Hydraulic Engineering
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Lecture-29
Introduction to Open Channel Flow and Uniform Flow (Contd.,)

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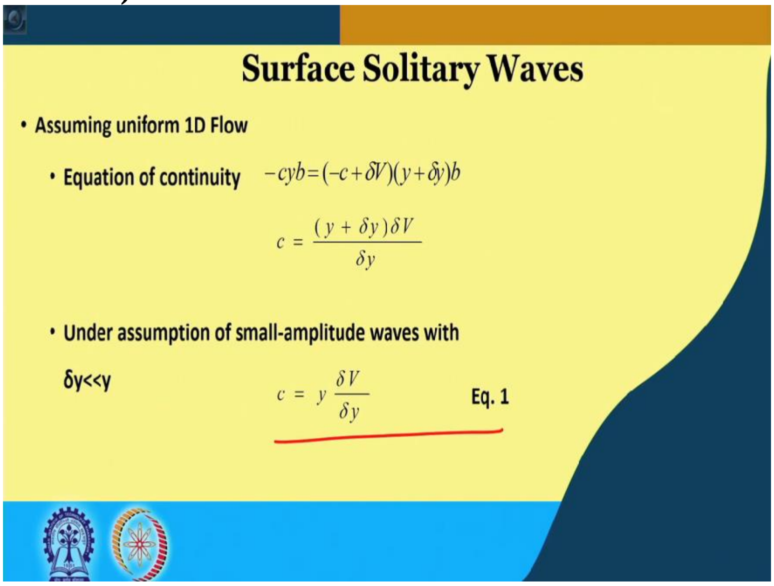

Welcome back students. In the last lecture we left off by deriving this equation 1, for surface solitary waves and we will continue from this point onwards. So, as we saw if when we applied the equation of continuity to that control volume, we arrived at an equation given by c is equal to y delta V / delta y.

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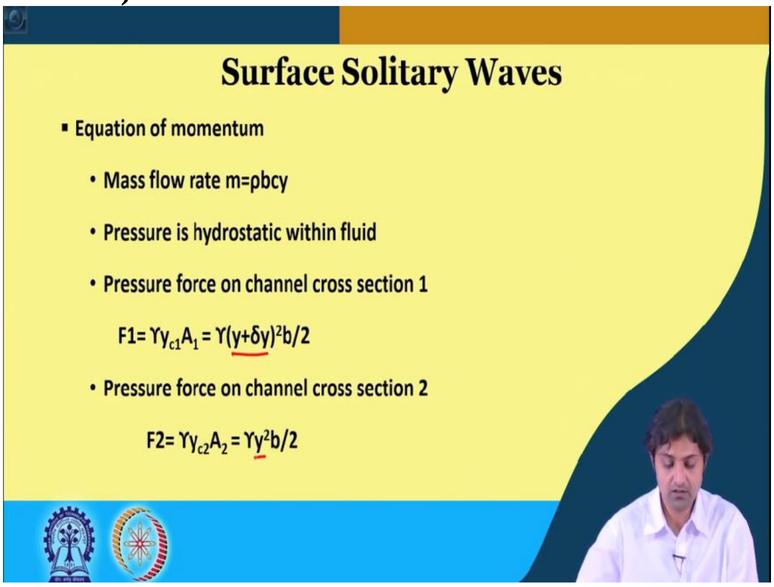

So now, we proceed further, we have applied equation of continuity now, we should also apply equation of momentum. So, how to apply? So, the mass flow rate m is given by ρbcy, ρ is the density and we need to have volume per unit time. So, b the width, y is the depth and c is the say let us say dx / dt, for example. So, this is width, this is length and this is height. So, this is the mass flow rate that is given m as rho b into c y.

So, we assume that the pressure is hydrostatic within the fluid and therefore, the pressure force on the channel in cross section 1 will be if you remember from your fluid statics class, it will be $\Upsilon y_{c1} A_1$ or gamma where the height was y + delta y. So, I will just show you, this was the height. So, if you remember the pressure diagram. So, it comes remember, so, this comes gamma into y + delta y wholes square b / 2.

Similarly, force on the channel, on the cross section 2, section 2 is the hand side of that control volume will be simply $\Upsilon y^2 b/2$. So, on the left hand side the high was y + delta y, that is, y + delta y whole square, on the right hand side was y, so it is y square b / 2.

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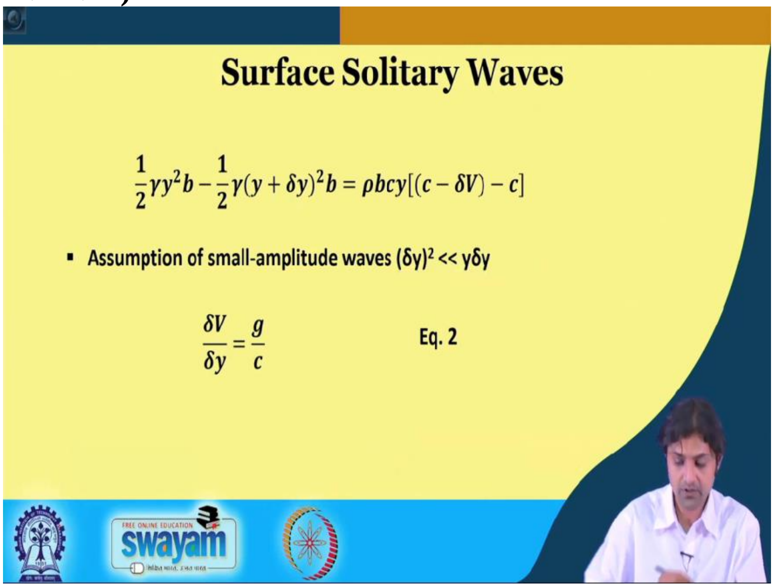

So, the we apply the change in momentum, we change, rate of change of momentum is the force. So, we obtain half gamma y square b, that is, force on the right hand side minus force on the left hand side is equal to rho b c y mass flow rate. You remember from momentum equation of fluid dynamics and this was obtained from Reynolds transport theorem. So, mass flow rate into the velocity change. So, the left hand side had c - delta v and the left hand side was and this was the right side.

So, if you assume that delta y square, so, delta y is small amplitude wave theory is already assumed. So, delta y is very less compared to y. So, therefore, delta y square will even be less. So, we can assume that delta y whole square is very less than y multiplied by delta y, and of course, this is also true, but it is also this is true as well, you know, y delta y is much larger than delta y whole square. So, if you put this and solve so, basically we try to write this one down here.

So, we just look at this term, and actually if we take half, half common, gamma, gamma common and this b, b common. So, the left hand side I am writing only just trying to, you know, half gamma, b is going to be common, it will be y square - y + delta y whole square, this is the left hand side. So, this will remain as it is, half gamma b, this will be y square - y square + delta y square + 2y delta y. So, half gamma b y square - y square - delta y square - 2y delta y.

So, this y square and y square will get cancelled, as we already said using the, this approximation, we can take one, this one out as well. So, it will be half gamma b of course, multiplied by - 2y delta y, this 2, 2 will get cancelled. So, it will be gamma b - gamma b y into delta y, now is equal to RHS. So, rho b c y, c, c get cancelled, and this becomes - delta v so, minus, minus get cancelled, b, b get cancelled, b, b get cancelled, one y will get cancelled.

So, gamma is rho g, so, we can write, rho g delta y is equal to rho c delta v. So, rho, rho also get cancelled. So, we can simply say, if we take delta y down, so, we can say delta v c multiplied by delta v / delta y is equal to g, or delta v / delta y is equal to g / c. See actually we have derived this now, going step by step, we are not going to do for every problem but I thought it would be easy for you to follow.

So, I will take down this ink now. So, under the assumption of small amplitude, I showed you how to go from here to here. So, because of this we have now this equation number 2. So, what are we going to do? So, we are going to equate equation 2 into equation 1. If you go back and see what the equation 1 was, equation 1 was c is equal to y delta V / delta y. So, this delta v / delta y we have obtained from equation number 2.

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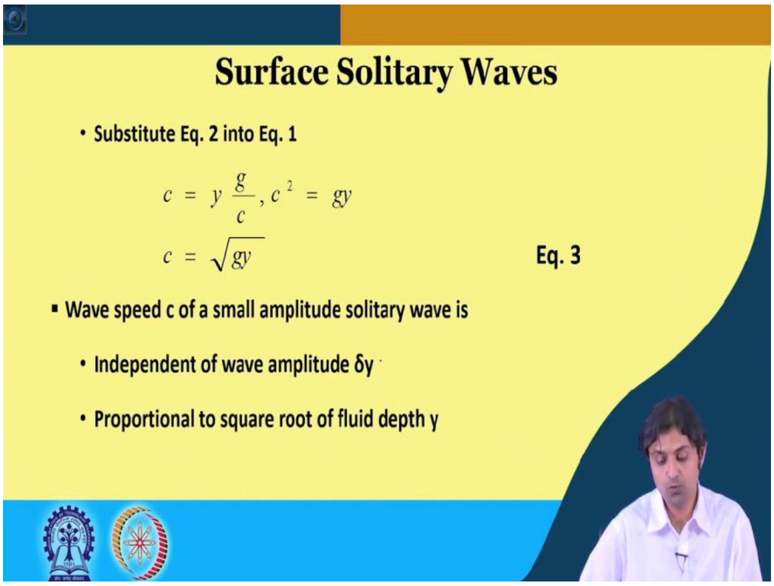

So, if we substitute equation 2 into equation 1, so c is equal to y delta V / delta y, we get c is equal to yg / c and c can go this side it becomes c square is equal to gy, or the final equation c is equal to $\sqrt{gy}$. Where c is speed of small amplitude surface solitary waves, g is acceleration due to gravity and y is depth of water in open channel. And this is equation number 3.

So, the result is that the waves speed c of a small amplitude solitary wave is it is independent of the wave amplitude, if you remember we have assumed that the amplitude of the wave was delta y. So, this c is does not depend upon delta y. It is also proportional to the square root of fluid depth y, and c is equal to under root gy. Which means it is proportional to square root of fluid depth y.

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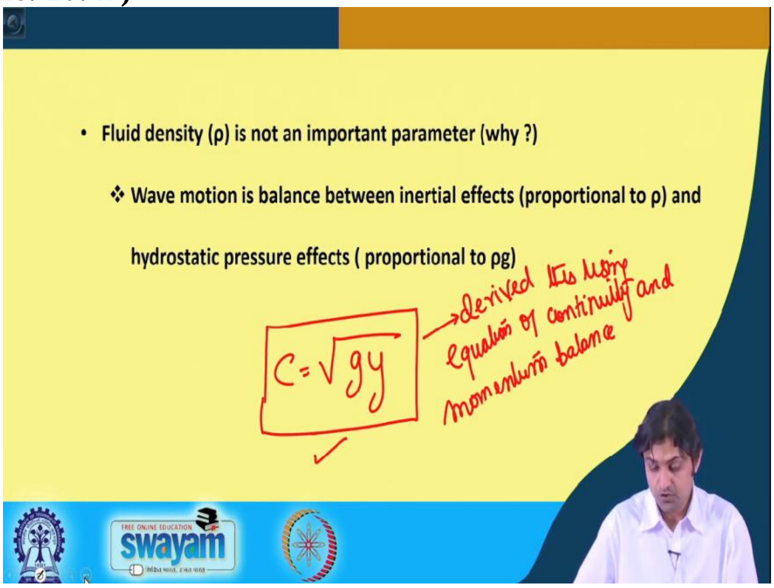

Here, fluid density is not an important parameter. And why is that? I think you think about it and we can later discuss it in the forum. So, actually, I have written down the reason, it was not supposed to, but it is okay. We will I will tell you; the reason is the wave motion is a balance between the inertial effects, which is proportional to rho and also the hydrostatic pressure effect, which is proportional to rho g.

In that way rho and rho gets cancelled out from both side and that is why in fluid density is not an important parameter in this wave of the speed of the wave. So, one important thing that we have, one important result that you must always remember is c is equal to under root gy and we had derived this using equation of continuity and momentum balance.

So, if you are given a question, this is what you are going to use for a single solitary waves that is the only question that you should be remembering to solve this question. So, I said that this was derived using the continuity and momentum balance. However, the same result can also be obtained using

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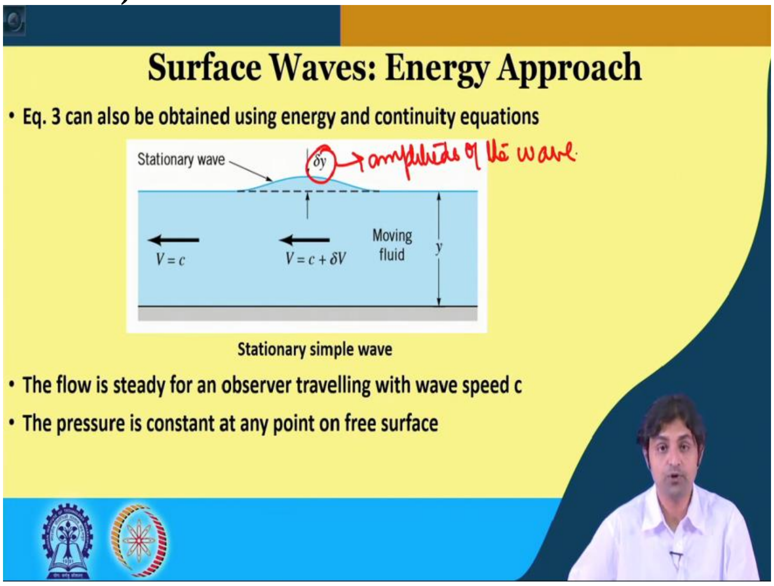

The energy balance approach. So, the equation number 3, c is equal to under root gy can also be obtained using energy and continuity equations instead of momentum. So, you see this simple stationary simple wave. So, when the observer is moving with the wave to him, this is, I mean, it is moving with the same wave speed. So, this wave will appear to be stationary to the observer, other things are same.

You see, the fluid is depth is y and the velocity behind him, you know, is c and that is moving in this direction because of the wave speed and the speed below the wave is c + delta V. And the flow is steady for an observer travelling with the wave speed c. And delta y is the amplitude of the wave. One of the other assumption is that the pressure is constant at any point on free surface, hydrostatic pressure assumption.

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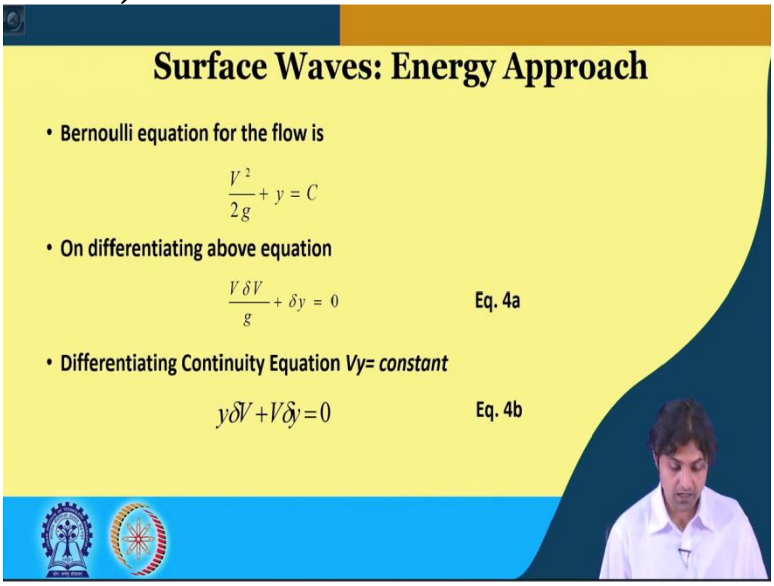

Now, we apply the Bernoulli's equation for the flow. Bernoulli's equation says that because there is the pressure is constant, so the pressure term will vanish, so it will become $\frac{V^2}{2g} + y$ is equal to constant. This is Bernoulli's equation So, if we differentiate this above equation, so what we get when we differentiate this with respect to y, so we get V square will become, so I will try to define 2V delta V and 2g will remain the same.

And y will become delta y is equal to 0 because constant when differentiated, we have differentiated with respect to y, so 2 and 2 gets cancelled. So, we get this equation, $\frac{V\delta V}{g} + \delta y = 0$, same as this one here. And we call this one equation 4a. Now, second step is we differentiate the continuity equation. Continuity equation is V y is equal to constant. So, we will apply the chain rule. So, we will get $y\delta V + V\delta y = 0$, same like this. This is equation 4b.

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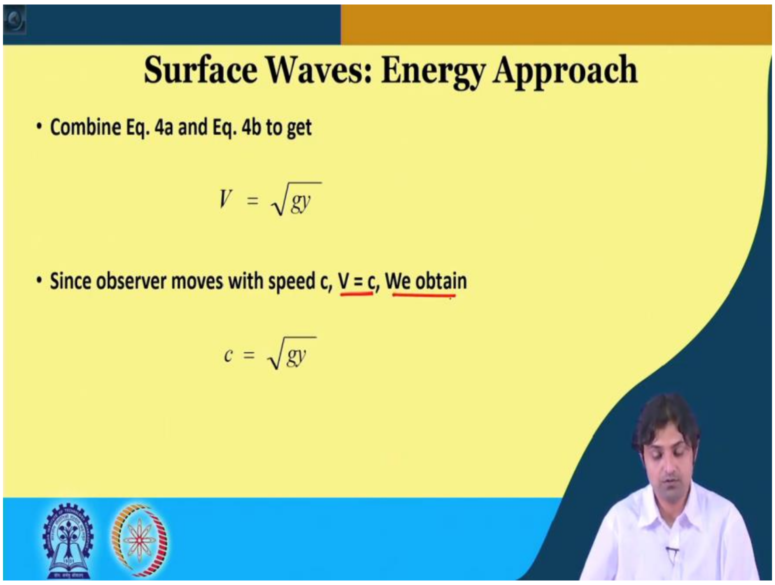

And if we combine, 4a and 4b, so let us try to combine 4a and 4b. So, what we do is, so the 4 a let us see, V / g is equal to - delta y / delta V from 4a. And from 4b, we can write, V delta y is equal to - y delta V. And if we bring this down it is become delta y / delta V is equal to - y / V. And we put this one here. So, we are going to get V / g is equal to minus of minus becomes plus, so this will become y / V and this will become V square is equal to gV.

Where, V is the speed of the wave. But yeah and that is what is written in the next slide. So combining equation 4a and 4b, we get V is equal to under roots gy. Since, the observer is moving with speed c that means V is equal to c. We will obtain c is equal to under the root gy which is the same equation that we have obtained from the other method, you know, momentum and continuity.

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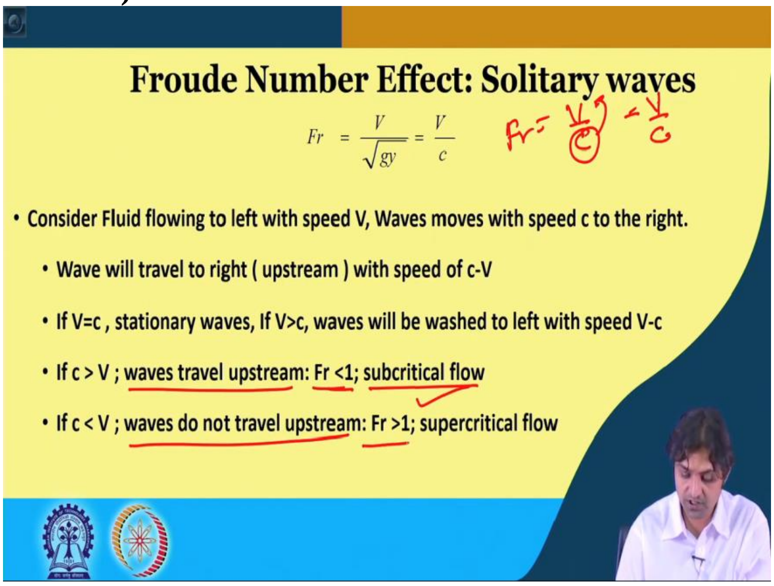

So, there is a Froude number effect also in solitary waves. Froude number as we discussed is given by V under root g y. This is the definition. But what have we obtained now using this solitary wave, that speed of the wave is under root g y and this we substitute here. So, we can write V / c. So, froude number for solitary wave is V / c, where c is the speed of the wave and V is the velocity of the fluid with the water with the with the velocity of the fluid, which will be definitely different from the waves speed.

So, if we consider a fluid which is flowing to the left with speed v. So, assume an open channel. So, I will assume an open channel without boundaries actually. So, the speed with left with speed V, this is fluid and there is a wave that is generated and is travelling with speed c to the right. Then wave will be able to travel to the right that means upstream with this speed c – V, because wave will be it will be on the fluid which is moving to the left with speed V and it has it own speed c.

So, the relative velocity will be c - V for the waves. If V is equal to c, then there will be formation of stationary waves that means, wave will not move at all because whatever the speed of the wave is to the right it will be cancelled out by the speed of the water flowing below it. If this V, the fluid speed is greater than c, c is the speed of the wave, waves will be travelling to the left, it will be washed off to the left with the speed V minus c, which is very normal. But if c is greater than V, then waves will travel upstream.

And in that case, Froude number will be less than one that means it is going to be a subcritical flow. This is an important result actually. So, if the speed of the wave is larger than the speed of the stream, I mean, stream as in water, the waves can travel upstream. And if we find out the Froude number, it is going to be one and it is going to be a subcritical flow. Let us try to see, so Froude number is V / c.

So, if c is greater than V, so what is going to be the Froude number, less than 1, so that means subcritical flow, very simple to see. If c is less than V, so that means V is greater than c waves do not travel upstream and because of this definition V / c, because V is greater than c Froude number is greater than one and this falls into regime of supercritical flow.

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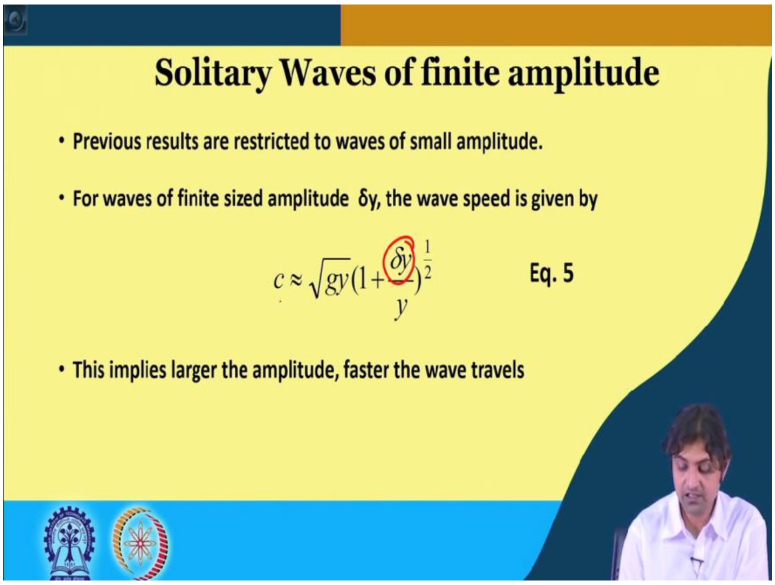

So, but the previous results that have been obtained are restricted to the waves of small amplitude, that is very important to remember. But if the waves are of finite size that means, they are no longer small amplitude, no longer small amplitude, then the wave speed actually is given by this, you can try to remember this formula, but, the derivation is not in the scope of this course. So, c is given by under the root of g y multiplied by one + delta y / y to the power half. And this is equation number 5. I think you should remember this.

Now, this if you see this equation, this will means if the amplitude delta y is larger, c will be larger. This means that the waves will going are going to travel fast. So, if the amplitude is larger the waves are going to travel fast. So, earlier we found out that small for all small amplitude the wave speed was not dependent upon the amplitude of the wave, but for a finite size solitary waves finites, Finite sized amplitude of solitary waves, the waves which has larger amplitude will travel faster.

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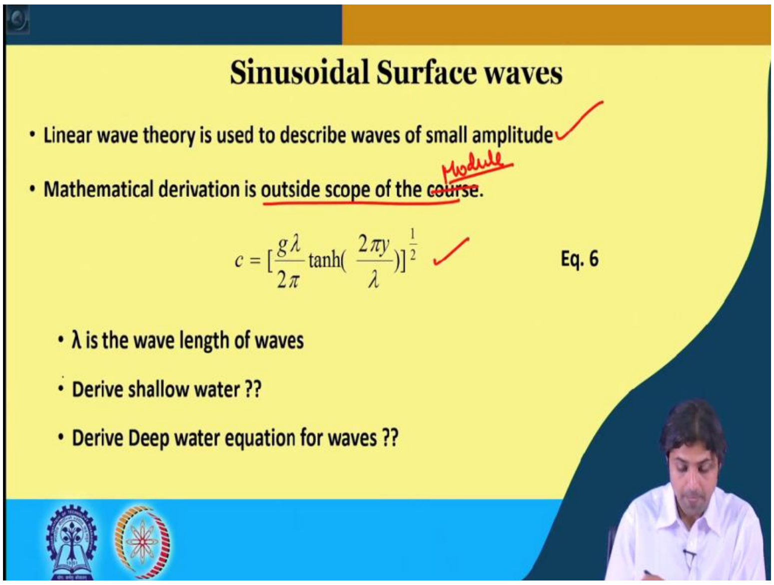

So, actually linear wave theory is used to describe waves of small amplitude. We are going to we will study the basics of linear wave theory as a part of inviscid flow. We will study the potential flow theory there and there you will see in detail this linear wave theory. This is going to be one of the last modules, one of the last modules of this course. So, right now, mathematical derivation is outside the scope of the course. Course means this module, not course, because we are going to derive it in later.

At the end of the course it is expected that you should be able to remember this value, but do not worry about this particular slide right away because we will cover this specifically in our last module, basics of a mechanics. And this is given as equation number 6. Here, lambda is the wavelength of waves. So, if you try to derive shallow water equation, shallow water means that the wave height, you know, the depth of the water is very less compared to the wavelength.

So, if you derive this, you will get, equations such as c is equal to under root gy. I am skipping all these because we are anyways going to study this in the last module.

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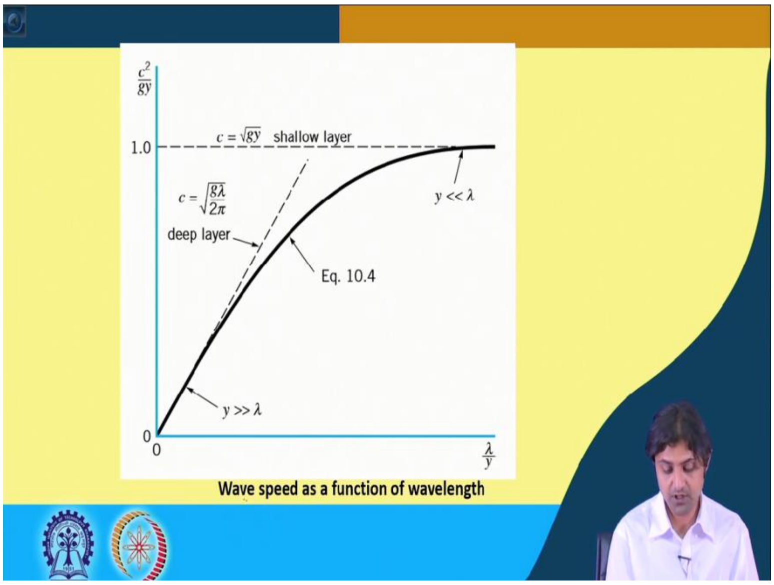

This is a graph that actually shows, you know, the curve that has been plotted lambda / y So, y is the depth and lambda is the wavelength and c is the speed of the wave, y is the depth and you see, this is the curve, different regimes have been shown. So, this is deep water. So, deep water when somebody says it is assumed that the depth of the water is much, much larger than the wavelength, shallow water means that the depth of water, depth and this is wave length.

So, we will see later, that the for deep water, this amplitude depends upon the, sorry, the V wave speed depends upon lambda, lambda is the wavelength. And in shallow water c is equal to under root g y, depends only on the water depth. But that is topic for, you know, later molecule. So, this as I said was the wave speed as a function of wavelength.

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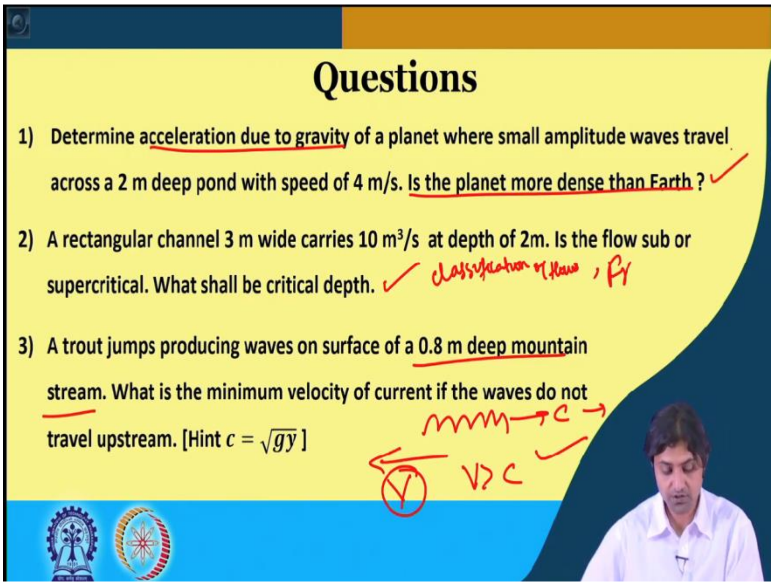

So, now there are some questions. It says that determine the acceleration due to gravity of a planet where small amplitude waves travel across a 2 meter deep pond with speed of 4 meters per second. The question is, there are 2 parts to it, we have to find the acceleration due to gravity. And the other part is, is the planet more dense than the Earth? This is a very vague question, but more importantly we know, we should know how to, you know, solve this question. There are 3 questions in we are which we are going to see.

The second one is, there is a rectangular channel 3 meter wide which carries 10 meters cube per second of water, at a depth of 2 meter. So, the question is, is the flow sub or supercritical and it is also asking what shall be the critical depth. This is related to the classification of flows. We are actually going to solve it. Here, we have to calculate Froude number, for example.

In the third question, again back to the waves. It says a trout, trout is a fish, that jumps producing waves on surface of a 0.8 meters deep mountain stream, which means that the depth of the river or a stream is 0.8 meter. It is asking, what is the minimum velocity of current if the waves do not travel upstream. So, basically, if there is a speed V of the stream in this direction and the waves are travelling in this direction with speed c, it is asking what is should be the minimum V, for which the waves should not be travelling to this, which means V must be greater than c all the time.

So, this is again a wave’s question. So, there are 3 questions, which we have to solve that will give us more confidence and practice about this particular, you know, the topic that we have done. And with this I would like to conclude today’s lecture. But in the beginning of the next lecture, we start with this particular slide again. I will take off and then we start solving this problem one by one. 1, 2 and 3. So, thank you so much for listening. See you in the next class.