


In which case

$$\eta = a \cos(kx - \sigma t) \quad (2.45)$$

$$u = \frac{agk}{\sigma} \frac{\cosh k(d+z)}{\cosh kd} \cos(kx - \sigma t) \quad (2.46)$$

$$a_x = agk \frac{\cosh k(d+z)}{\cosh kd} \sin(kx - \sigma t) \quad (2.47)$$

$$w = \frac{ag}{\sigma} k \frac{\sinh k(d+z)}{\cosh kd} \sin(kx - \sigma t) \quad (2.48)$$

$$a_w = -agk \frac{\sinh k(d+z)}{\cosh kd} \cos(kx - \sigma t) \quad (2.49)$$


So, in this case eta will be a cos k x - sigma t, the u will change u and a x and w and a w, so, if we assume a different velocity potential you remember we had to velocity potentials. So, we started doing all the calculation with the first velocity potential but instead of the first day we started with the second we will obtain this set of the wave kinematic parameters.

(Refer Slide Time: 20:36)

WATER PARTICLE DISPLACEMENT UNDER PROGRESSIVE WAVE


- The expression for individual horizontal and vertical water particle displacements is Obtained as follows.

$$\delta_x = \int u dt = \frac{H}{2} \frac{\cosh k(d+z)}{\sinh kd} \cos(kx - \sigma t) \quad (2.50)$$

$$\delta_z = \int w dt = \frac{H}{2} \frac{\sinh k(d+z)}{\sinh kd} \sin(kx - \sigma t) \quad (2.51)$$

Let $\delta_x = D \cos(kx - \sigma t)$ where $D = \frac{H}{2} \frac{\cosh k(d+z)}{\sinh kd}$

Handwritten notes:
 $\frac{F_x}{D} = \cos(kx - \sigma t)$
 $\frac{F_z}{D} = \sin(kx - \sigma t)$



So we have studied the velocity potential, we found out the velocities under the progressive where we have found out the acceleration now importantly we have to find out water particle displacement is nothing but integral u times g T w times d T in extend that direction respectively. So, the expression of individual horizontal and vertical particle displacement is integral u dt u we already know in terms of h before.

So, the final results comes out to be $\frac{h}{2} \cos kx + z \sin kx$ into $\cos kx - \sigma t$. Similarly, the vertical displacement δz is given by $\frac{h}{2} \sin kx + z \sin kx$ into $\sin kx - \sigma t$ these are the periodic terms. So, once you derive the velocity potential everything can be found out. Now, if we say $\delta x = \frac{h}{2} \cos kx - \sigma t$ and we said D is $\frac{h}{2} \cos kx + z \sin kx$ and similarly.

(Refer Slide Time: 22:00)

$\delta_z = B \sin(kx - \sigma t)$ where $B = \frac{H}{2} \frac{\sinh k(d+z)}{\sinh kd}$

$$\cos^2(kx - \sigma t) = \left(\frac{\delta_x}{D}\right)^2 : \sin^2(kx - \sigma t) = \left(\frac{\delta_z}{B}\right)^2$$

Since, $[\cos^2(kx - \sigma t) + \sin^2(kx - \sigma t) = 1]$, we have

$$\left(\frac{\delta_x}{D}\right)^2 + \left(\frac{\delta_z}{B}\right)^2 = 1 \quad (2.52)$$

Equation of water particle displacement

This equation of an ellipse showing that the water particles moves in an elliptical orbit.

Where, D = Semi major axis (horizontal measure of particle displacement)

B = Semi minor axis (vertical measure of particle displacement)

We assume δz is $B \sin kx - \sigma t$ where B is a different quantity called $\frac{h}{2} \sin kx + z$ divided by $\sin kx$ therefore, we can write $\cos^2 kx - \sigma t$ is δx by D whole squared, let us go back. So, what we have said is this is D and this is B . So, if we take D down so, it will become δx by $D = \cos kx - \sigma t$ and if we take D down here it becomes δz by $D = \sin kx - \sigma t$. So, $\sin^2 \theta + \cos^2 \theta = 1$ therefore, this is what we have used.

So, what we write is δx by D whole squared + δz by B whole squared = 1, this is in general what type of equation if D is not equal to be elliptical. So, this is an equation of analysts showing that water particle moves in in an elliptical orbit. So, we have proved particle moved in an elliptical orbit. Here D is the semi major axis at the horizontal measure of the particle displacement and B is these semi minor axis that is the vertical measure of the particle displacement. So the vertical the particle displacement this is an important equation.

(Refer Slide Time: 24:28)


➤ Shallow water Condition:
For $\frac{d}{L} < \frac{1}{20}$ we have used $\cosh k(d+z)$ and $\sinh k(d+z)$

$$\begin{aligned} \sinh k(d+z) &\longrightarrow k(d+z) \\ \sinh kd &\longrightarrow kd \end{aligned}$$

Hence, $D = \frac{H}{2} \cdot \frac{1}{kd}$

$$B = \frac{H}{2} \cdot \frac{k(d+z)}{kd} = \frac{H}{2} \cdot \frac{(d+z)}{d}$$

- Hence, the water particles move in elliptical orbits (paths) in shallow and intermediate waters with the equation of the form



Now we analyze this displacement in shallow water. So what happens in shallow water for d by L less than 1 by 20 we have 0.05 we have used $\cos hkd + z$ and $\sin hkd + 2 \sin hkd + z$ goes to $k d + z$ and $\sin hkd$ goes to kd . Hence, D will be what if we as you remember D was this equation D was h by $2 \cos hkd + z$ and $\sin hkd$ in shallow water this becomes $kd + z$ divided by kd . So this capital D becomes h by 2 into 1 by kd and B becomes h by 2 $k d + z$ by kd hence the water particle in shallow water moved in analytical orbit in shallow and intermediate waters with the equation of the form.

(Refer Slide Time: 25:36)

$$\left(\frac{\delta_x}{\frac{H}{2} \frac{1}{kd}} \right)^2 + \left(\frac{\delta_z}{\frac{H}{2} \frac{(d+z)}{d}} \right)^2 = 1 \longrightarrow (2.53)$$


➤ Deep water condition:

For the case $\frac{d}{L} > \frac{1}{2}$

$$D = \frac{H}{2} \frac{\cosh k(d+z)}{\sinh kd} = \frac{H}{2} \left(\frac{e^{k(d+z)} + e^{-k(d+z)}}{e^{kd} - e^{-kd}} \right)$$

As 'd' (depth of water of d/L) is very large $e^{-k(d+z)}$ and e^{-kd} will be very small compared to $e^{k(d+z)}$

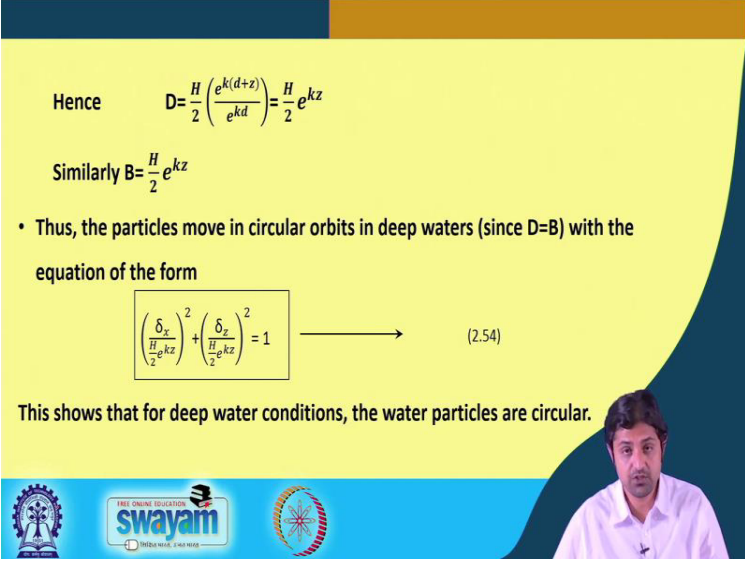
deep



If discuss just simply substituted in shallow and intermediate shallow and intermediate waters with this D and B but in deep water for the case d by L greater than half D becomes h by 2 e to

the power $k d + z + e$ to the power $-k d + z$ divided by e to the power $k d - e$ to the power $-k d$ as D is very large e to the power $-k d + z$ and e to the power $-k d$ will be very small correct because this deep now, so, if it $-\sin$ they will be very small compared to e to the power $k d + z$. So, this can be taken away.

(Refer Slide Time: 26:25)



Hence
$$D = \frac{H}{2} \left(\frac{e^{k(d+z)}}{e^{kd}} \right) = \frac{H}{2} e^{kz}$$

Similarly $B = \frac{H}{2} e^{kz}$

- Thus, the particles move in circular orbits in deep waters (since $D=B$) with the equation of the form

$$\left(\frac{\delta_x}{\frac{H}{2} e^{kz}} \right)^2 + \left(\frac{\delta_z}{\frac{H}{2} e^{kz}} \right)^2 = 1 \quad \longrightarrow \quad (2.54)$$

This shows that for deep water conditions, the water particles are circular.

And therefore, we can write h by $2 e$ to the power $k d + z$ divided by e to the power $k d$ and simply h by $2 e$ to the power $k z$. This is important. Similarly, if we do the same process with B , we will again get h by $2 e$ to the power $k z$ because deep water that is going to happen again. Therefore, you see, both B and D are same. So, in an ellipse if both the major and minor x is the same, it is nothing but a circle.

Therefore, the particles move in circular orbits in deep water since these equal to be with the equation of the form δx divided by h by $2 e$ to the power $k z + \delta z$ h by $2 e$ to the power $k z$ whole squared $= 1$. So, see these things are very important these are the type of questions which you can expect what type of orbit in deep water then you have to take mark the correct 1 as a circular orbit elliptical orbit. This shows that for deep water condition the water particle trajectories are circular.

(Refer Slide Time: 28:08)

- The amplitude of the water particle displacement decreases exponentially along with the depth. The water particle displacements becomes small relative to the wave height at a depth equal to one half the wave length below the SWL. The variation of the water particle displacements under different depth conditions is illustrated in Fig 2.6

Fig. 2.6 Schematic representation of fluid particle trajectories

So, you see, this is the representation the schematic representation of fluid particle trajectories the amplitude of the water particle displacement it first of all it decreases exponentially along the water depth the water particle displacement becomes small relative to the wave height at a depth equal to one half the wavelength below this still water level that is the deep water the variation of the water particle that displacement under the depth condition is illustrated in this.

(Refer Slide Time: 28:55)

Solution to the Dispersion equation

- An approximate solution for wave number k in the dispersion relationship given by eq. (2.29)
- For a given σ and d proposed by Hunt (1979) can be solved directly for kd .

$$(kd)^2 = y^2 + \frac{y}{1 + \sum_{n=1}^6 d_n y^n} \quad (2.55)$$

Where $y = \frac{\sigma^2 d}{g} = k_0 d$ and

$d_1 = 0.666666666$	$d_2 = 0.35555555$	$d_3 = 0.160846508$
$d_4 = 0.0632098765$	$d_5 = 0.0217540484$	$d_6 = 0.0065407983$

The celerity can be obtained as

$$\frac{c^2}{gd} = \left[y + (1 + 0.6522y + 0.4622y^2 + 0.0864y^4 + 0.0675y^5)^{-1} \right]^{-1} \quad (2.56)$$

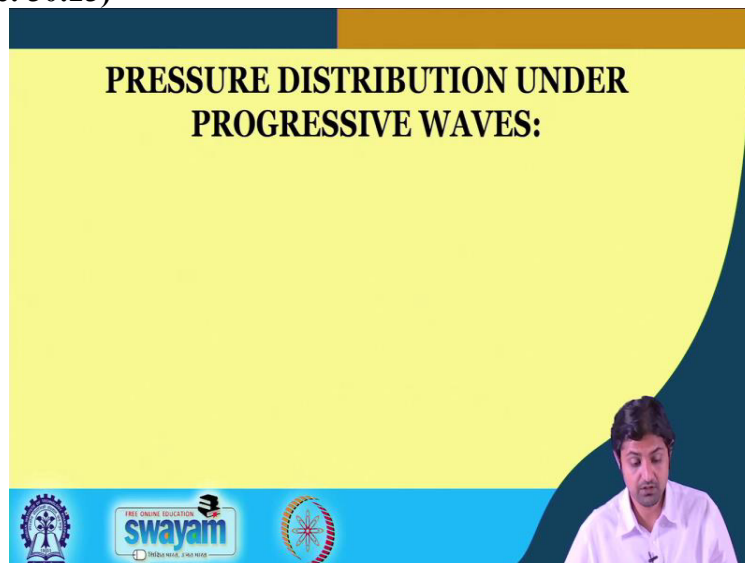
Which is accurate to 0.1% for $0 < y < \infty$

So, there is 1 last thing before the end of this lecture, I have included in the slide but does not have any significance for this particular course. You see the dispersion relationship, I said there are 2 solutions. One is either using the wave tables and the second is trial and error. But some

scientists like Hunt he came up with the direct solution for $k d$. He said $k d^2 = y^2 + y$ divided by $1 + n$ goes from 1 to 6 $\ln Y_n$.

And this final equation can be used in MATLAB or Excel to determine for different y so if you can put it on a computer code using MATLAB or Excel, you the you There is no need for you know, iteration or any other method. So this is just for information purpose. You do not need do not need to remember maybe the name you know the if somebody is interested in doing further research or going for a master's program or you know, something related to research they can use this equation as such. So, with this actually this equation is accurate for 0.1% for y going from 0 to infinity.

(Refer Slide Time: 30:23)



So, I think this is a fine point to stop. In the next lecture, we will conclude this module of invested flow that is wave mechanics and we start with the pressure distribution and the progressive waves from the next lecture and finish this module. And this will also be the end of the course, but I will talk about it in the last lecture of this course called hydraulic engineering. Thank you so much for listening. See you in the next lecture.