Fluid Mechanics
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Lec 29: The Navier-Stokes Equation part 2

Good morning. Let us start today's class on Navier-Stokes equations. In the last class, we have derived the Navier-Stokes equations, which is the four equations, mass conservation equations and the linear momentum equations. As we have the four equations, as well as we have four dependent variables like velocity field u, v, w. and the pressures for incompressible flow. So, solving that equations you can get the solutions velocity field and the pressure field for incompressible Newtonian fluids.

Please always have a remember what are the assumptions we have when you are deriving basic fluid equations today as we are discussing about Navier-Stokes equations. Today I will talk about the Navier-Stokes equations, how we do the approximations of Navier-Stokes equations for a simplified fluid flow with giving a series of illustrations that how we can simplify these Navier-Stokes equations. Let us look it back Navier-Stokes equations which looks like this. If you look at this Navier-Stokes equations which is in Cartesian coordinates if I can write in vector forms which is easy to remember it is del cross dot v divergence of velocity vectors is 0 for incompressible fluid flow and that is the equations of continuity equations and that when for incompressible flow that is what you can look it as a component wise we will get this component is equal to 0.

That is what we get it for the mass conservation equations in differential forms, which gives us that for incompressible flow, so we have the gradient of the velocity field, the scalar field of u with respect to x. v respect to y and w respect to z that is the basic way you can just remember it the continuity equations either you can remember these equations which is very simple things is velocity divergence the del dot v if you do not remember that you can always remember this part if you have the scalar components of a velocity field of u v and w which is then x, y and z directions. So, the incompressible continuity equations in partial differential form will have a gradient of u, the partial gradient of u in the x directions plus the partial gradient of v in the y directions, partial gradient of z in the z velocity w directions. That is the very simple way we can remember either this form or you can remember this form which is the continuity equations, the mass conservation equations for incompressible flow, that is what it again I have to report it is a incompressible flow, mass conservation equations which you get it when the density is a more less constant, density does not vary significantly for that cases we can get it a simple conservation equations, mass conservation equations which we can physically also interpreted this part. Now, if you look at the next the linear momentum equations as we have derived this is linear momentum equations in vector forms.

So, in vector forms it is comes like rho So, it is the accelerations part is equal to is rho g grade p pressures into mu Laplace operators of velocity field. that is what you have to just take it you can always have a look it is mass into accelerations component for unit volume that is what is components is acceleration mass into acceleration equal to the force component. This is the gravity force, this is the pressure force component, these are force components what we are getting it because of viscous stress components because of the viscous stress component. Here we have a the dynamics viscosity or the coefficients of viscosity we are assumed to be a constant that is what the isothermal case. The temperature does not change drastically that is the simplifications what we have done it that the mu does not change with the time does not change with the time that is what it happens for isothermal cases.

So, if you look it that way we have a certain assumptions here the Newtonian fluid. So, assumptions what you have the new TNNN fluid that is the one of the assumptions which we have considered it. We have considered the isothermal case the mu is a constant, we have considered incompressible flow where the rho is a constant. So, all the three assumptions are giving us equations which expanded in the x directions you will get this part that is same vector equations I should have a practice to you should have a practice to expand the vector forms that is always you can practice it. So, this is what if I write that equations in x directions I will get it.

this is what I will get a local acceleration term I will get the convective acceleration terms. So, convective acceleration term you can always remember it if I am doing del u by del x that means it is multiplied with u you have a del u by del y multiplied with v del y del z multiplied w always if you just expand the terms you will get it that part. So, this is the local acceleration, this is the convective acceleration. But if I interpret these equations as a mathematicians, this is what the non-linear terms, this is what non-linear terms because there is a multiplication of u and del u by del x, v del u by del y. So, there is a non-linear terms, it is there in a convective accelerated terms.

We have a pressure gradient part, we have a gravity force and we have the Laplacian operator of the velocity in the u directions. If I just expand it, I will have this form. So, advantage of mu is a constants. So, advantage of Laplace's operator is a linear equations that is what we can get a analytical solution for that. We can get analytical solution for these because it is a linear equations even if it is a second order equations looks like that, but we can have this.

So, this is a linear equations partial differential equations. of u in a three directions of x, y, z, three directions of x, y, z. So, you just to have to interpret these equations. This is the vector forms. And if I write in x directions, I will have a local accelerations, the convective accelerations which will have a non-linear terms, I will have a pressure gradient, the gravity force components I can look it does this gravity force dominate in that flow field what I am considering.

the viscosity does it dominate it. So, all these terms I have to look it which are having a dominancy part. How I can simplify these equations for examples if I consider the mu is very very close to 0. So, it may have the problems I will give you the illustrations how the reasons where you can see that the mu is a close to 0 or If you have these conditions or the velocity does not change drastically, the Laplace operators of the velocity fields does not change drastically, we can neglect this part. We can neglect this part.

So when you neglect that part, it really gives a very simplified equation. If I read it again, the equations that I have the linear momentum equations I just again writing for you linear momentum equations. Vector forms what we get it the rho d v d t this is the accelerations component minus rho g del phi plus the love as operators of v. Laplandian operators of V. This were the vector forms linear momentum equations.

As I was discussing is that if it is a conditions where these terms are 0 in a particular fluid flow, if these terms is 0 then these equations becomes comes as a very linear form of very simplified of equations that is what we called the Euler equations that means we have rho d v by d t is equal to rho g that is what is still we are using this vector operators del into phi. So, this is what the regions where the viscous does not dominate it. So, that the reasons we can apply the Euler equations which have only these two force components gravity force and the pressure force mass into accelerations that is the basic equations we have. Now, if you look it where you I have the problems of when we look at this dv dt as I discuss it that the we have the problems with this non-linear terms. So, this is a non-linear terms That is what is the problems to look for the analytical solutions.

No doubt, today we have a computational fluid dynamics with us. We can solve this equation. It is not that complex. As I discussed that we are doing for a very simplified cases like Newton fluid, incompressible, isothermal, also the laminar flow. But when you go for a very complex flow like turbulent flow, you talk about temperature changing, we can have another equations on energy conservation.

That is we are not discussing under this undergraduate courses level. So, if you look at that also for the turbulence flow. So, we will have a very complex equations we will get it. Today, we can solve that equations. All these complex equations we can solve using computational fluid dynamics.

But in this course, we will try to simplify the equations to try to get analytical solutions for that. That is what the strategy. If it's that, one of the cases that we can identify the problems where the viscosity doesn't play the major roles. So we can identify the reasons within the fluid domains where viscosity does not play any dominant roles. If you find out that reasons, we will have a basic equation solver is the Euler equations.

It is a simple Euler equations will get it. Now, if you look at these terms, if I look at this d v by d t, if I just expand it I will have a in vector forms. So, I will have these components is a local accelerations component which is with respect to time and I will have a components V dot grade. Please look at some of the vectors calculus part or vector relations in any of the engineering mathematics books. I am not going more details.

The same components I can write in this form. If I expand it in x directions, I will get these terms. That is not a big thing. So, always you can d dot del to the v, that is what is we are getting it. Now, if you look at these equations, which is having these components.

Here there is a non-linear terms, there is a partial derivative with respect to time. So these terms will be 0, the conditions where you have a steady flow. So if I have a steady flow these components become 0. We're just looking at how this component we can handle it. That's the strategy of today's class, that how these convective accelerated terms we can simplify for a few of the cases.

So if you can solve that part, that's what we tried to simplify these convective acceleration terms. We'll talk about how these Euler equations starting from these Euler equations, we can derive very basic equations as you know it is the Bernoulli's equations. That is the today's class. We will talk about how we can derive these Euler equations along the streamlines that we can derive as a Bernoulli's equation. So, if you look at that whenever you apply the Bernoulli's equations as we discussed previous classes that we do a series of assumptions.

And based on that series of agent terms, we apply the linear momentum equations along a streamlines. That is the Bernoulli's equations. Those things we will discuss more, but please remember it all the things are derived from very basic equations is the Navier-Stokes equations. From that in a stepwise we simplify the problems and the simplified solutions is as far with a certain there is a series of assumptions which we will discuss by phase by phase to get the Bernoulli's equations. which as I discussed many times these equations we misused many times also used many times.

So, many of the times you apply the Bernoulli's equations also you do not know are the assumptions behind that. I do believe it, I do encourage you that please go through the Bernoulli's equations in the next levels as we derived earlier class along the streamlines. Today class we will talk about with a simplifications of Navier-Stokes equations with series of approximations we can get the Bernoulli's equations that the things we will discuss more. Let me go to very interesting part is that many of the times also we can write the same form for cylindrical coordinate systems. We are not going to discuss in this class, but please have a practice to write this mass conservation equations and r components.

As you know it is a cylindrical coordinates we have a r theta and z. So, these equations are there. It looks complex equations, no doubt about that, but please has to look at these equations which also is the same form of what we discuss of, this is the same vectorial form of what we discuss for the Cartesian coordinate system. Now, let us come back to very simple examples. Like for examples, we have a The heart blockage, you know it.

This is very common in artery systems who have a heart blockage. That means if I have a blockage, okay, this is pipe flow. and the blockage. If it is a symmetry this blockage is a symmetry that means blockage what is happening in this side this side is same. So, if I draw the streamlines for this so you can understand it the middle streamline will go like this and other flow patterns will be change it like the first one will come like this come like this go like this.

come like this go like this if it is a symmetry we can draw the streamlines that is what. So, these examples if you can consider is a blockage is there that is the blockage part we are looking it we there how the streamlines are coming the flow is coming from top to bottom flow directions here and these are the all the streamline. In case of symmetric vortex that is what is in a biological science you could locate heart blockage in natural science where we are talking about river with a bridge we can have a constraint, we can reduce the dimensions of the river systems. So, width of the river systems as we do it we are constrained it. So, if you look at that we can draw the streamlines and we can have the interpretations that what the boundary conditions here, the flow boundary conditions and what is the boundary conditions here that is what we can do it.

We can look at that what is the boundary conditions from upstream and what is the boundary conditions. So, this can be as small as at a as big as a bridge you know on a river systems where you are constrained it. So, this is what a symmetric case. which we can easily draw on it and we can conduct the experiment we can get it that how the streamlines patterns are there. So, if I know the streamline patterns I will get the velocity field and I will get it the pressure field as I get the velocity pressures and density field I can find out stress field I also can find out the shear stress field all the field I can get it to know it much of force is acting on these band sites if you talking about like in a artery systems I can talk about the blood or we talk about air if I considering a part of engines or you talk about water or the sediment for a river.

So, the flow the medium the flow fluid will be change it whether it is a blood or the waters or the air depending of the problems what we are considering or depending of the size whether you have a big size like a river or you are talking about very tiny small sizes like artery in a human body. So, if you look at that we have a these type of conditions. So, you my objective here is to know it try to understand the fluid flow and try to simplify the fluid flow that is the strategy of a fluid flow specialist. So, simplify the problems that flow is coming, going out and this is the no flow boundary conditions and in case of the symmetry, I will have the streamlines and from the streamlines, I can find out using this Navier-Stokes equation solvers, I can get the velocity, pressures, density. stress components, the shear stress component acting on the walls that's what will be generate the force of that.

My fluid can be a blood, can be air, can be waters any mix of this water with the sediments and all. So that that's not a big issue. The issue is that we have to simplify the problem. The same problem if it's not having a symmetry like asymmetric problems like what you are saying it here. So I just that means the connector between these two is not in symmetric.

So, this is a symmetric case which is easily can understand it. When you go for axisymmetric case as you can understand it your streamlines will behave like this and this. The streamline of this will come with this like and this. So, if you look it that way the same only if you are changing blockage. This is symmetric, this is not a symmetric blockage.

This same thing, you can understand it, heart blockage. If it's a symmetric, that's for different conditions. If you have a, not a symmetric, one side of the walls, you will have more depositions, another side you have the less deposition. your the streamline patterns of the blood flow that is what it changes it as it changes it your velocity distribution change it it is that is what if you see this the velocity distributions will not be symmetric you can see this is the velocity this is the velocity this is the streamlines and you can see zone of the rotations zones or formations are there as compared to this case the rotations only happen in very smaller regions. So, you can see that flow fields the vortex the will be generated because of the blockage, because of this blockage you will have this not non-symmetric cases you will have this vortex generations.

So, the distributions of you can just compare it the distributions of pressures the distributions of this pressure this is the pressure diagrams that is what we will also change it and the velocity will change it the vortex formations will change it that is the fluid mechanics strength. It can apply for biomechanics, it can be applied for turbo machineries, it can apply for any civil engineering structures and all. So basic ideas want to convey it, you always will try to simplify the problems. To have a knowledge by drawing the streamlines, drawing the anticipated boundary conditions. That knowledge has to be developed with by solving a series of the problems that what type of flow patterns will be there.

As the symmetric case and unsymmetric case, how will you change it? As I will change these flow Reynolds numbers, as it expected it, these patterns will change it. If I change the flow Reynolds numbers or if I increase the flow Reynolds numbers. whether if you will go for laminar or turbulence, the flow separation zones, all will change it. The patterns, whatever, getting it, that what will change it. in case of from the symmetric to asymmetric as I change the flow Reynolds numbers that what will change it the flow can change from laminar to turbulence there can be change of having the flow separations.

So, if you just talk about this that the human body having the arteries if you have more or less is a circular pipes. So, in that case when you have a this slight fit blockage how your flow patterns within the arteries are changing it. As you go for different Reynolds numbers and you can understand these the biomechanics which is really interesting today's world and fluid mechanics plays much more really say applications today to solve many of the problems but in this course anyway I will not go to much that levels but I just want to encourage you understand the fluid flow problems not to just solve navistic equations solvers we have approximate solvers we have from by using computational fluid dynamics, but understanding the problems always need how many fluid mechanics problems you solve it. That's my encouragement. That's the reasons we have been following one of the two best books, Sinjal Sembala and Appam White.

So please go through this, solve these exercise problems, try to gain the knowledge which you can use in many of the fields. It's not just to look at how to pass the exams. Not looking at that, we have developed the course which is Taking that, this is the throat of the channels where it can have a flow rapidly accelerated. That's what we discussed a lot.

Near the throat, you can look at the streamlines. As the streamlines, the width of the streamlines are decreasing patterns, it clearly shows that the accelerated zones. There could be a flow separations, that is what is there. There could be a flow separations at the downstream of blockades. Large recycling regions or vortex, you can see how the vortexes are happening, how these pressures are going to change it, that is very interesting things between the left and right walls. So that is the reason whenever you have any sort of the problems in health, it gives really a lot of symptoms, that you have to try it, that just you think it, why do we have the fevers, why you have an increasing of the temperature of your body which is having 98, close to 98 percent of the water bodies, the fluid systems, that you try to understand it.

I am not going to that level of biomechanics here, but please try to look the fluid mechanics, it is much higher level, not only solving these navier stupefaction. Now, let we get how do you solve the problems as I said it one of options with you us by numerical solutions. which is computational fluid dynamics. I am just introducing to you very simple way, but there are a series of courses will be available on the CFD course. So, if you use the numerical solutions of the CFD, now for these navier strict equations, I will get it the approximate solutions of U, V, W.

This is what I am designating. These are the approximate solutions, the density and the pressures. These are not exact solutions. These are approximate solutions. More details you will get it when you learn what is this computational fluid dynamics, but we get the approximate solutions. But technically, many of the times we should simplify this Navier Stoke equations as we are discussing it that we should simplify the Navier Stoke equations.

Let me give a simple example of the tidal energy, which nowadays very famous for the cases of getting the renewable energies. The tidal energy harvesters are nowadays very interesting components are there. Like for examples, you will have a bed. You construct a turbine and the flow comes like this. This is the tidal which comes from these, this is the upstream, this is the downstream and you try to look at this is the air, this is the waters, You try to solve these problems using computational fluid dynamics or you try to solve analytically to know it how much of energy harvesters will be there if I just put a turbine below a sea level where the tidal waves will rotate these turbines and we can harvest the energy.

That is what is the tidal energy. So, you will have a wave as you know it So, if you look at this way there are lot of interesting problems we can solve using the CFD. Always you have to look at this appropriate assumptions as I said it I repeatedly telling it the Navier Stoke equations what I have solved what we have derived it that is what is only for incompressible new TNN and laminar flow. So, we have not considered the turbulent flow we have not considered temperature variabilities, we have not considered density variabilities, we just consider the Newtonian fluids. So, even if that we should know what is the simplifications we are doing it, what is the boundary conditions we are imposing here. Like for example, if you look at what is the boundary conditions we are imposing here, whether the velocity which is coming with the time it changes it, what is the boundary conditions we are What is the boundary conditions we are imposing here on the bed of the sea? We can use the no flow boundary condition.

What are the boundary conditions we are imposing at the interface between air and waters? This is the interface between the air and waters that is what is the free surface boundary and what is the boundary conditions we are applying here. So we need to understand it that what type of boundary conditions we are going to impose it for this tidal energy harvesters that if I have the velocities at the is coming it with the functions of the t I can know it how this tidal waves comes it what is their periodicity, I can amplitudes all I can know it, but I should know it there will be no flow boundary condition this is a free surface and also I should have the boundary conditions at this point so that we can define the problems that is what we can look it. As I discuss it that We will not look it to solve this Navier-Stokes equations as it because it is a non-linear two-dimensional equations. We try to simplify these equations. That is what I will give you some of the examples how we can simplify these equations.

Then once you simplified it, most of the times we convert these partial differential equations. with all these assumptions, we make it ordinary differential equations. That is what the strength we make it, that with all these assumptions, which is a partial differential equations, we make it to a ordinary differential equations, which very easy to integrate it with subjecting the boundary conditions, we can solve these integrations constants. That is the strategy what we follow it.

What we do it? simplify, scale the problems. That is the reasons I am trying to tell you that please try to scale the problems. If you scale the problems, you can simplify it. That is what it is. Like for examples, if I just want to draw the streamline of these flow patterns, even if I do not solve it, if I just used a streamline, so I can draw a streamlines.

I can draw a streamlines. I can draw the streamlines and I can try to understand it, how it is happening it. So, always a sketch of problems, try to find out what are the boundary conditions, what type of plume it is, it is a one-dimensional, two-dimensional, three-dimensional, which are the components we should consider in the Navier-Stokes equations, which are the components we can neglect it, it does not they do not dominate it. So, if you do it basic our idea from partial differential equations to convert to a ordinary differential equations with respect to velocity or the pressures and we just do the time integrations of that then the constant of the integrations we get it by applying this boundary conditions, by applying this boundary conditions we can get the constants that is what is the solutions. So, if you look at that as a summary I can say it that in when you use the CFD computational fluid dimension tools we get the approximation field of u v w and the density and the pressure. But same way, when you try to get analytical solutions, it is your way how many terms you neglect it, what are their assumptions you are putting it, so that you can simplify these equations, which is nonlinear, second order partial differential equations to a simple ordinary differential equations and you solve it.

So these are two arts we have. One is the tools, using that getting the approximate solutions. Another is based on your knowledge, based on fluid mechanics knowledge, you simplify the Navier-Stokes equations make it to a simple ordinary differential equations then you solve it that the strategy we follow it. Now, if you will go for the next part which is the boundary conditions which I just introduced to you that first conditions is no slip boundary condition. I think this is what I very Intentionally I introduce, this is in the first class of fluid mechanics class.

So we introduce the no slip boundary conditions. That means that's the basic properties of the fluid flow. That means if a fluid is moving, velocity of V, the fluid attached to these also will be moving the fluid particles attached to this, the fluid particles this fluid V f and another ones moving with V m. So, both because no slip conditions both has to move it the same velocity that is what is the no slip boundary conditions because of that as you know it when you have a plate with a uniform That is a very simplified conditions. You have a plate, you have uniform flow, this as equivalent that is a uniform wind flow is happening it and you have a plate as it expected is that. So, because of this plate you will have a low velocity zone and viscous components will be dominated here.

And your the streamlines will go like this, like this, like this, like this. So, if I plate is stationary the velocity fluid particles energy start should equal to the 0. So, velocity along this will be 0, along this will be 0. And these are the reasons we will discuss more is boundary layers concept what we will be talk a bit more. So, you will have the boundary layers the beyond the regions the flow is irrotational flow is irrotational that is the concept. Then second part is interface boundaries like you have a two fluids for example, we have the open channels and the flow is coming you have a water, you have a air.

This is what the interface boundary conditions, this is what interface boundary condition. Interface boundary conditions, if I zoom it, take a small elements, take a small element, if I draw it as the no slip boundary conditions will be there between two medium and B, the velocity component of V A, B, V, as vectorically they should be the same. Magnitudes and the scalar components should be the same. Not only that at the interface the shear stress acting on the surface T tau S A and tau S B that should be equal. That is the conditions of the interface between two liquids or liquid and gas between two gases.

So, you will have the interface between two conditions at that locations definitely because there is no slip boundary conditions because both are moving it. So, the velocity at medium A should be equal to velocity of medium B, B equal to V A and also the shear stress will be equal to the acting that should be the equilibrium. That is the shear stress acting on the A medium and B mediums on the surface will be the equal. So, if you look at that, that is what the basic equilibrium equations in terms of no slip boundary conditions and the equilibrium equations for the pre-surface boundary conditions. That is what you can understand it here, here it is A and B, here it is also you come with B and A, that is slight bit change, no problem.

Now if you come to very basic things as we always look at interface between the air and waters that is many of the problems we solve it. So, in that case if I look at u water should be equal to u air that is ok. So that means at the interfaces levels I will get it the velocity of water is equal to velocity of air at the three surface levels. Also I will get it the shear stress working on waters that is what will be shear stress working on air that should be. But, if I use the Newton's laws of viscosities that means I can put it mu waters is the gradient of velocity in the y directions at the water that is what we will mu air into gradient of velocity of u at the air.

So if you look at that, that is what I just used the Newton's laws of viscosity, relationship between stress and the velocity gradient, that is what I have. Now if you look at the problems as you know it, the dynamic viscosity of the water will be much larger as compared to the dynamic velocity of waters. For example, m waters as larger than 50 times of m waters. That is the reasons. So if somebody is 50 times or larger, then definitely this will be the smaller than the 50 times.

The gradients will be the smaller than the 50 times. So it's a 50 times larger. If I equating these two part, so that will be the smaller than the 50 times. that is the reasons we consider it and always you can draw the velocities which is very interestingly here that if there is a interface and if I am to draw the velocity distributions the in air I will have a very fast the gradient to support these boundary conditions But in case of waters, we will have the less because this is what is 50 times lesser than that. So I will have a velocity distributions like this.

And this is the inter-surface. And I'm getting the velocity distributions like this part. Now if you try to understand it that thus how we are getting this part. So that means that component is has to be very less if you can try to understand it that is the reason very simple boundary conditions we put it. the pressure or the waters, liquid is equal to waters and shear stress becomes a zero.

That is the conditions we will prevail it. Just try to interpret it these components. You can always get it the shear stress acting on the at this free surface should equal to zero or very close to zero values. Then we can have these problems. Same way another components many of the times we solve it. boundary conditions. What we looking it like many of types we have the flow it is comes like velocity distributions like this.

If you have these conditions and it is a symmetric if you take it this surface it is just a symmetric mirror image. If it is a mirror image and the symmetric cases no doubt looking this velocity distributions if this is the u and this is the y easily we can say that the gradient at that symmetric lines will be the 0, that is will be happening because looking this velocity distributions, looking the symmetric lines, you can say that along the symmetric lines the del u by del y will be the 0, that is the slope, slope at this point will be the 0 because that is the picking points. Now if you look it what will be the v, v also will be the 0. So, if I have a symmetric problems instead of solving total problems we just solve this part subjecting the boundary conditions of the v equal to 0 and the gradient of the u in the y direction becomes 0. So similar way, many of the cylindrical flow, pipe flow also we simplified it, considering the symmetry of the problems.

But please try to understand it. how what is we are expecting the symmetric flow whether all these conditions are fulfilled for the symmetric conditions otherwise you will have a difficulties to apply these symmetric conditions which help us to simplify the problems from three dimensional to two dimensional or make it a problems very simple way to solve if we can really identify the symmetry conditions, boundary conditions and apply it. So, that is the components what we look at. So, let us go for a as I discussed that using the Navier-Stokes equations we can simplify it find out these basic derivations of the Bernoulli's equations from Euler equations that derivations part I am showing it how will this mathematics components are available in f m white book. So, please go through that I will be conceptually I will try to explaining you that how we can derive from Navier's equations to Euler equations. From Euler equations we can derive it what is the Bernoulli's equations that is the part we will discuss now.

If you look at the basically we have the Euler equations which for frictional less or less viscosity, that viscosity is not dominate much. Those conditions where the viscosity does not dominate much, that is the reasons where you have a less frictions. If you have that, that is the equations is the Euler equations as we look it d y by dt mass into accelerations per unit volume is equal to rho g grade p, that is the equations. That is the equations, that the equations were this. We can look at this component as earlier I wrote it, this component which is the accelerations component, I can put it local and convective accelerations.

Just look at the vector relationship. So, you have a convective accelerations components what you will get it that is what is here that is what is here. Now, if you look it that this relationship which is very interesting we are looking it how we can make it this component simplified it. From the vector algebra any of the books if you look it vector algebra we can split the sequences to two parts that is what again I am writing it. So, dot product with del V with a vector algebra you can look at any of the vector algebra books we can write it del gradient of half V square is all our vectors curl V cross V that is the new curl V is is cross products between del and the v that is what is called v that is what is fluid verticity that is what is the fluid verticity. So, if you look it if I just expand this I will get these terms which is always we are familiar with half v square.

and we have the terms which is coming its tau star cross V. This is a cross products between tau which is a curl V, cross product between V again you have the V. So that expansions we are getting it here. That is expansions we are getting it here. These components if you can try to remember it, this component is there in the Bernoulli's equations.

That is why the idea to take it this form so that we can close to the terminology of Bernoulli's equations. We know if there is a pressure component in Bernoulli's equations, there is a V square component there. and you have the rho g is there. So, we can get it three the heads what we are talking about But there is a new component which we are getting it by deriving these equations that is a cross between velocity curl and the V-vectors. This part I can simplify it as is given in FM White books. We can simplify these terms assuming it I have this velocity field and there is a dr So, the elemental distance of the d r and if I try to integrate that is this is equations I am just replacing it.

So, I just try to integrate these equations rho d v by d t minus rho g del phi integrating these equations, integrating these equations with del dot product of dr, dr is the vectors component is there. So, if I do this I will get these terms, I will get these terms interesting component of this as well as I will get g and dr components. the major conflict for me is these terms. So, I will try to look it at what conditions these terms become 0. If these terms is 0 which is having curl vectors cross V vectors which is totally non-linear components what I am getting it.

if I can say that this is becomes 0 then it is for me it is easy for integrating these terms because these are easy to integrate it along this field they are. So, only these terms at what conditions this becomes 0. If you look at that that is what is gives us all these assumptions what we have in Bernoulli's equations. These terms plays the major roles to say which are the conditions will have a the this cross V vectors is 0.

So, even if it is there I am just repeating it to understand it which are the conditions these terms becomes 0. One is flow at the rest we do not bother about that anyway we have a governing equations for that when you have a V equal to 0 that is not a interest for us because anyway we are not looking for equations for that tau will be 0 because when you have a flow is irrotational. When you have flow is irrotational, just look it, flow is not an outer rotating it, there is no velocity variation, the flow is more or less non-irrotational or irrotational flow that is what is the tau will be 0 that the conditions this is also will be 0. that is our conditions will be the curl vectors indicate for us the. So, when you have a irrotational flow which is very easy things that is not a big issue. But you have a d r is perpendicular to that as you have the dot products this can we have a specialized conditions rare condition.

But d r is parallel to V that means we have a velocity field d r is this it is a parallel to the V what is meaning that? That means this the line is the streamline. This is the line the streamline. If the DER is a parallel to V that means this is the line is representing the streamline. So if that is the conditions also it is quite valid. So we are talking about or we are going to derive the Bernoulli's equations with a basic assumption is that along or streamlines because these terms becomes 0.

The tau cross B is equal to 0 when we want to integrate along the streamline that is the reasons previous classes I did a integration along the streamline we derived this linear momentum equations. But from the Navier-Stokes equations with these assumptions like Euler equations then looking this vectorical expand of the law Euler equations the convective acceleration terms again we are coming to a conditions fulfilling that along the streamlines that tau cross V is equal to 0.

So, tau cross V is equal to 0. So, that is the along the streamlines it is valid. Again, I am emphasize that. So, not to use the Bernoulli sequence just like that. If you draw a streamlines, if you know the streamlines, along the streamlines you can use the Bernoulli sequences. That is no problem at all. That is the basic definition, the assumptions are there.

So, if you look at that, that means tau cross V, you will be equal to 0. If that components are 0, then I think you have very simple things to do the integrations. G dj is there and d rho half, this is this. So, along the streamlines, if I consider it, streamline and having points 1 and 2 this is the streamline S1. If I consider this still it has unsteady components the integrations between 1 and 2 the streamline 1 and 2 that is how V1 2 square V1 just do the integrations you will get this part.

Further if you simplified it that is a steady flow. If you have a unsteady flow, always you can start these Bernoulli's equations, these components. That is what we should practice it that if you are not solving the steady flow, if you are solving these unsteady flows, you can start these components as well as you can start these three components along the streamline to get it how the velocity variations there, how the pressure is there. but in case of the steady flow that is again assumptions you have brought it steady flow, streamlines, frictionless, I think it is too idle. We are thinking it is too idle good boys to follow everything starting from steady streamlines friction slates. Within these type of assumptions we are getting equations which is you know from 11th, 12th levels, okay, that's what is Bernoulli's equations.

But try to understand it, fluids are not as good as or as, it's naughty, it's naughty, it doesn't follow these boundary conditions what you talk about, steady, streamlines, they're frictionless, okay, Newtonian fluids, incompressibles as so many, the laminar flows and all. So many of the fluid flow problems which does not follow this. So you try to solve the problems which are very simplified problems. Having all these series of assumptions, again I am talking about Bernoulli's equations that steady Along the streamlines, frictionless, Newtonian fluid, isothermal conditions, the laminar conditions, all these conditions if you fulfill it.

It is, I think, it is a simple problem, but we solve it. No doubt we, that is the reason we use the Bernoulli's equation, many of the places where these assumptions are not valid. We get the results, we pass the exams, that is not a big issue, but please try to understand it, the fluids are as nerdy as we, so it does not follow these conditions what we define it. Thank you.