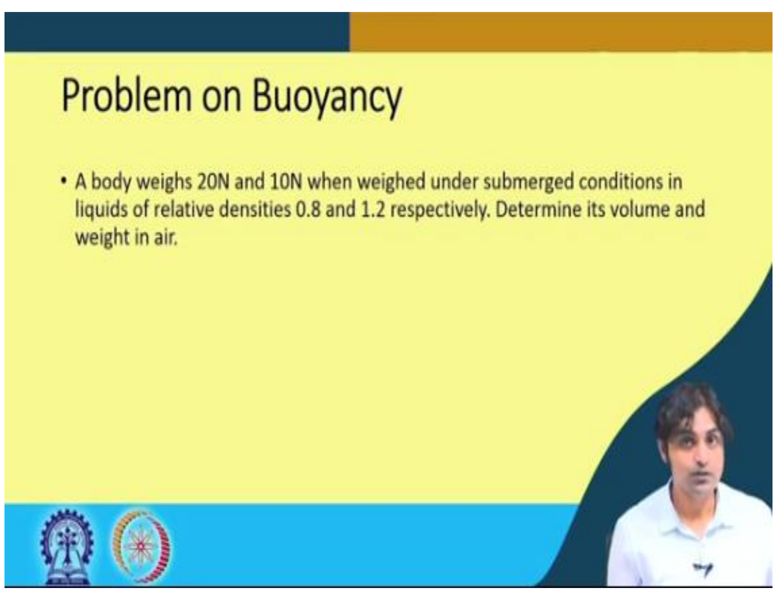

The problem says, the body weighs 20 Newton and 10 Newton when weighed under submerged conditions in the liquid of relative densities 0.8 and 1.2 respectively. Determine its volume and weight in the air. So, yeah, what I am going to do again a screen it is very simple problem, but I think it will give you a thorough brush up. Let, I have not selected the pen. Now, I will select the screen, white screen.

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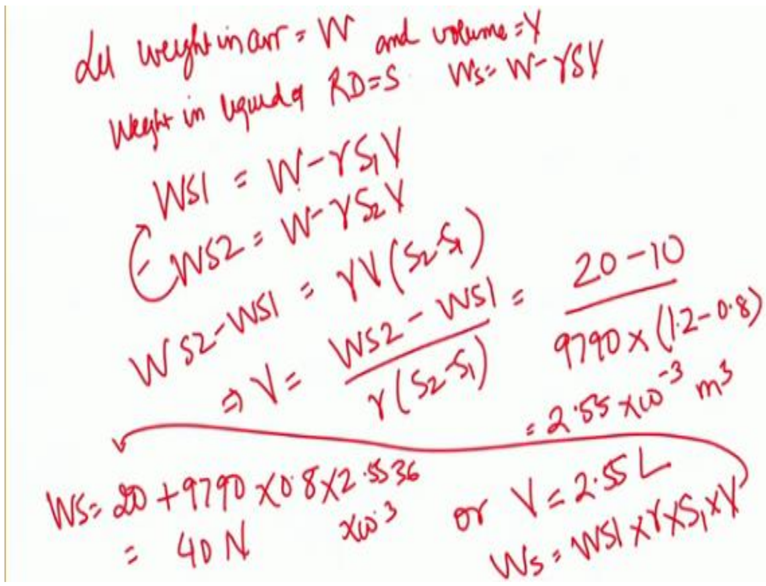

So, let weight in air be W and volume is equal to V. Weight in liquid of relative density is equal to S be W s is equal to W - ϒ S V. So, the Ws1 is going to be W - ϒ S1 into V, WS2 is going to be W - ϒ S2 into V. So, what we do is we simply subtract first from the second. So, it will be WS2 - WS1 is equal to ϒ V S2 - S1, simply, this – this. and Therefore, we can get V is equal to W S2 - WS1 divided by ϒ S2 - S1 and we can simply substitute the values 20 - 10 in our case, ϒ we already know 9790 into S2 - S1 is 1.2 - 0.8 and this comes out to be 2.55 into 10 to the power - 3 meter cube or the volume in liters can be written as 2.55 liters and WS can be written as WS1 into ϒ into S1 into V. So, I will take this calculation here. So WS can be written as 20 you know + 9790 into 0.8 into 2.5536 into 10 to the power - e actually, we are using this equation here. So, it should be W S is equal to W WS weight in air W S. So, it will be W yeah, so 20 will add actually and this will come out to be 40 Newton. So, yeah, that WS1 we have already found out because it was 20.

So, we are using this equation, so this is not very true. So to calculate weight we are using this equation actually, fine. So this solves our question number last. So, with this we conclude the week one where we have studied the basics of fluid mechanics one this week. Next week we will be exploring fluid kinematics, fluid dynamics and the Bernoulli's equation and we will conclude the basics of fluid mechanics one that would be required for studying the other important chapters of hydraulic engineering. So, this is all for this week. Thank you so much. See you next week. Bye.