Fluid Mechanics
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Lec 24: Mass Conservation Equation- I

Very good morning all of you. Today we are going to discuss on differential analysis of fluid flow which is very interesting chapters in the book of Senjal Chembala and also the F. M. White book which is the foundations of the computational fluid dynamics and So, considering that let us start the differential analysis of the fluid flow. This concept is used for developing computational fluid dynamics algorithms. Looking that let us as we discuss about integral approach in the previous classes which we generally use as a control volume.

We use as a control volume considers there is a dicks is mountain over a decks and we have the control volumes to just to estimate how much of force is acting on these dicks. So, if you look at these problems, we consider a control volumes and we try to look at the velocity components, what could be expected inflow, outflow, also the outflow here and what could be the velocity vectors and based on that we apply the mass considerations and the momentum equations to estimate the force, the gross force acting on this. So acting on this, Dix fires. So if you look at that, when you do these type of things, the interior part in the control volumes, we do not know anything about this.

That means the interior part of this control volume we consider as a black box. That means we do not have any knowledge on the interior part of this control volume. This is what the integral approach to consider a control volume. Apply this mass conservation and momentum equations to get the force component of this. In this case, as I again I am to repeat it to tell it, in this case we do not know the interior within this control volume, the velocity field, the velocity field.

or the pressure field. So that is the reasons we call it the within the control volume it is just a black bus. We do not have any knowledge on the velocity and the pressure field. But if you look at the same problems if I go for a next levels where each point within the flow domains that is what is the flow domains. If I want to know it what will be the pressures and what will be the velocity field that is the differential approach.

That is what is the differential approach. That means instead of having so big control volumes to know the gross characteristic of the force component or the mass conservation properties we are looking it that within the flow domains if I divide into the a number of points for each points I should know it what is the pressure and the velocity and what is the density variations. If I know that that is what is the differential analysis components. This is the integral analysis component where we consider a gross control volumes to obtain the force components by applying mass and the linear momentum conservation equation. How we want to get it? The pressure fields, the velocity field and the density field for these Dix problems.

The basic idea comes here is that the my control volume which is have the dimensions of like if it is the x this is the y and this is the z coordinate. If this is the x y and the z coordinates and this is the dimensions let we consider it is dx dy and dz that is my very simplified a parallel fight control volumes having a dimensions of dx dy dz. If my dx is tending towards 0. The dy tendings towards 0 and the dz tending towards 0. That means what I am looking at the my control volume tendings towards a infinitely small the control volumes converging towards a point value.

If that is the conditions that is the conditions that means dx dy dz tending towards a point. Then I can write the differential equations for analysis of this mass considerations and the linear momentum equations will give me differential equations format and that differential equation format can help me to obtain this the pressure field the velocity field and the density field throughout this domain. That is what is the difference between the integral approach and the differential analysis. So looking that the basic concept is that the for given control volumes the dimensions of the control volumes we have been reducing it. d x d y d z all these dimensions converging towards a point very infinitely small.

So if you do that that if you reduce your control volumes to a smaller contour volume as smaller as that it is close to a points. So, in that case we get a set of a partial differential equations that is what we will going to derive it set of the partial differential equations that is the partial differential equations differential equations. for mass and linear momentum. So, if I write this partial differential equations for mass and linear momentum, the basically I will get the four basic equations. 4 basic partial differential equations.

So, we will get 4 partial differential equations from that one is mass conservation equations and the 3 equations which is a vector forms of linear momentum okay. So basically we will get four partial differential equations which will be the one will come from mass conservation equations, three vector form equations of linear momentum equations with a three directions in Cartesian coordinates is x, y, z that four equations, but these four equations are coupled equations. That means it has the four dependent variables like the density, the velocity components are interlinked within these four basic equations. So solving these equations we can get the value of density and three basic scalar component of velocity as well as the pressure field. So basic idea is ours to derive the mass conservation equations.

So the basic concept and this partial differential equations part that is what I have given in this introduction okay. Let us come for the mass conservation equations for a control volume which is infinitely small. So when you go for that let us as we discuss about the Reynolds transport theorems in previous classes we will apply this Reynolds transport theorems here but only our control volume is infinitely small. So considering that if I put it for a control volumes which is a infinitely small okay which is infinitely small and for that case if I substitutes the extensive properties b is equal to m and the beta is equal to 1 that is what we derived it in previous classes. If I consider that the basic equations what I will get it that is very the conservation of mass for infinitely small control volume that does not matter it for us but if you look at the Reynolds transport theorems then I will get these equations which is as you know this part what we will get it that.

So if you look at that if I write from this Reynolds transport theorems for these control volumes I will have a the the partial derivative of rho with respect to t and the control volumes integrals the volume integrals of this control volumes plus I will have the integral the surface integrals rho v dot and dA okay. So this is from the Reynolds transport theorems we can apply the b equal to m and beta equal to 1 we will get the one the volumetricals which is the having the partial derivative of density with respect to time this is the dV the volumetricals part and here I will have this the dot products of velocity and this normal vectors rho d a as a surface integral this part. But if I go for a simple problems like I have the inflow if I consider there is a mass inflow is coming in mass flow outflow is going out. So, this is the inflow and this is what my domain. This is the reasons I do not have a no mass flow.

So if I have a there is a part of the control surface mass inflow is coming into this control volume and it is going out. See if I follow that so this surface integral part we can simplified it and to write as you can write as very simple way that the control volumes d v is equal to sum of mass flux in minus sum of mass flux out. So, if you look at this part which is very simplifications part for a control volumes that rate of change of the mass within the control volume. The how much of mass is changing within the control volume that is supposed to equal to rate at which mass inflows into this control volume the minus the rate of the mass outflow from the control valve. So this is very basic equations of mass conservations.

This is the part is talking about or indicating us the change of the mass within the control valve. This is the talking about the mass flux which is going into these control volumes. This is the mass flux talking about how much of mass flux going out from this. So the minus of that too should be have a change in the storage part. So if you look at this way from the Reynolds transport theorems we can write a very simple way the mass conservation equations in terms of the density and mass inflows going in and coming out.

But before going for more derivations I just want to highlight it that if you look at very basic equations that the mass flux which is a mass per unit time that is what we define it as kg per second. So kg per second how much of mass flux is coming it is kg per second that what will be rho times of q. So rho is a density which is mass per unit volume and you have the volumetric volume rate which is a discharge is volume per unit time that is what will give us mass per time. So if you look at these equations, which are very simple equations, that means we can get the mass flux. If I know the discharge, if I know the density, I can know this mass flux is coming in or going out.

But when we are considering infinitely small control volumes, we are looking at the mass flux rho Q as rho A into V. Or I am looking at the max flux per unit area, the surface area is equal to rho into v. This is what we have to remember it when you derive this complex non-linear equations of mass conservation and momentum conservation equations. Again I am just to revise is that if you look at this mass conservation equations if I write in terms of Reynolds transport theorems which where I use The capital B extensive property is mass M.

The beta will be the 1. If I substitute on this Reynolds transport theorems, anyway you know it, this part, the system part will be the 0. But there will be a control volume part which will have this change of the mass storage within this control volume. And the net the mass flux is going through this control surface through this control force net. Net means we are looking at the minus of the mass influx going out and in that is what will give us very simple form on these ones. But if I look it this mass flux which is a kg per second which is rho times of q that what if I rearrange it and if I write it the mass flux per unit surface area that is what will have a rho times of the v.

That is the rho times of v that is what we will use so frequently for deriving the mass conservation equations. Let us come for this deriving the mass conservation equations. The two ways we can derive it. One is a GUS theorem or divergence theorem concept. Another one as you know it is as a considering a infinitely small control volume.

The first part let us come for deriving mass conservation equations for infinitely small control volumes. So, looking at infinite small control volumes, so we can derive this mass conservation equations two ways. One is the divergence theorems and to apply the divergence theorems, we are again going back to the Reynolds transport theorems which establish the relationship between the mass the change in the mass in the control volumes and the net outflux of the mass going through this control surface. The same concept will take it to derive the mass conservations based on the Gauss theorems. This theorem is very interesting theorem as not used only this the fluid flow also the magnetic flow and all.

So let us look at the what is the the divergence between the two vectors. And mostly as we are using the fluid mechanics, we are looking at divergence field of velocity field. What do we understand on that? Let us consider these figures. So there is a flow where I have the velocity component is low here. As it is going, the velocity vectors are increasing it.

So if you look at this part and if you look at how the velocity divergence changes it, that is quite interesting when you talk about the divergence concept. Like if I have the velocity, this is a two-dimensional flow, so I can have a velocity scalar component u and the small v will have a two scalar components okay. For these velocity vectors of v I have a two scalar component of u and the v and if I look at this part and you know the del vector is del by del x i del by del y j for two dimensional case okay. So for the two dimensional case we know. So we are looking the divergence vector of this.

Now if you look it for this case 1 for the case 1 where we have the smaller velocity as go up I have the higher velocity. If I look it this component del u by del x will be the positive del v by del y will be the positive this v is a small v. So if I have that then your divergence factors of these will be the positive value. So it is showing it the flow is outwards okay flow is the outwards and there is a presence of the source there is a presence of the source if I get the divergence of velocity vectors is positive value that is indicating that way. But if I come for this case okay where I have a sink where the velocity is more as coming to the closures the velocity reductions happens it.

So in that case del u by del x is less than 0 and del v by del y less than 0 and if I do the velocity divergence that is what will be the negative value. That is what will be the negative value. Let us interpret the physically what do we mean by the divergence of the velocity vectors. If it is a positive value it indicates the source and flow is outwards. If it is a negative value it indicates the flow is inward as well as there would be a sink points.

but the special case like this where you have your del y del x is 0 del v by del y is equal to 0 and you have this component is also 0. So that is what is quite the flow like this. So by understanding this divergence concept for the case 1 which is will have a flow divergence is positive indicates outward flow having a source The divergence negative indicates us for there would be a sink flow is inwards and you can have a parallel flow type of things where the flow divergence indicates us the 0 value. The same concept we are going to apply for a control volumes where the mass flux is coming in and the going out.

mass flow is coming in and going out. Here we are instead of g vectors we are looking at the components of the velocity components. Let us come back to the Gauss theorems. What it does it says it is okay. This is a conservation of mass conservation equations that it helps us to transform a volume integral of a divergence of a vector. That is what we are talking about, the divergence of the velocity field into a surface integral over a surface which defines the volume.

That is what is the basic definition of Gauss theorem. it says that we can transform a volume integrals of divergence of a vector into a surface integrals on the surface which defines the volume. Mathematically if I write it this form I will get it this part that if there is a volume integrals this is volume integrals of a divergence of g vectors, g vectors into the d v that is what is equal to surface integrals, surface integrals of g vector into n d a. So, that is what is the Gauss theorems. So, this is what the Gauss theorems talking about that if I have the volume integrals of a divergence of a vectors.

G is a vectors what I am looking at the divergence of vectors over the volume of these control volumes. What I will get it? It establish a relationship between volume integrals and the surface integrals where I can get it g dot n. This is the surface integrals of dot product of g and n and you will have the dA, the surface integrals part. Now, if you look at these equations, equations 2 and this is what the equations 1, which is from the Reynolds transport theorems. Now, interestingly, what I am looking at, I am not looking for the change this control volume part, which is, anyway, I don't have much problems.

I just want to change this control surface part into this control volume part. Instead of g, I have to use the p values. So I have to change from equations 1 if we can change we can change surface integral part surface integral part then we can write it very interestingly the control volumes rho rho t d v plus this is what I will be talking to these control volumes levels. So if I apply that I will get it as del rho v del v equal to 0.

So just repeat it that way. as we understand from the Gauss theorems which is a divergence theorems that we can establish the volume integral of the divergence of a vectors to a surface integrals area integral of the surface. The same concept I have just I have used it from basic Reynolds transport equations to derive this part over this control volumes that is what we can look it because this is the common same derivations I can write it this form is. So, we can write it these ones in very compact forms so if i look at this part see if some things if your integrals that is what is becomes a zero okay if i doing a volume integrals of something is equal to zero that means this component is supposed to be zero so that means if i can write it the finally the mass conservation secretions from Gauss diversals theorem it comes out to be very compact form is del this is the del operators v is equal to 0. This is what mass conservation equations. mass conservations, the equations which is three-dimensional forms, it is the three-dimensionals and these conservations equations is applicable for compressible, incompressibles.

So, all sort of the flow these conservations equations is applicable which we derived from the divergence theorems in very compact forms in terms of the two terms are there here is a density and the velocity vectors. So, this is what the mass conservation equations derived by using divergent theorem. Bye.