Chapter 24

REINFORCED CONCRETE BEAMS; Part I

24.1 Introduction

1 Recalling that concrete has a tensile strength (f'_t) about one tenth its compressive strength (f'_c), concrete by itself is a very poor material for flexural members.

2 To provide tensile resistance to concrete beams, a reinforcement must be added. Steel is almost universally used as reinforcement (longitudinal or as fibers), but in poorer countries other indigenous materials have been used (such as bamboos).

3 The following lectures will focus exclusively on the flexural design and analysis of reinforced concrete rectangular sections. Other concerns, such as shear, torsion, cracking, and deflections are left for subsequent ones.

4 Design of reinforced concrete structures is governed in most cases by the *Building Code Requirements for Reinforced Concrete*, of the American Concrete Institute (ACI-318). Some of the most relevant provisions of this code are enclosed in this set of notes.

5 We will focus on determining the amount of flexural (that is longitudinal) reinforcement required at a **given section**. For that section, the moment which should be considered for design is the one obtained from the **moment envelope** at that particular point.

24.1.1 Notation

6 In R/C design, it is customary to use the following notation

24.1.4 Basic Relations and Assumptions

13 In developing a design/analysis method for reinforced concrete, the following **basic relations** will be used:

1. Equilibrium: of forces and moment at the cross section. 1) $\Sigma F_x = 0$ or Tension in the reinforcement = Compression in concrete; and 2) $\Sigma M = 0$ or external moment (that is the one obtained from the moment envelope) equal and opposite to the internal one (tension in steel and compression of the concrete).

2. Material Stress Strain: We recall that all normal strength concrete have a failure strain $\epsilon_u = .003$ in compression irrespective of f'_c.

14 Basic **assumptions** used:

Compatibility of Displacements: Perfect bond between steel and concrete (no slip). Note that those two materials do also have very close coefficients of thermal expansion under normal temperature.

Plane section remain plane $\Rightarrow$ strain is proportional to distance from neutral axis.

24.1.5 ACI Code

15 The ACI code is based on limit strength, or $\Phi M_n \geq M_u$ thus a similar design philosophy is used as the one adopted by the LRFD method of the AISC code, **ACI-318: 8.1.1; 9.3.1; 9.3.2**

16 The required strength is based on (**ACI-318: 9.2**)

$$\boxed{\begin{aligned} U &= 1.4D + 1.7L \\ &= 0.75(1.4D + 1.7L + 1.7W) \end{aligned}} \tag{24.1}$$

17 We should consider the behaviors of a reinforced concrete section under increasing load:

1. Section uncracked

2. Section cracked, elastic

3. Section cracked, limit state

The second analysis gives rise to the Working Stress Design (WSD) method (to be covered in Structural Engineering II), and the third one to the Ultimate Strength Design (USD) method.

24.2 Cracked Section, Ultimate Strength Design Method

24.2.1 Equivalent Stress Block

18 In determining the limit state moment of a cross section, we consider Fig. 24.1. Whereas the strain distribution is linear (**ACI-318 10.2.2**), the stress distribution is non-linear because the stress-strain curve of concrete is itself non-linear beyond $0.5f'_c$.

19 Thus we have two alternatives to investigate the moment carrying capacity of the section, **ACI-318: 10.2.6**

24 We have two equations and three unknowns (α, β_1, and β). Thus we need to use test data to solve this problem[1]. From **experimental tests**, the following relations are obtained

f'_c (ppsi)	< 4,000	5,000	6,000	7,000	8,000
α	.72	.68	.64	.60	.56
β	.425	.400	.375	.350	.325
β_1	.85	.80	.75	.70	.65

Thus we have a general equation for β_1 (**ACI-318 10.2.7.3**):

$$\boxed{\begin{array}{rcll} \beta_1 & = & .85 & \text{if } f'_c \leq 4,000 \\ & = & .85 - (.05)(f'_c - 4,000)\frac{1}{1,000} & \text{if } 4,000 < f'_c < 8,000 \end{array}} \tag{24.2}$$

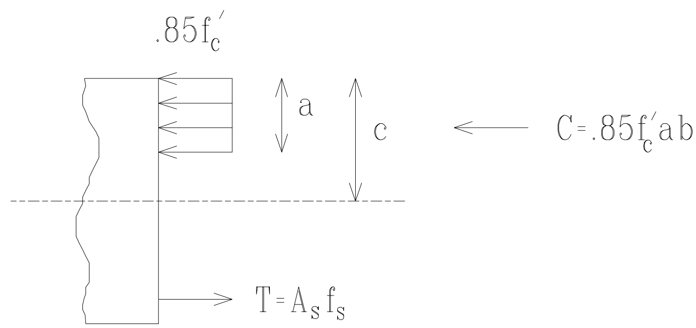

Figure 24.2: Whitney Stress Block

24.2.2 Balanced Steel Ratio

25 Next we seek to determine the **balanced steel ratio** ρ_b such that failure occurs by simultaneous yielding of the steel $f_s = f_y$ and crushing of the concrete $\varepsilon_c = 0.003$, **ACI-318: 10.3.2** We will separately consider the two failure possibilities:

Tension Failure: we stipulate that the steel stress is equal to f_y:

$$\left.\begin{array}{rcl} \rho & = & \frac{A_s}{bd} \\ A_s f_y & = & .85 f'_c ab = .85 f'_c b \beta_1 c \end{array}\right\} \Rightarrow c = \frac{\rho f_y}{0.85 f'_c \beta_1} d \tag{24.3}$$

Compression Failure: where the concrete strain is equal to the ultimate strain; From the strain diagram

$$\left.\begin{array}{rcl} \varepsilon_c & = & 0.003 \\ \frac{c}{d} & = & \frac{.003}{.003+\varepsilon_s} \end{array}\right\} \Rightarrow c = \frac{.003}{\frac{f_s}{E_s} + .003} d \tag{24.4}$$

[1]This approach is often used in Structural Engineering. Perform an analytical derivation, if the number of unknowns then exceeds the number of equations, use experimental data.

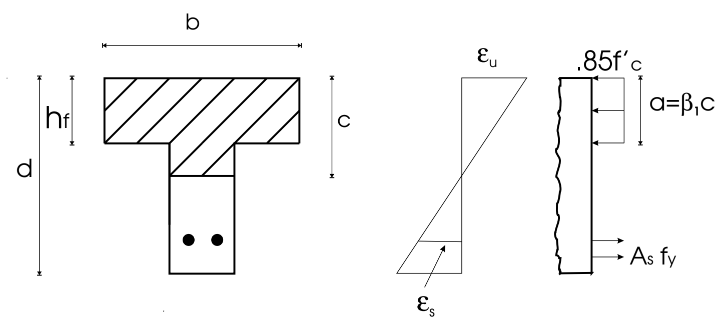

Figure 25.3: T Beam Strain and Stress Diagram

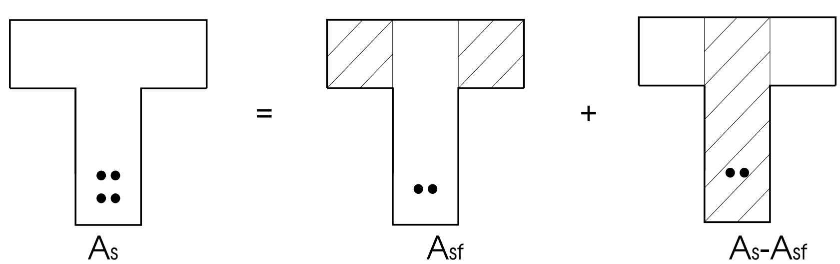

Figure 25.4: Decomposition of Steel Reinforcement for T Beams

Solution:

1. Check requirements for isolated T sections

   (a) $b_e = 30$ in should not exceed $4b_w = 4(14) = 56$ in$\surd$

   (b) $h_f \geq \frac{b_u}{2} \Rightarrow 7 \geq \frac{14}{2}\surd$

2. Assume Rectangular section

$$a = \frac{(12.48)(50)}{(0.85)(3)(30)} = 8.16 \text{ in} > h_f$$

3. For a T section

$$\begin{aligned}
A_{sf} &= \frac{.85f'_c h_f(b-b_w)}{f_y} \\
&= \frac{(.85)(3)(7)(30-14)}{50} = 5.71 \text{ in}^2 \\
\rho_f &= \frac{5.71}{(14)(36)} = .0113 \\
A_{sw} &= 12.48 - 5.71 = 6.77 \text{ in}^2 \\
\rho_w &= \frac{12.48}{(14)(36)} = .025 \\
\rho_b &= .85\beta_1 \frac{f'_c}{f_y}\frac{87}{87+f_y} \\
&= (.85)(.85)\frac{3}{50}\frac{87}{87+50} = .0275
\end{aligned}$$

4. Maximum permissible ratio

$$\begin{aligned}
\rho_{max} &= .75(\rho_b + \rho_f) \\
&= .75(.0275 + .0113) = .029 > \rho_w\surd
\end{aligned}$$

5. The design moment is then obtained from

$$\begin{aligned}
M_{n1} &= (5.71)(50)(36 - \tfrac{7}{2}) = 9,280 \text{ k.in} \\
a &= \frac{(6.77)(50)}{(.85)(3)(14)} = 9.48 \text{ in} \\
M_{n2} &= (6.77)(50)(36 - \tfrac{9.48}{2}) = 10,580 \text{ k.in} \\
M_d &= (.9)(9,280 + 10,580) = 17,890 \text{ k.in} \rightarrow \boxed{17,900 \text{ k.in}}
\end{aligned}$$

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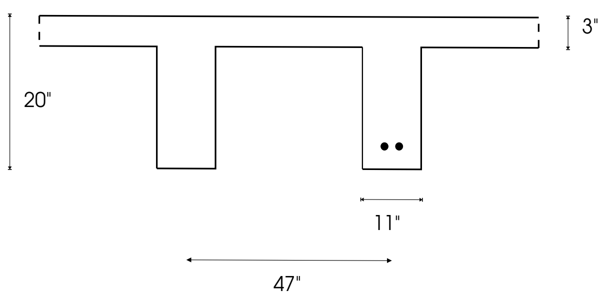
■ Example 25-2: T Beam; Moment Capacity II

Determine the moment capacity of the following section, assume flange dimensions to satisfy ACI requirements; $A_s = 6\#10 = 7.59$ in^2; $f'_c = 3$ ksi; f_y=60 ksi.



Solution:

1. Determine effective flange width:

$$\left.\begin{array}{lcl} \frac{1}{2}(b-b_w) \leq 8h_f & & \\ 16h_f + b_w = (16)(3)+11 & = & 59 \text{ in} \\ \frac{L}{4} = \frac{24}{4}12 & = & 72 \text{ in} \\ \text{Center Line spacing} & = & 47 \text{ in} \end{array}\right\} b = 47 \text{ in}$$

2. Assume $a = 3$ in

$$\begin{array}{rcl} A_s & = & \frac{M_d}{\phi f_y (d-\frac{a}{2})} = \frac{6{,}400}{0.9)(60)(20-\frac{3}{2})} = 6.40 \text{ in}^2 \\ a & = & \frac{A_s f_y}{(.85)f'_c b} = \frac{(6.4)(60)}{(.85)(3)(47)} = 3.20 \text{ in} > h_f \end{array}$$

3. Thus a T beam analysis is required.

$$\begin{array}{rcl} A_{sf} & = & \frac{.85 f'_c (b-b_w) h_f}{f_y} = \frac{(.85)(3)(47-11)(3)}{60} = 4.58 \text{ in}^2 \\ M_{d1} & = & \phi A_{sf} f_y (d - \frac{h_f}{2}) = (.90)(4.58)(60)(20 - \frac{3}{2}) = 4,570 \text{ k.in} \\ M_{d2} & = & M_d - M_{d1} = 6,400 - 4,570 = 1,830 \text{ k.in} \end{array}$$

4. Now, this is similar to the design of a rectangular section. Assume $a = \frac{d}{5} = \frac{20}{5} = 4.$ in

$$A_s - A_{sf} = \frac{1,830}{(.90)(60)\left(20 - \frac{4}{2}\right)} = 1.88 \text{ in}^2$$

5. check

$$\begin{array}{rcl} a & = & \frac{1.88)(60)}{(.85)(3)(11)} = 4.02 \text{ in} \approx 4.00 \\ A_s & = & 4.58 + 1.88 = \boxed{6.46 \text{ in}^2} \\ \rho_w & = & \frac{6.46}{(11)(20)} = .0294 \\ \rho_f & = & \frac{4.58}{(11)(20)} = .0208 \\ \rho_b & = & (.85)(.85)\left(\frac{3}{60}\right)\left(\frac{87}{87+60}\right) = .0214 \\ \rho_{max} & = & .75(.0214 + .0208) = .0316 > \rho_w \surd \end{array}$$

6. Note that 6.46 in^2 (T beam) is close to $A_s = 6.40$ in^2 if rectangular section was assumed.

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