Hydraulic Engineering
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Lecture- 15
Laminar and Turbulent Flow (Contd.)

Welcome back to this lecture of laminar and turbulent flow. We have left the last lecture before introducing the topic of shear stresses in turbulent flow.

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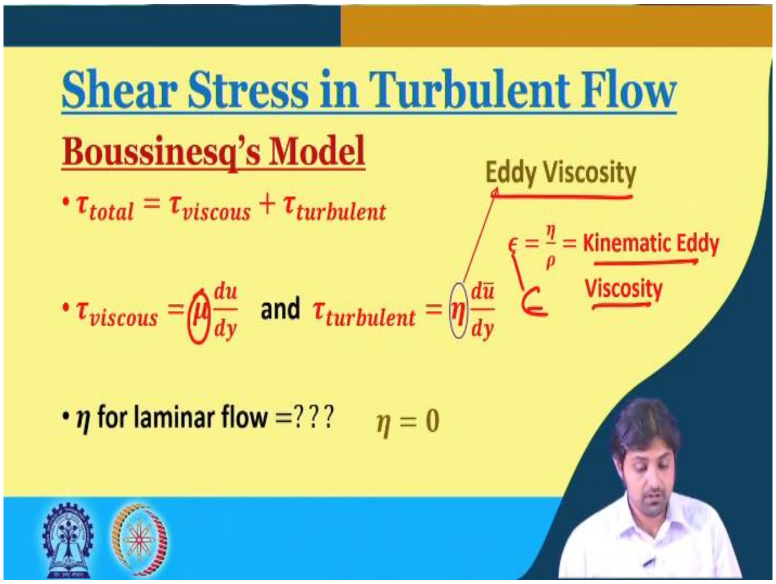

So, we are going to continue with this particular topic. So, shear stress in turbulent flow. We are going to talk about a model that is called Boussinesq's model, where the total shear stress, in case of laminar flow it was due to the viscosity viscous. Sorry. Yeah, that was only due to the viscous. But in a turbulent flow, there is an additional component of shear stress that happens because of the turbulence in the flow.

So, therefore, the shear stress in total is much, much larger than the viscous flow. At least it is definitely larger than the viscous flow because there is shear stress that is associated with turbulence too. So, Boussinesq's says as

$$\tau_{viscous} = \mu \frac{du}{dy}$$

for laminar flow. Therefore, the shear stress due to do the turbulence component is

$$\tau_{turbulent} = \eta \frac{d\bar{u}}{dy}$$

. Here, you see, this is similar. So, instead of μ there is something called η a new coefficient of viscosity, and this is called eddy viscosity.

So, we are not going to the derivation right now, at some point we can see these derivations when appropriate chapter comes. But now you have to take that the shear stress due to turbulence is eddy viscosity du / dy, very similar to the shear stress in the laminar flow. The coefficient is therefore different. And if we want to write a kinematic eddy viscosity then we write it by epsilon, for example. So, this can be written as eta / rho, similar type of definition as laminar flow.

What is eta for laminar flow, for example, or mu for laminar flow? That you already know, we have been doing that, it was 10 to the power - 3 Pascal second, No okay. So, eta for laminar flow will be 0.Yes. So, but mu for this laminar flow is 10 to the power - 3 Pascal second and if the flow is laminar eta is going to be 0 because it is related to the turbulent viscosity.

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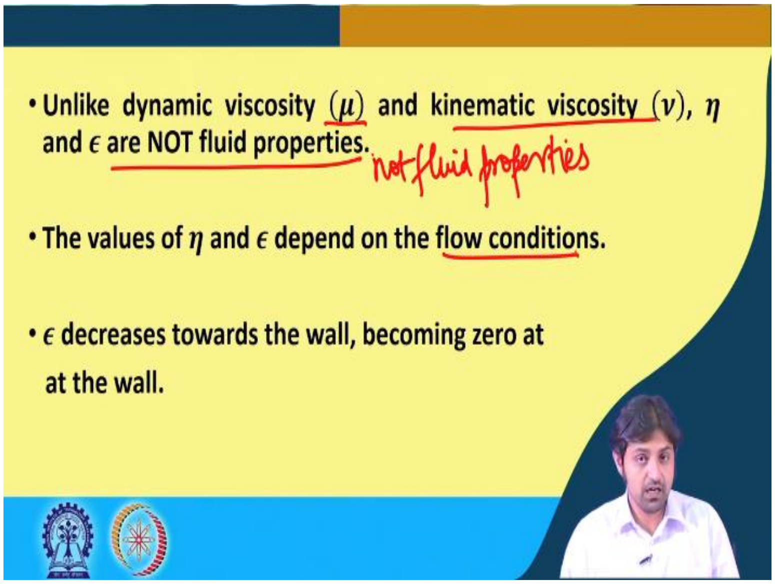

So, unlike the dynamic viscosity mu and kinematic viscosity nu, eta and epsilon are not fluid properties, they are not fluid properties. The values of eta and epsilon are dependent on the flow conditions. So, epsilon decreases towards the wall becoming 0 at the wall. So, the epsilon, that is, the eddy, kinematic eddy viscosity decreases as you move towards the wall and becomes 0 at the wall.

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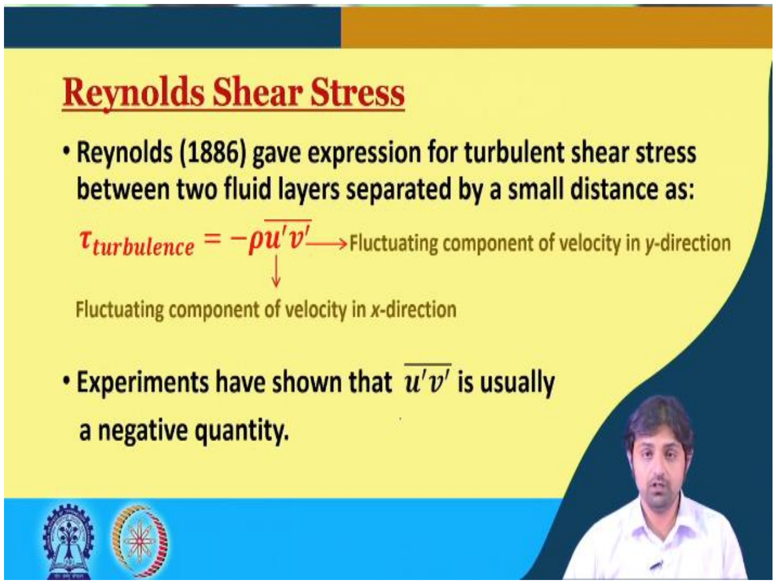

Now, coming to what is Reynolds shear stress. So, Reynolds in 1886 gave expressions for turbulent shear stress between two fluid layers separated by a small distance. And he said that the shear stress due to turbulence can be written as, minus rho u prime v prime whole bar. Actually, it is not an assumption, but this can actually be derived, which we will do at some point in this hydraulic engineering course, but not now.

So, you have to understand, Reynolds shear stress is given by

$$\tau_{turbulence} = -\rho\overline{u'v'}$$

and it does not have only one component, it has minus u dash w dash, it will have minus v dash w dash. So, there are different, there are some normal shear stress, but this is one of the shear stress component. Whereas, what is u prime? That is the fluctuating velocity component in x direction, v prime is fluctuating component of velocity in y direction. Experiments show that u prime v prime is usually a negative quantity. Therefore, the tau turbulence or minus rho u prime v prime whole bar is total positive quantity, it has negative correlation that we will see.

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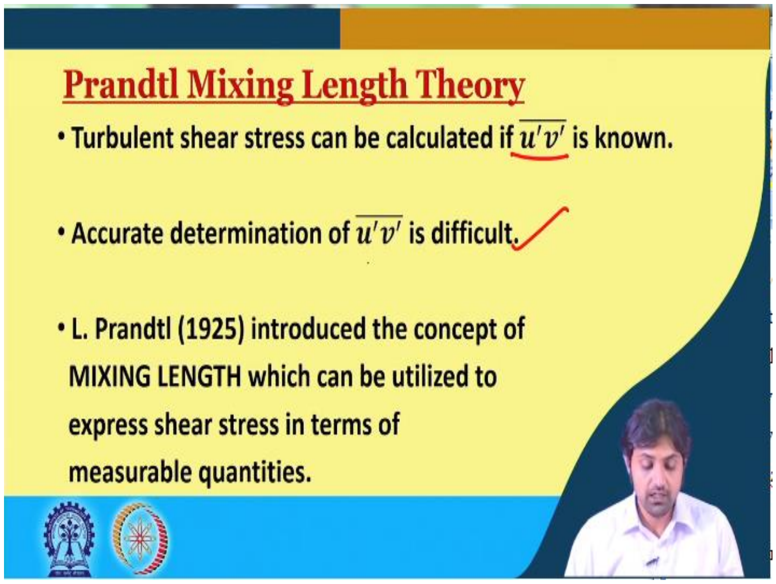

Now, there is a concept of Prandtls mixing length theory. So, turbulence shear stress can be calculated if this thing is known, u prime v prime whole bar is known. Because as we see, the tau turbulence by Reynolds was given by minus rho u prime v prime whole bar. So, what a nice thing it would be if we can calculate u prime v prime bar because that is unknown until now. So, accurate determination of u prime v prime whole bar is very difficult. Therefore, in 1925 Prandtls introduced the concept of mixing length, which can be utilized to express the shear stress here, in terms of some measurable quantity.

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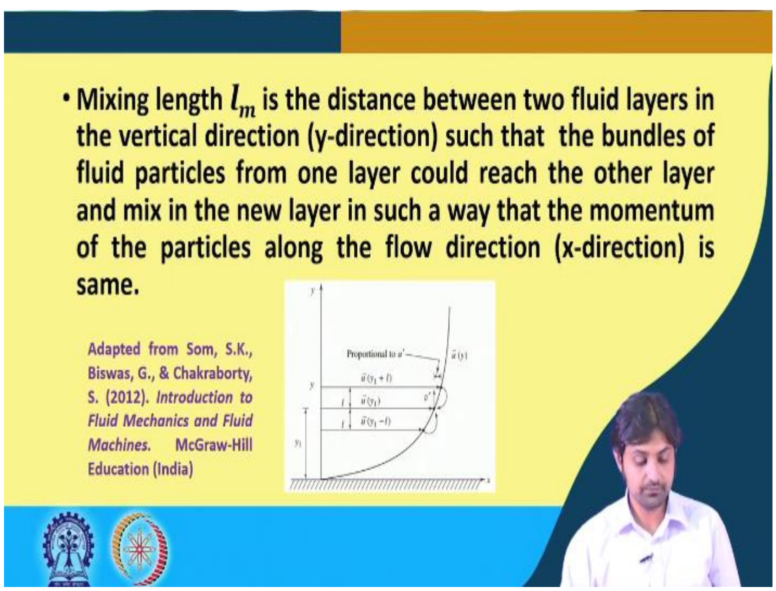

So, mixing length lm, he said it can be described in terms of mixing length l m. He said mixing length l m is the distance between 2 fluid layers in the vertical direction, in the y direction, such that, the bundles of fluid particles from one layer could reach the other layer and mix in the new layer in such a way that the momentum of the particle along the flow direction is the same. So, he related it to mixing.

And he said that the mixing length is the distance between 2 fluid layers in the vertical direction, such that, the bundles of fluid particles from one layer could reach the other layer and the mixing can happen, something like this. So, this is the velocity profile and he says he divided the fluids in 2 layers and this he says is the mixing length. We are going to explore this in more detail in the next slide.

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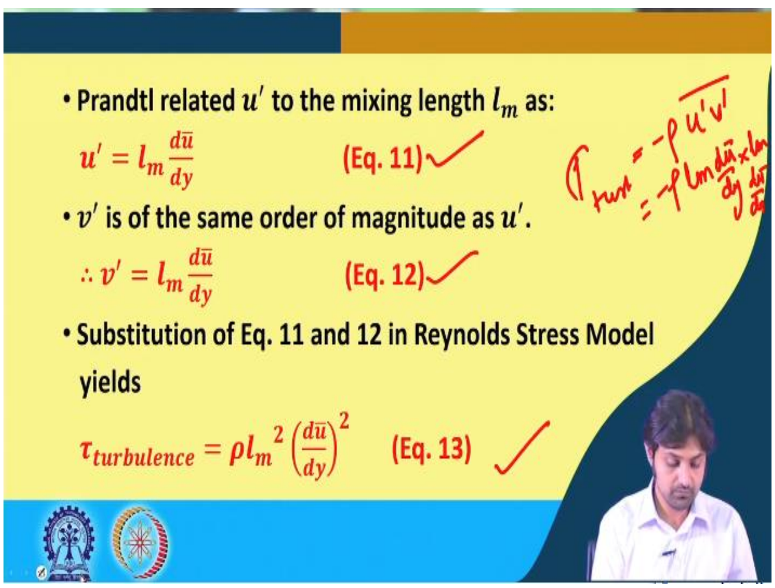

So, Prandtl related u prime to mixing length lm. He said that this u prime, as you can see in the figure here, he said proportional to u prime but I am going to write it in the next slide. He related u prime to the mixing length lm as, he said this u prime can be written as, mixing length lm multiplied by the gradient of the average velocity. So, he said u prime, this is very important to know, lm as the du bar / dy.

This is what his assumption was, where v prime is also of the same order of magnitude as u bar and similarly, this can also be written as v prime is equal to lm du bar / dy, similar type of equation. And if you substitute equation 11 and equation 12 in Reynolds stress model, which was it was tau turbulent is equal to minus u prime v prime whole bar you get, so, minus and minus will become positive it will become tau turbulence is

$$\tau_{turbulence} = \rho {l_m}^2 \left(\frac{d\bar{u}}{dy}\right)^2$$

You can just substitute and see, l m du bar / dy multiplied by l m du / dy bar, it is the same thing. So, it becomes rho l m square du bar by dy, this is equation 13. So, this is one of the, this is the mixing length theory of the Prandtl.

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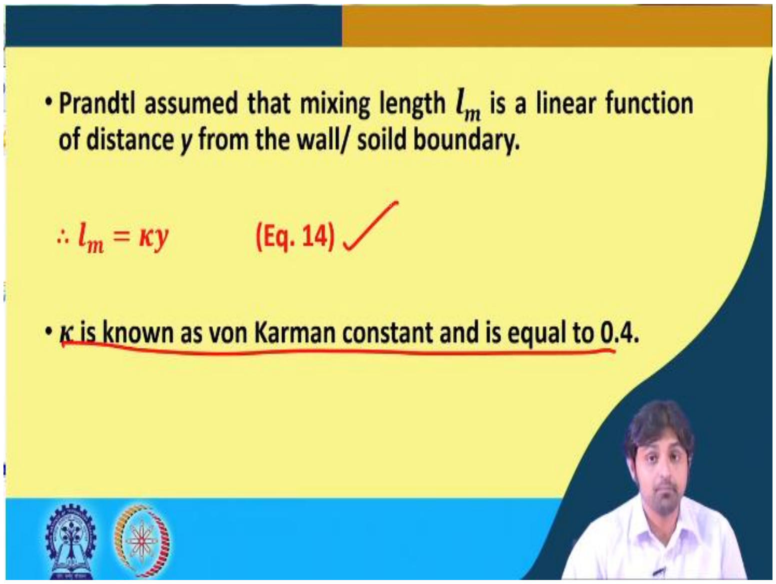

So, Prandtl also assumed that the mixing length lm is a linear function of distance y from the wall or any solid boundary. Therefore, he said l m can be written as ky. So, if you look go back and see here now rho is known, du / dy can be calculated because we are dealing in terms of average velocity, which can be measured. Now the only unknown is l m. So, how do we find this l m now? So, the problem is becoming less and less complex we are going from one variable to the other. Now, the only unknown thing is l m.

So, Prandtl needed to relate this to something. So, he assumed that lm is a linear function of distance y and he said l m is equal to kappa into y, where kappa is known as von Karman constant and it has been found to be equal to 0.4. So, he said mixing length is 0.4 times y, this is quite an important result. So, now, we know everything, for example. Shear stress in turbulent flow was related in terms of u prime v prime which was made by Prandtl as, u prime and v prime or of the same order of magnitude and is equal to, which was equal to, which was proportional to du dy l m and l m is equal to kappa into y.

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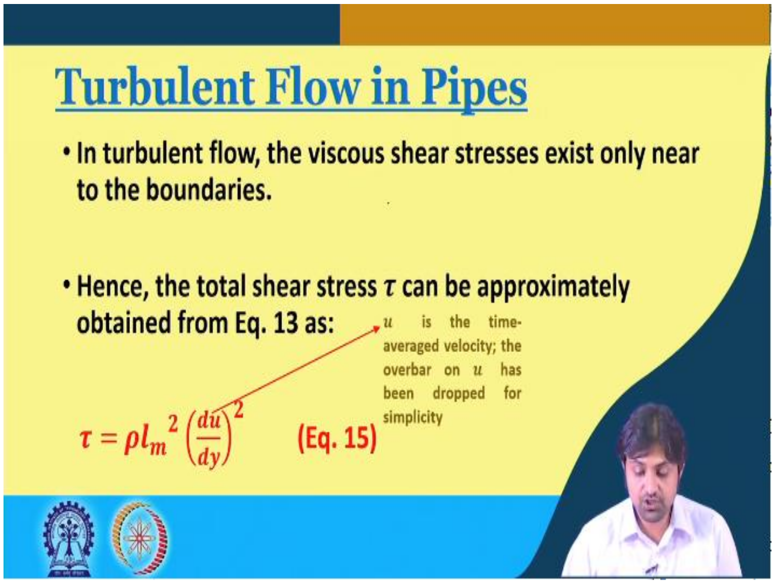

So, now we will see the turbulent flowing pipes now. So, in turbulent flow the viscous shear stresses exists only near the boundary and most of the region is dominated by the turbulence. So, near the boundary the viscous shear stress will act and that are the only places where its existence is. Hence, the total shear stress can be approximately obtained from equation 13 as the total, you know, this was so, this was equation 13.

So, we call this now equation 15 but because most of the shear stress in turbulent flow is due to the turbulent shear stress. So, we can neglect the viscous shear stress. We say tau is equal to rho lm square du / dy whole square. Where, u is the time-averaged velocity, the over bar on u has been dropped for just for simplicity.

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