Hydraulic Engineering
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Lecture - 27
Dimensional Analysis and Hydraulic Similitude (Contd.,)

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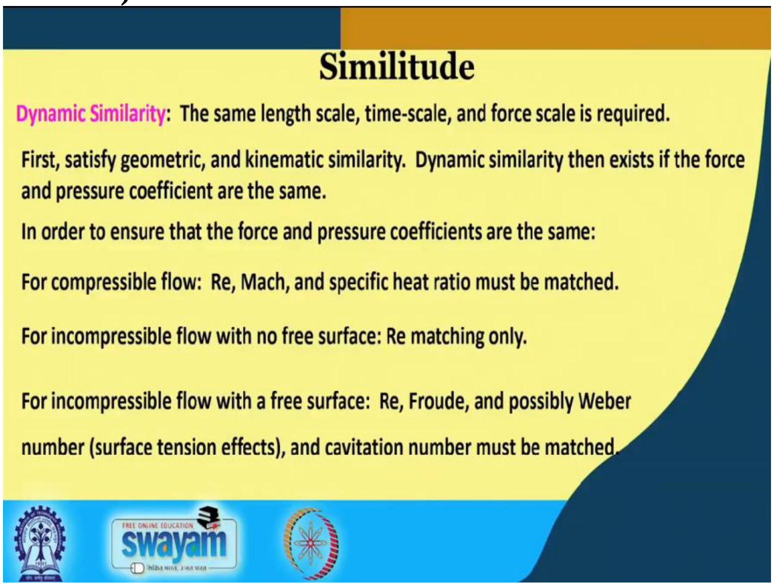

Welcome back to the last lecture of this module. We concluded at the similitude last time, where we discussed the dynamics similarity. And we start with the extension of similitude, that is, model scales.

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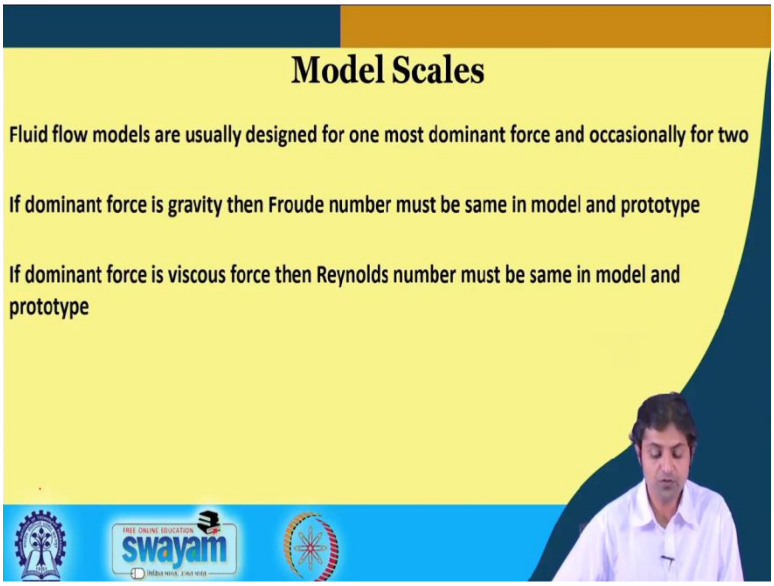

So, the fluid flow models are usually designed for one most dominant force and occasionally for 2. Suppose, if the dominant force here is gravity then Froude number must be the same in model and prototype. If the dominant force iss viscous force then Reynolds number must be the same in model and prototype.

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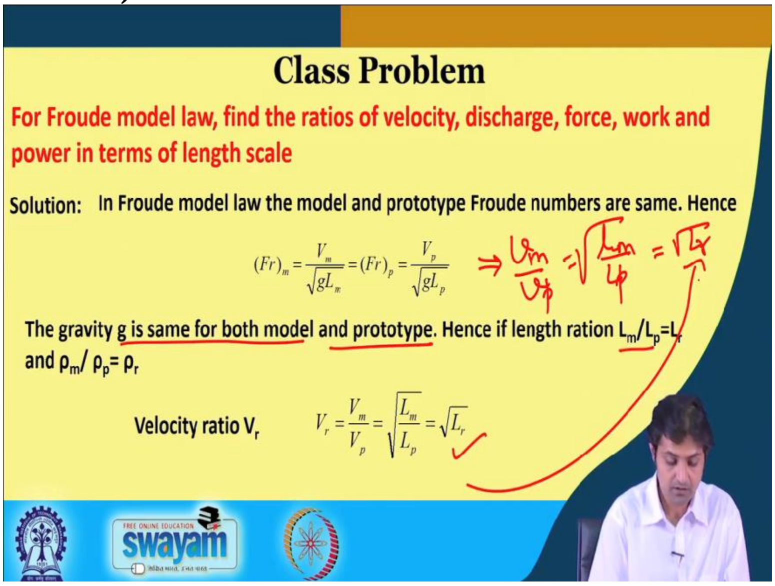

So, we start by solving a problem. The question is, for Froude model law, find the ratios of velocity, discharge, force, work and power in terms of length scale. So, the way, the solution will grow, so, in Froude model law, the model and prototype Froude numbers are the same. That is why, you know, it is called the Froude Model Law. So, Froude in the Model m, Fr m will be, $(Fr)_m = \frac{V_m}{\sqrt{gL_m}}$, should be equal to Froude number of prototype is equal to $(Fr)_p = \frac{V_p}{\sqrt{gL_p}}$. So, very simple.

So, if we been, it will be V m / V p is equal to under root Lm / Lp, g will get cancelled. Anyways, for now, I will take away this. So, the gravity g is same for both model and prototype. Hence, if the length ratio Lm / Lp is Lr, what we said here was, Vm / Vp is equal to under root Lm / Lp. So, the velocity ratio, Vr will be under root Lr. This is what we get, they have done the same thing here by pen.

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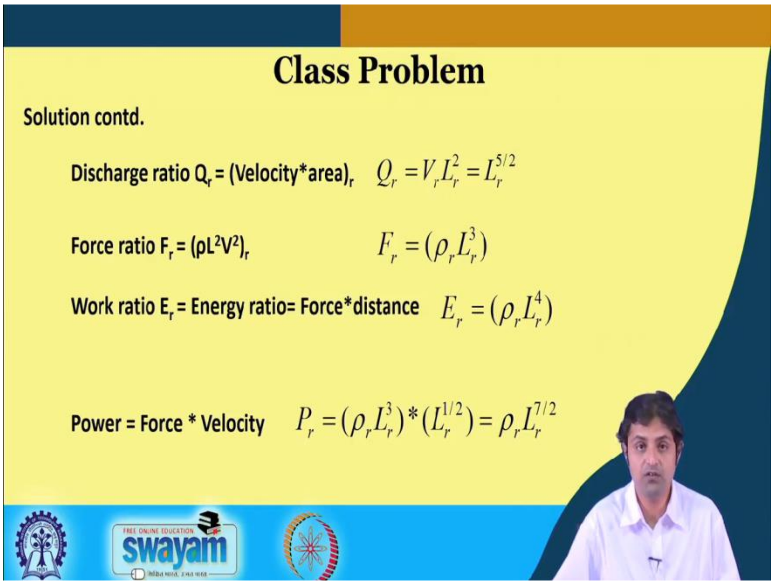

Now, the discharge ratio Q r. So, discharge ratio Qr will be the ratios of velocity into area. So, I will just. So, velocity is given by Vr and area is the length, I mean, the length whole square. So, Vr we have already found out that Vr was Lr to the power half, in the previous slide, multiplied by Lr square so it becomes Lr to the power 5 / 2, as indicated here.

Force ratio. Force ratio will be the ratio of densities multiplied by L r square into velocity square. So, ratio of densities we said was ρ r into L r square and Vr was under root L r whole square, so it will be ρ r L r cube. But anyways I will take this down myself. So, force ratio is going to be ρ r L r cube, as we just derived. What about the energy ratio? Energy ratio is force into distance, so Fr into L r. So, it becomes ρ r L r to the power 4. So, ρ r into L r to the power 4

Now, power is force into velocity. So, it is force ratio into V r. So, it is force is ρ r L r cube into V r is under root L r. So, it becomes ρ r L r to the power 7 / 2. So, same thing, ρ r into L r to the power 7 / 2.

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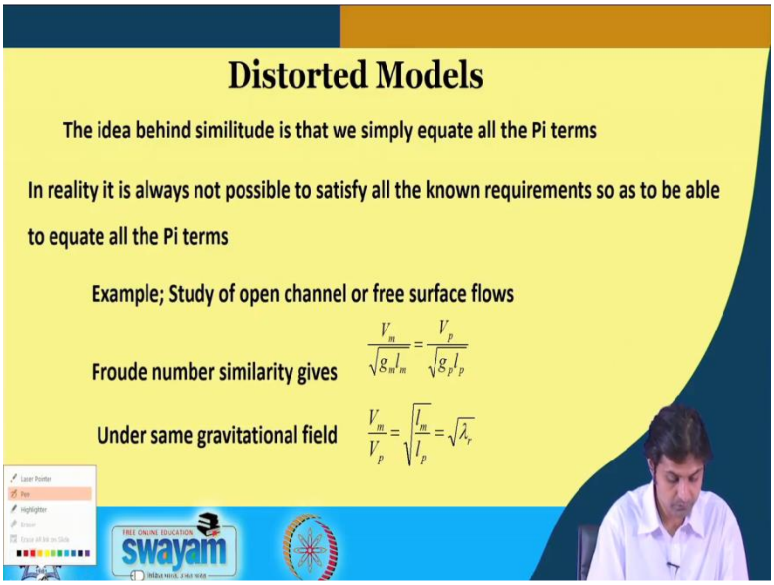

Now, there are something called distorted models. Those were an example with, you know, proper modelling, where all the laws were taken into account. There are something called distorted models. So, idea behind similitude is that, we simply equate all pi terms. But in reality, it is not always possible to satisfy all the known requirements so as to be able to equate all pi terms. In reality, it becomes very difficult to equate all these terms.

Example; Study of open channel or free surface flows, we will see through an example. So, in this case, the Froude number similarity will give V m / under route g m l m is equal to V p under root g p l p. And if we have the same gravitational field, just considering, this will give Vm / Vp. So, the ratio of the velocity will be lambda. So, under root of length ratio, that we have seen in the previous example so, under root of length ratio.

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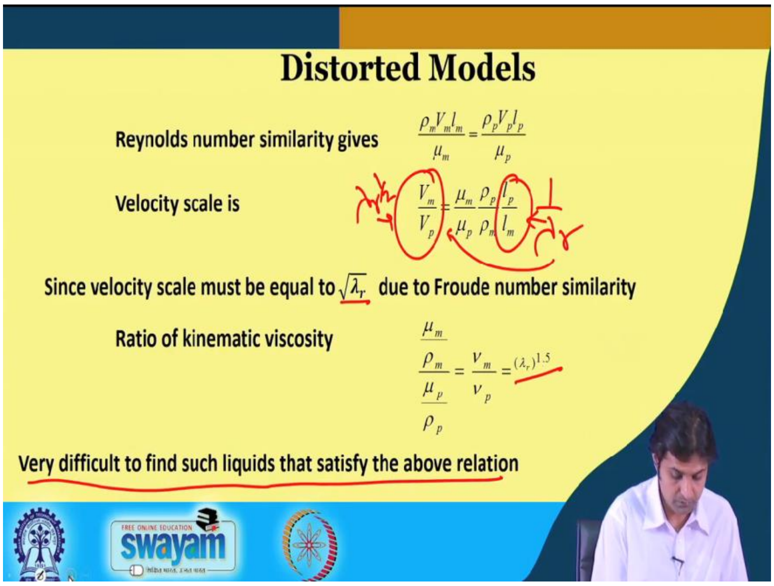

Now, if because both Froude number and Reynolds number should be the same, if we do the Froude Reynolds number stability we will have Reynolds number equated for Model Reynolds number and this is prototype Reynolds number. So based on that the velocity scale is going to be Vm / Vp will be mu m / mu p into ρ p / ρ m into Lp / Lm. So, we bring V p down here and take other terms that side.

Since, velocity scale must be equal to lambda. So, we will equate this to lambda r and this is already lambda r. So, ratio of kinematic viscosity is going to be mu m / ρ m / mu p / ρ p is equal to nu m / ρ p is equal to, this will come this side, because this was lambda r to the power half and this was here, so, it becomes lambda r to the power 3 by 2. And in reality, it is very difficult to find such liquids that satisfy the above relation. So, it might not be always possible to find the fluids having this viscosity ratio.

So, in practical, ideally, if we can have all the type of similarities, that is the best case solution. But in reality, it is it might not be always, you know, I mean, we might not be ableto find it all the time.

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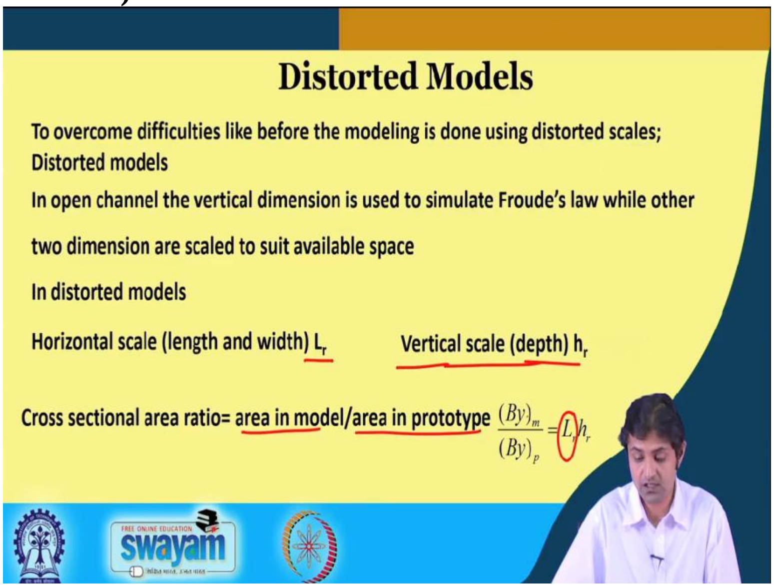

So, to overcome difficulties like before modelling, I mean, the one that we did, we have we use distorted scales or distorted models. So, what is done here? In open channel the vertical dimension is used to simulate Froude laws while the other 2 dimensions are scaled to suit the available space. So, in Froude law, in Froude number the vertical dimension so basically, for Froude law we use only the vertical dimensional scaling and for the other 2 Dimension X and Y, we use the geometric scaling, for example.

So, in distorted scale models, horizontal scale length and width is given by L r. A vertical scale can be different, in this case it can be h r. And ideally, L r should be equal to h r, but in distorted models we do not do that. Cross sectional area will be the area in the model divided by area in the prototype, if we try to calculate the cross sectional area. So, it will be width into y divided by B into y p. So, width of model by width by prototype is given by the length ratio.

Whereas, the vertical cross section y m / y p is given by h r. So, cross sectional is given by L r into h r area. So, it is not a L r square, it is L r into h r.

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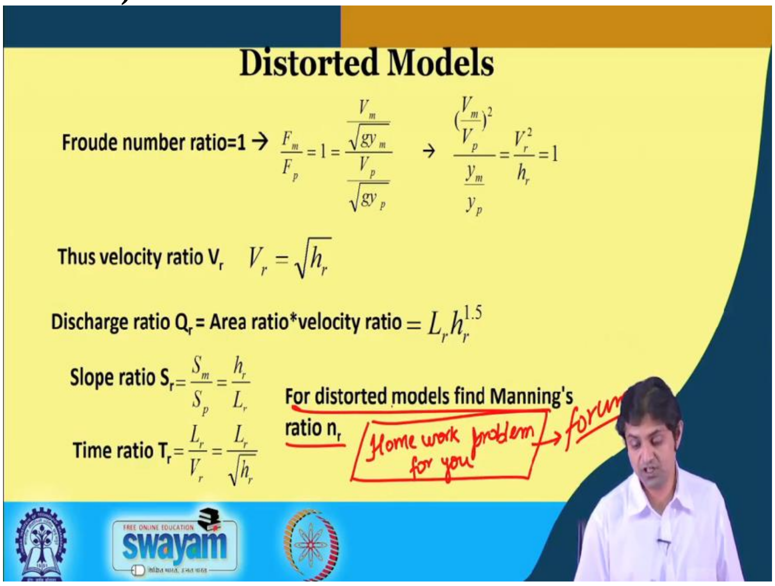

Froude number ratio is one. So, this give us, F m / F p is equal to 1 and this is V m. So, that is what it means, F r is equal to 1. So, through this, so implies V m / V p is equal to under root y m / y p. And what is the ratio of y m / y p? Is h r, so, it is under root h r, but anyways, we will see. So, V m / V p whole square / y m / y p is equal to V r square / h r, another way of solving and we get the same result.

So, the velocity ratio is under root of h r. Discharge ratio is area ratio and velocity ratio. Area ratio was L r h r into velocity ratio is h r to the power half. So, it becomes L r into h r to the power 3 / 2. Slope ratio is Sm / Sp is equal to its y / x. So, it is h r / Lr, one side is the depth, one is the x and y dimensions. Time ratio is Lr / Vr. So, L r is L r and V r is under root h r. So, it becomes L r divided by under root h r.

So, for distorted models find, you know, sorry. So, for distorted models find Manning's ratio n r, this is a homework problem for you. So, you can find and post your solutions in the forum. So, for distorted models only you have to find the Manning's ratio. So very simple if you know the Manning's number formula, you will just look it up.

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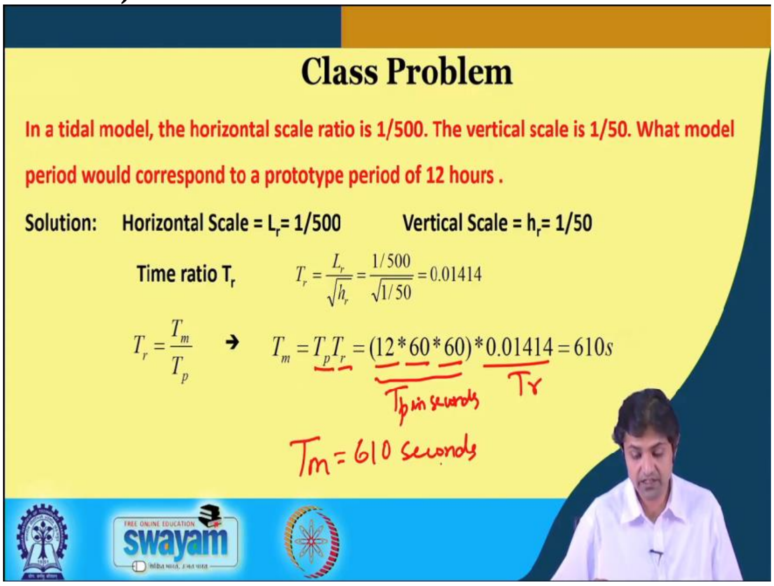

So, we are going to solve one problem. In a tidal model it is given, that the horizontal scale ratio is 1 / 500. So, horizontal scale ratio is L r is 1 / 500 and the vertical scale is, so that means, h r is 1 / 50. What model period would correspond to a prototype period of 12 hours. Very good question and we will see how do we solve this problem. So, horizontal, as I wrote horizontal scale Lr is given by 1 / 500 and vertical scale h r is given by 1 / 50.

So, time ratio is L r / under root h r. We can also see that from the previous slide formula for distorted, previous slide formula. So, it is L r / under root h r. L r ratio is 1 / 500 and h r is, under root hr is 1 / 50. So, this comes out to be 0.01414. Now, the time ratio is given by T m / T p and this T r we already know. And it says that the prototype period of 12 hours which implies Tm will be T p into T r. Prototype is 12 hours into 16 minutes into 60 seconds multiplied by Tr, Tp in seconds.

And this when you multiply, it will give model time period of almost 610 seconds. So, this means a model second of 610 seconds would correspond to a prototype period of 12 hours in a tidal model. So, this is the problem that we have seen now. So, after that we are going to solve some problems that relates to, you know, these similitude and model scale, some more problems.

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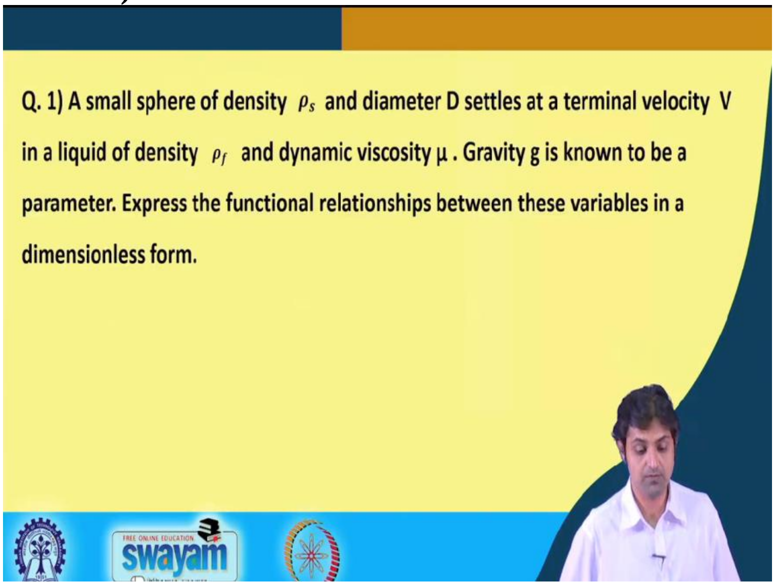

So, first problem, which we have already actually done in dimensional analysis. So, we go to problem number 2. What does it say? It says that a model boat, 1 / 100 size of its prototype has 0.12 Newton of resistance when simulating a speed of 0.15, sorry, speed of 5 meters per second of the prototype. Water is the fluid in both the cases. What is the corresponding resistance in the prototype? We can actually neglect these frictional forces. So, we will start as always. So, this is question number 2. We are going to the white screen.

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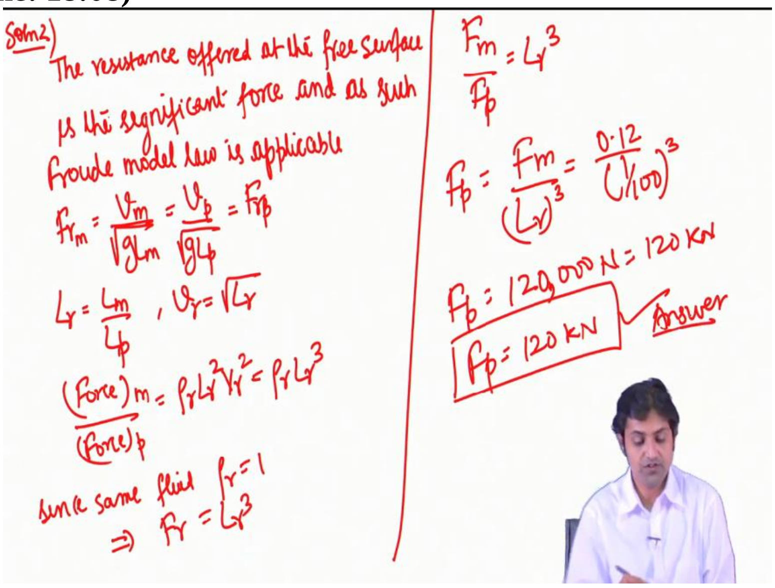

So, the resistance offered at the free surface is the significant force and as such Froude model law is applicable. So, Froude model law, Fr m is V m under root gl m is equal to V p / gl p is equal to Froude number in prototype. L r is L m / L p and V r is going to be under root L r, that we have already seen using the Froude model law. And force by model by force by prototype came out to be ρ r L r square, if you remember the formula, it will be ρ r L r cube.

So, since same fluid is there, therefore, ρ r is equal to one. This implies, F r is L r whole cube, or in other terms, for F m / F p is equal to L r cube. So, Fp in prototype, force in model divided by L r cube. Now, we substitute the values, F m is 0.12 Newton and this is 1 / 100th L r. So, this is going to be, 120,000 Newton or 120 kilo Newton. So, force in prototype is going to be 120 kilo Newton. This is the answer.

So, here you have seen how we have applied this model similitude to obtain the force in the prototype. So, we will solve another problem, problem number 5.

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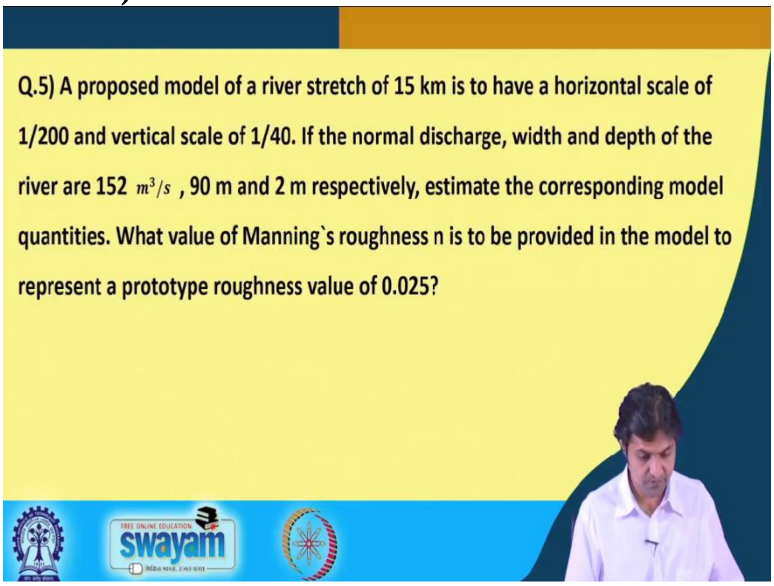

This is on the same concept. So, the question here states, a proposed model of a river stretch of 15 kilometres is to have horizontal scale of 1 / 200 and a vertical scale of 1 / 400. What does this question say? I mean, what does this indicate? This indicates that we are dealing with distorted scales because we have a different vertical scale, sorry, a different, we have a different horizontal scale and a different vertical scale.

So, proceeding back to the reading the question, if the normal discharge, width and depth of the river are 152 meter cube per second, 90 meter and 2 meters, so discharge is 152 meter cube per second, width is 90 meter and depth is around 2 meter. We have to estimate the corresponding model quantities. What value of Manning's roughness n is to be provided in the model to represent a prototype roughness value of 0.025?

So, basically, this question will demonstrate all the distorted scale, including the Manning's roughness question that I gave you to solve at home. So, to solve that, I will go to the, you know, white screen.

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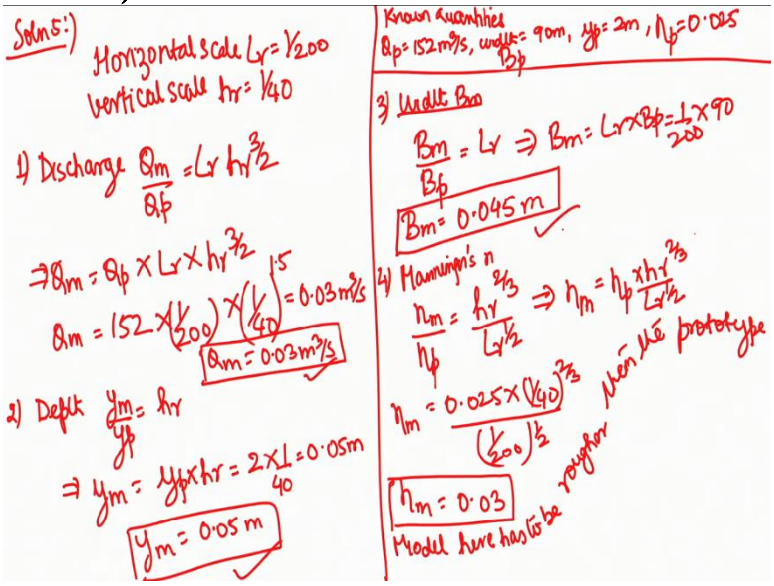

So, I will just write solution 5 here. So, we have been given, so I will just draw a line here as well, a horizontals scale, L r is 1 / 200, vertical scale, h r is 1 / 40, that is the question. So, first thing to solve is discharge. So, Q m / Q p we have already found out, it was Lr into hr to the power 3 / 2. So, implies Q m is going to be Q p into L r into h r to the power 3 / 2 and Q p is 152 meter cube per second multiplied by 1 / 200 and h r is 1 / 40 to the power 1.5 and this will give 0.03 meter cube per second in model.

Second thing is depth, y m / y p is h r, simple. So, this implies y m is equal to y p into hr or y p here given is, so y p the depth was actually, so I should have actually first. So, I will also write down the known quantities. Sorry, for this. I will write down the known quantities here. Q prototype was given as, 152 meter cube per second, width was given as, 90 meter and y p was given as, 2 meter. So, y p was 2 meter into h r was 1 / 40 and it comes out to be 0.05 meter. So, that means in model it is 0.05 meter, y m and Q m was 0.03 meter cube per second.

Now, the third thing is width, B m we have to find. So, B m / B p is the length ratio. So, this implies, B m is, so width is B p basically. So, B m is L r into B p. So, L r 1 / 200 into B p was 90 meters. So, B m comes out to be 0.045 meters. So, now, Manning's n, this is important. So, Manning's n, n m / n p, you have to find out the formula but it comes out to be, h r to the power 2 / 3 divided by L r to the power half. This implies, model n m is Manning's number in prototype into h r to the power 2 / 3 L r to the power half.

So, in the known quantities we were also told the prototype roughness was given, that was 0.025. So, this is 0.025 into h r was 1 / 40 to the power 2 / 3 divided by 1 / 200 to the power half. So, eta m comes out to be 0.03. So, prototype roughness was 0.025 and the model roughness is 0.03. So, model here, has to be rougher then the prototype. So, if we have seen that everything is reducing the discharge in model has reduced, the y has reduced, the depth, the width has reduced.

But the special thing to note is that the Manning’s n or the roughness, so model here has to be more rough than the prototype. It is an interesting result, some things to ponder over. And with this, you know, I would like to finish this module. We have some more questions but we can, you know, take it up in either forum if you have come up with anything else or in the assignment that you are given. So, thank you so much for attending this module. I will see you in the next week.