

Hydraulic Engineering
Prof. Mohammad Saud Afzal
Department of Civil Engineering
Indian Institute of Technology Kharagpur

Lecture - 27
Dimensional Analysis and Hydraulic Similitude (Contd.,)

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Similitude

Dynamic Similarity: The same length scale, time-scale, and force scale is required.

First, satisfy geometric, and kinematic similarity. Dynamic similarity then exists if the force and pressure coefficient are the same.

In order to ensure that the force and pressure coefficients are the same:

For compressible flow: Re , Mach, and specific heat ratio must be matched.

For incompressible flow with no free surface: Re matching only.

For incompressible flow with a free surface: Re , Froude, and possibly Weber number (surface tension effects), and cavitation number must be matched.

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Welcome back to the last lecture of this module. We concluded at the similitude last time, where we discussed the dynamics similarity. And we start with the extension of similitude, that is, model scales.

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Model Scales

Fluid flow models are usually designed for one most dominant force and occasionally for two

If dominant force is gravity then Froude number must be same in model and prototype

If dominant force is viscous force then Reynolds number must be same in model and prototype

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So, the fluid flow models are usually designed for one most dominant force and occasionally for 2. Suppose, if the dominant force here is gravity then Froude number must be the same in model and prototype. If the dominant force is viscous force then Reynolds number must be the same in model and prototype.

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Class Problem

For Froude model law, find the ratios of velocity, discharge, force, work and power in terms of length scale

Solution: In Froude model law the model and prototype Froude numbers are same. Hence

$$(Fr)_m = \frac{V_m}{\sqrt{gL_m}} = (Fr)_p = \frac{V_p}{\sqrt{gL_p}} \Rightarrow \frac{V_m}{V_p} = \sqrt{\frac{L_m}{L_p}} = \sqrt{L_r}$$

The gravity g is same for both model and prototype. Hence if length ratio $L_m/L_p = L_r$ and $\rho_m/\rho_p = \rho_r$

Velocity ratio V_r $V_r = \frac{V_m}{V_p} = \sqrt{\frac{L_m}{L_p}} = \sqrt{L_r}$

So, we start by solving a problem. The question is, for Froude model law, find the ratios of velocity, discharge, force, work and power in terms of length scale. So, the way, the solution will grow, so, in Froude model law, the model and prototype Froude numbers are the same. That is why, you know, it is called the Froude Model Law. So, Froude in the Model m , Fr_m

will be, $(Fr)_m = \frac{V_m}{\sqrt{gL_m}}$, should be equal to Froude number of prototype is equal to

$(Fr)_p = \frac{V_p}{\sqrt{gL_p}}$. So, very simple.

So, if we been, it will be V_m / V_p is equal to under root L_m / L_p , g will get cancelled. Anyways, for now, I will take away this. So, the gravity g is same for both model and prototype. Hence, if the length ratio L_m / L_p is L_r , what we said here was, V_m / V_p is equal to under root L_m / L_p . So, the velocity ratio, V_r will be under root L_r . This is what we get, they have done the same thing here by pen.

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Class Problem


Solution contd.




Discharge ratio $Q_r = (\text{Velocity} \cdot \text{area})_r$ $Q_r = V_r L_r^2 = L_r^{5/2}$

Force ratio $F_r = (\rho L^2 V^2)_r$ $F_r = (\rho_r L_r^3)$

Work ratio $E_r = \text{Energy ratio} = \text{Force} \cdot \text{distance}$ $E_r = (\rho_r L_r^4)$

Power = Force * Velocity $P_r = (\rho_r L_r^3) * (L_r^{1/2}) = \rho_r L_r^{7/2}$



Now, the discharge ratio Q_r . So, discharge ratio Q_r will be the ratios of velocity into area. So, I will just. So, velocity is given by V_r and area is the length, I mean, the length whole square. So, V_r we have already found out that V_r was L_r to the power half, in the previous slide, multiplied by L_r square so it becomes L_r to the power $5/2$, as indicated here.

Force ratio. Force ratio will be the ratio of densities multiplied by L_r square into velocity square. So, ratio of densities we said was ρ_r into L_r square and V_r was under root L_r whole square, so it will be $\rho_r L_r$ cube. But anyways I will take this down myself. So, force ratio is going to be $\rho_r L_r$ cube, as we just derived. What about the energy ratio? Energy ratio is force into distance, so F_r into L_r . So, it becomes $\rho_r L_r$ to the power 4. So, ρ_r into L_r to the power 4

Now, power is force into velocity. So, it is force ratio into V_r . So, it is force is $\rho_r L_r$ cube into V_r is under root L_r . So, it becomes $\rho_r L_r$ to the power $7/2$. So, same thing, ρ_r into L_r to the power $7/2$.

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Distorted Models


The idea behind similitude is that we simply equate all the Pi terms

In reality it is always not possible to satisfy all the known requirements so as to be able to equate all the Pi terms

Example; Study of open channel or free surface flows

Froude number similarity gives $\frac{V_m}{\sqrt{g_m l_m}} = \frac{V_p}{\sqrt{g_p l_p}}$

Under same gravitational field $\frac{V_m}{V_p} = \sqrt{\frac{l_m}{l_p}} = \sqrt{\lambda_r}$



Now, there are something called distorted models. Those were an example with, you know, proper modelling, where all the laws were taken into account. There are something called distorted models. So, idea behind similitude is that, we simply equate all pi terms. But in reality, it is not always possible to satisfy all the known requirements so as to be able to equate all pi terms. In reality, it becomes very difficult to equate all these terms.

Example; Study of open channel or free surface flows, we will see through an example. So, in this case, the Froude number similarity will give $V_m / \sqrt{g_m l_m}$ is equal to $V_p / \sqrt{g_p l_p}$. And if we have the same gravitational field, just considering, this will give V_m / V_p . So, the ratio of the velocity will be λ_r . So, under root of length ratio, that we have seen in the previous example so, under root of length ratio.

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Distorted Models


Reynolds number similarity gives $\frac{\rho_m V_m l_m}{\mu_m} = \frac{\rho_p V_p l_p}{\mu_p}$

Velocity scale is $\frac{V_m}{V_p} = \frac{\mu_p \rho_p l_p}{\mu_m \rho_m l_m}$

Since velocity scale must be equal to $\sqrt{\lambda_r}$ due to Froude number similarity

Ratio of kinematic viscosity $\frac{\mu_m}{\rho_m} = \frac{V_m}{V_p} = (\lambda_r)^{1.5}$

Very difficult to find such liquids that satisfy the above relation



Now, if because both Froude number and Reynolds number should be the same, if we do the Froude Reynolds number stability we will have Reynolds number equated for Model Reynolds number and this is prototype Reynolds number. So based on that the velocity scale is going to be V_m / V_p will be μ_m / μ_p into ρ_p / ρ_m into L_p / L_m . So, we bring V_p down here and take other terms that side.

Since, velocity scale must be equal to λ . So, we will equate this to λ and this is already λ . So, ratio of kinematic viscosity is going to be $\mu_m / \rho_m / \mu_p / \rho_p$ is equal to ν_m / ρ_p is equal to, this will come this side, because this was λ to the power half and this was here, so, it becomes λ to the power 3 by 2. And in reality, it is very difficult to find such liquids that satisfy the above relation. So, it might not be always possible to find the fluids having this viscosity ratio.

So, in practical, ideally, if we can have all the type of similarities, that is the best case solution. But in reality, it is it might not be always, you know, I mean, we might not be able to find it all the time.

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Distorted Models

To overcome difficulties like before the modeling is done using distorted scales;
Distorted models
 In open channel the vertical dimension is used to simulate Froude's law while other two dimension are scaled to suit available space
 In distorted models
 Horizontal scale (length and width) L_r Vertical scale (depth) h_r
 Cross sectional area ratio = $\frac{\text{area in model}}{\text{area in prototype}} = \frac{(By)_m}{(By)_p} = L_r h_r$

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So, to overcome difficulties like before modelling, I mean, the one that we did, we have we use distorted scales or distorted models. So, what is done here? In open channel the vertical dimension is used to simulate Froude laws while the other 2 dimensions are scaled to suit the available space. So, in Froude law, in Froude number the vertical dimension so basically, for Froude law we use only the vertical dimensional scaling and for the other 2 Dimension X and Y, we use the geometric scaling, for example.

So, in distorted scale models, horizontal scale length and width is given by L_r . A vertical scale can be different, in this case it can be h_r . And ideally, L_r should be equal to h_r , but in distorted models we do not do that. Cross sectional area will be the area in the model divided by area in the prototype, if we try to calculate the cross sectional area. So, it will be width into y divided by B into y_p . So, width of model by width by prototype is given by the length ratio.

Whereas, the vertical cross section y_m / y_p is given by h_r . So, cross sectional is given by L_r into h_r area. So, it is not a L_r square, it is L_r into h_r .

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Distorted Models

Froude number ratio = 1 $\rightarrow \frac{F_m}{F_p} = 1 = \frac{\frac{V_m}{\sqrt{g y_m}}}{\frac{V_p}{\sqrt{g y_p}}} \rightarrow \frac{\left(\frac{V_m}{V_p}\right)^2}{\frac{y_m}{y_p}} = \frac{V_r^2}{h_r} = 1$

Thus velocity ratio $V_r = \sqrt{h_r}$

Discharge ratio $Q_r = \text{Area ratio} \times \text{velocity ratio} = L_r h_r^{1.5}$

Slope ratio $S_r = \frac{S_m}{S_p} = \frac{h_r}{L_r}$

Time ratio $T_r = \frac{T_m}{T_p} = \frac{L_r}{V_r \sqrt{h_r}}$

For distorted models find Manning's ratio n , *Home work problem for you* \rightarrow forum

Froude number ratio is one. So, this give us, F_m / F_p is equal to 1 and this is V_m . So, that is what it means, F_r is equal to 1. So, through this, so implies V_m / V_p is equal to under root y_m / y_p . And what is the ratio of y_m / y_p ? Is h_r , so, it is under root h_r , but anyways, we will see. So, V_m / V_p whole square / y_m / y_p is equal to V_r square / h_r , another way of solving and we get the same result.

So, the velocity ratio is under root of h_r . Discharge ratio is area ratio and velocity ratio. Area ratio was $L_r h_r$ into velocity ratio is h_r to the power half. So, it becomes L_r into h_r to the power $3 / 2$. Slope ratio is S_m / S_p is equal to its y / x . So, it is h_r / L_r , one side is the depth, one is the x and y dimensions. Time ratio is L_r / V_r . So, L_r is L_r and V_r is under root h_r . So, it becomes L_r divided by under root h_r .

So, for distorted models find, you know, sorry. So, for distorted models find Manning's ratio n_r , this is a homework problem for you. So, you can find and post your solutions in the forum. So, for distorted models only you have to find the Manning's ratio. So very simple if you know the Manning's number formula, you will just look it up.

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Class Problem

In a tidal model, the horizontal scale ratio is 1/500. The vertical scale is 1/50. What model period would correspond to a prototype period of 12 hours .

Solution: Horizontal Scale = $L_r = 1/500$ Vertical Scale = $h_r = 1/50$

Time ratio T_r $T_r = \frac{L_r}{\sqrt{h_r}} = \frac{1/500}{\sqrt{1/50}} = 0.01414$

$T_r = \frac{T_m}{T_p} \rightarrow T_m = T_p T_r = \frac{(12 \times 60 \times 60) \times 0.01414}{\text{Tp in seconds}} = 610s$

$T_m = 610 \text{ seconds}$


So, we are going to solve one problem. In a tidal model it is given, that the horizontal scale ratio is 1 / 500. So, horizontal scale ratio is L_r is 1 / 500 and the vertical scale is, so that means, h_r is 1 / 50. What model period would correspond to a prototype period of 12 hours. Very good question and we will see how do we solve this problem. So, horizontal, as I wrote horizontal scale L_r is given by 1 / 500 and vertical scale h_r is given by 1 / 50.

So, time ratio is $L_r / \text{under root } h_r$. We can also see that from the previous slide formula for distorted, previous slide formula. So, it is $L_r / \text{under root } h_r$. L_r ratio is 1 / 500 and h_r is, under root h_r is 1 / 50. So, this comes out to be 0.01414. Now, the time ratio is given by T_m / T_p and this T_r we already know. And it says that the prototype period of 12 hours which implies T_m will be T_p into T_r . Prototype is 12 hours into 16 minutes into 60 seconds multiplied by T_r , T_p in seconds.

And this when you multiply, it will give model time period of almost 610 seconds. So, this means a model second of 610 seconds would correspond to a prototype period of 12 hours in a tidal model. So, this is the problem that we have seen now. So, after that we are going to solve some problems that relates to, you know, these similitude and model scale, some more problems.

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Q. 1) A small sphere of density ρ_s and diameter D settles at a terminal velocity V in a liquid of density ρ_f and dynamic viscosity μ . Gravity g is known to be a parameter. Express the functional relationships between these variables in a dimensionless form.



So, first problem, which we have already actually done in dimensional analysis. So, we go to problem number 2. What does it say? It says that a model boat, 1 / 100 size of its prototype has 0.12 Newton of resistance when simulating a speed of 0.15, sorry, speed of 5 meters per second of the prototype. Water is the fluid in both the cases. What is the corresponding resistance in the prototype? We can actually neglect these frictional forces. So, we will start as always. So, this is question number 2. We are going to the white screen.

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Soln 2)
The resistance offered at the free surface is the significant force and as such Froude model law is applicable

$$Fr_m = \frac{V_m}{\sqrt{g L_m}} = \frac{V_p}{\sqrt{g L_p}} = Fr_p$$

$$L_r = \frac{L_m}{L_p}, V_r = \sqrt{L_r}$$

$$\frac{(Force)_m}{(Force)_p} = \frac{\rho_r L_r^2 V_r^2}{\rho_r L_r^3} = \frac{\rho_r L_r^2 V_r^2}{\rho_r L_r^3}$$


Since same fluid $\rho_r = 1$
 $\Rightarrow Fr = L_r^3$

$$\frac{F_m}{F_p} = L_r^3$$

$$F_p = \frac{F_m}{(L_r)^3} = \frac{0.12}{(1/100)^3}$$

$$F_p = 120,000 \text{ N} = 120 \text{ kN}$$

Answer
 $F_p = 120 \text{ kN}$

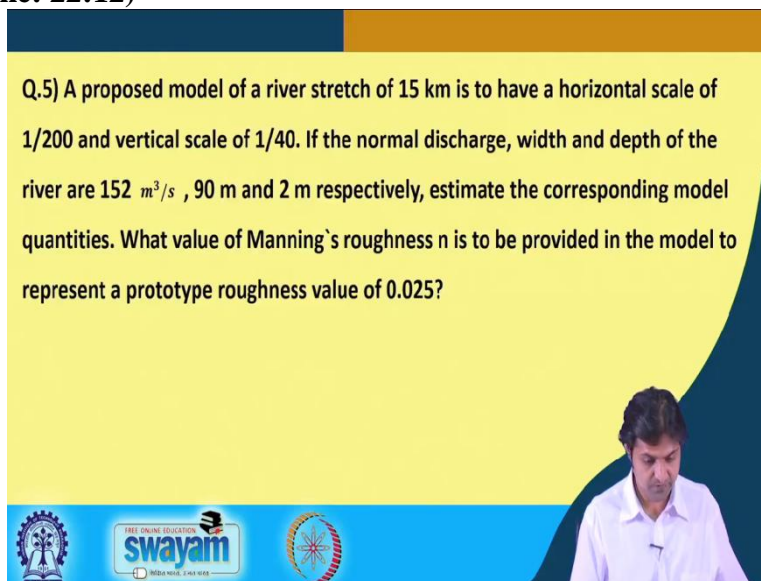


So, the resistance offered at the free surface is the significant force and as such Froude model law is applicable. So, Froude model law, Fr_m is V_m under root $g L_m$ is equal to $V_p / \sqrt{g L_p}$ is equal to Froude number in prototype. L_r is L_m / L_p and V_r is going to be under root L_r , that we have already seen using the Froude model law. And force by model by force by prototype came out to be $\rho_r L_r^2 V_r^2$, if you remember the formula, it will be $\rho_r L_r^3$.

So, since same fluid is there, therefore, ρ_r is equal to one. This implies, F_r is L_r whole cube, or in other terms, for F_m / F_p is equal to L_r cube. So, F_p in prototype, force in model divided by L_r cube. Now, we substitute the values, F_m is 0.12 Newton and this is $1 / 100$ th L_r . So, this is going to be, 120,000 Newton or 120 kilo Newton. So, force in prototype is going to be 120 kilo Newton. This is the answer.

So, here you have seen how we have applied this model similitude to obtain the force in the prototype. So, we will solve another problem, problem number 5.

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Q.5) A proposed model of a river stretch of 15 km is to have a horizontal scale of $1/200$ and vertical scale of $1/40$. If the normal discharge, width and depth of the river are $152 \text{ m}^3/\text{s}$, 90 m and 2 m respectively, estimate the corresponding model quantities. What value of Manning's roughness n is to be provided in the model to represent a prototype roughness value of 0.025?

This is on the same concept. So, the question here states, a proposed model of a river stretch of 15 kilometres is to have horizontal scale of $1 / 200$ and a vertical scale of $1 / 400$. What does this question say? I mean, what does this indicate? This indicates that we are dealing with distorted scales because we have a different vertical scale, sorry, a different, we have a different horizontal scale and a different vertical scale.

So, proceeding back to the reading the question, if the normal discharge, width and depth of the river are 152 meter cube per second, 90 meter and 2 meters, so discharge is 152 meter cube per second, width is 90 meter and depth is around 2 meter. We have to estimate the corresponding model quantities. What value of Manning's roughness n is to be provided in the model to represent a prototype roughness value of 0.025?

So, basically, this question will demonstrate all the distorted scale, including the Manning's roughness question that I gave you to solve at home. So, to solve that, I will go to the, you know, white screen.

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Soln 5:

Horizontal scale $L_r = 1/200$
Vertical scale $h_r = 1/40$

1) Discharge $\frac{Q_m}{Q_p} = L_r h_r^{3/2}$
 $\Rightarrow Q_m = Q_p \times L_r \times h_r^{3/2}$
 $Q_m = 152 \times \left(\frac{1}{200}\right) \times \left(\frac{1}{40}\right)^{1.5} = 0.03 \text{ m}^3/\text{s}$
 $Q_m = 0.03 \text{ m}^3/\text{s}$

2) Depth $\frac{y_m}{y_p} = h_r$
 $\Rightarrow y_m = y_p \times h_r = 2 \times \frac{1}{40} = 0.05 \text{ m}$
 $y_m = 0.05 \text{ m}$

3) Width B_m
 $\frac{B_m}{B_p} = L_r \Rightarrow B_m = L_r \times B_p = \frac{1}{200} \times 90$
 $B_m = 0.045 \text{ m}$

4) Manning's n
 $\frac{n_m}{n_p} = \frac{h_r^{2/3}}{L_r^{1/2}} \Rightarrow n_m = n_p \times \frac{h_r^{2/3}}{L_r^{1/2}}$
 $n_m = \frac{0.025 \times \left(\frac{1}{40}\right)^{2/3}}{\left(\frac{1}{200}\right)^{1/2}}$
 $n_m = 0.03$
 Model here has to be rougher

Known quantities
 $Q_p = 152 \text{ m}^3/\text{s}$, width = 90m, $y_p = 2 \text{ m}$, $n_p = 0.025$

So, I will just write solution 5 here. So, we have been given, so I will just draw a line here as well, a horizontal scale, L_r is $1/200$, vertical scale, h_r is $1/40$, that is the question. So, first thing to solve is discharge. So, Q_m / Q_p we have already found out, it was L_r into h_r to the power $3/2$. So, implies Q_m is going to be Q_p into L_r into h_r to the power $3/2$ and Q_p is 152 meter cube per second multiplied by $1/200$ and h_r is $1/40$ to the power 1.5 and this will give 0.03 meter cube per second in model.

Second thing is depth, y_m / y_p is h_r , simple. So, this implies y_m is equal to y_p into h_r or y_p here given is, so y_p the depth was actually, so I should have actually first. So, I will also write down the known quantities. Sorry, for this. I will write down the known quantities here. Q_p prototype was given as, 152 meter cube per second, width was given as, 90 meter and y_p was given as, 2 meter. So, y_p was 2 meter into h_r was $1/40$ and it comes out to be 0.05 meter. So, that means in model it is 0.05 meter, y_m and Q_m was 0.03 meter cube per second.

Now, the third thing is width, B_m we have to find. So, B_m / B_p is the length ratio. So, this implies, B_m is, so width is B_p basically. So, B_m is L_r into B_p . So, L_r $1/200$ into B_p was 90 meters. So, B_m comes out to be 0.045 meters. So, now, Manning's n , this is important. So, Manning's n , n_m / n_p , you have to find out the formula but it comes out to be, h_r to the

power $2/3$ divided by L_r to the power half. This implies, model n_m is Manning's number in prototype into h_r to the power $2/3$ L_r to the power half.

So, in the known quantities we were also told the prototype roughness was given, that was 0.025. So, this is 0.025 into h_r was $1/40$ to the power $2/3$ divided by $1/200$ to the power half. So, η_m comes out to be 0.03. So, prototype roughness was 0.025 and the model roughness is 0.03. So, model here, has to be rougher than the prototype. So, if we have seen that everything is reducing the discharge in model has reduced, the y has reduced, the depth, the width has reduced.

But the special thing to note is that the Manning's n or the roughness, so model here has to be more rough than the prototype. It is an interesting result, some things to ponder over. And with this, you know, I would like to finish this module. We have some more questions but we can, you know, take it up in either forum if you have come up with anything else or in the assignment that you are given. So, thank you so much for attending this module. I will see you in the next week.