Hydraulic Engineering
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Lecture - 53
Viscous Fluid Flow (Contd.)

Welcome back students to the final lecture of this module viscous fluid flow where we are deriving the Navier–Stokes equations.

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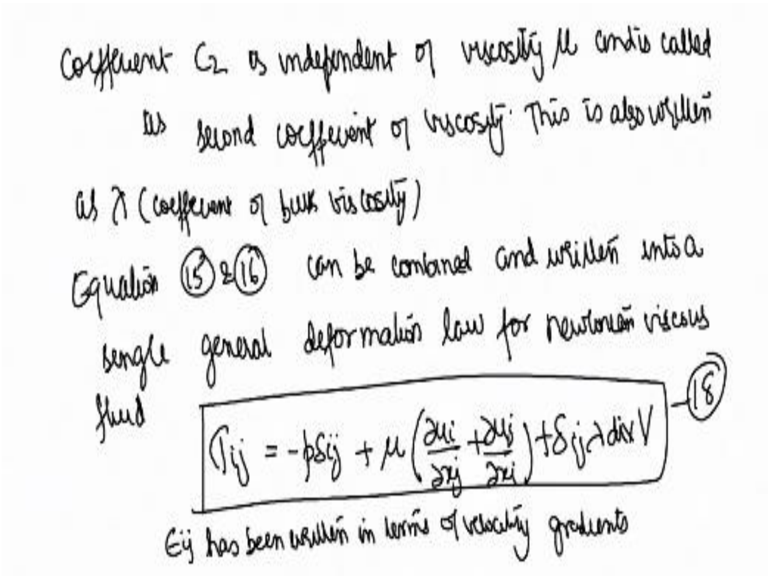

So in the last lecture we wrote the general deformation law for Newtonian Viscous Fluid okay this equation we call it equation number 18.

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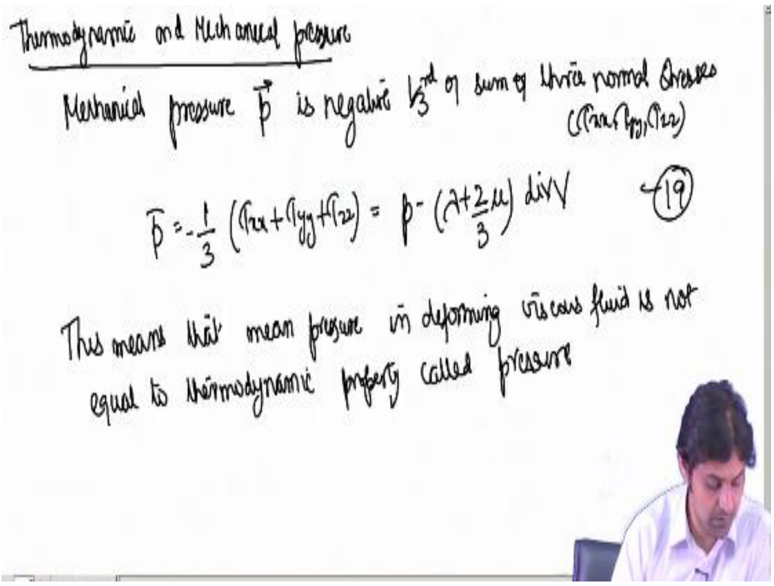

So proceeding forward so we will talk a little bit before writing the Navier–Stokes equations we will talk about the difference between the thermodynamic and mechanical pressure. So do you think they both are same, no they are not. So mechanical pressure so the pressure that we derive we find out during the Navier–Stokes equations or any other such equation is the thermodynamic pressure.

So the mechanical pressure p bar is negative one-third of sum of three normal stresses all right. What are the three normal stress τ_{xx}, τ_{yy} and τ_{zz}. So p bar can be written as – of 1/3 τ_{xx} + τ_{yy} + τ_{zz}. or it is written lambda + 2/3 mu divergence of V and this is equation number 19. If you substitute the value of τ_{xx}, τ_{yy} and τ_{zz} during the previous equation this is what you are going to get okay.

So what does this mean? This means that mean pressure in deforming viscous fluid is not = thermodynamic property called pressure okay.

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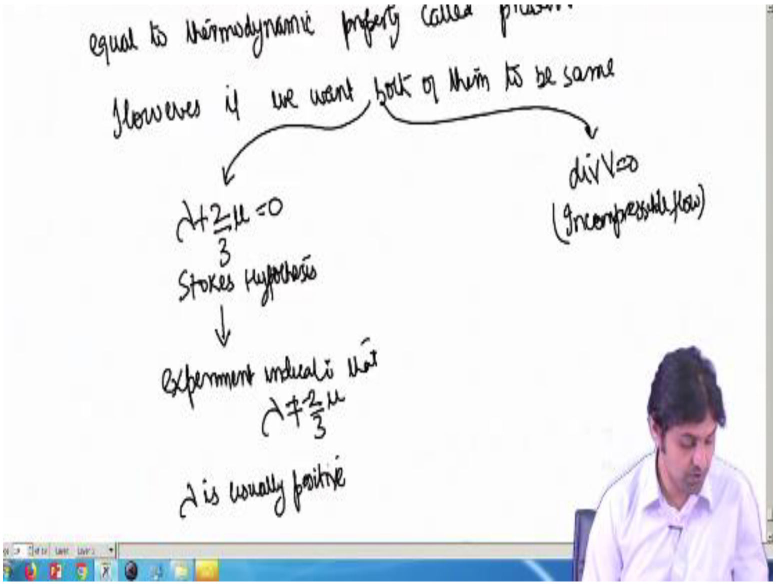

However, if we want both of them to be same two different ways right. One is if you (()) (03:56) look at the above equation p bar will be = p either lambda + 2/3 mu = 0 or divergence of V = 0. So let us say =0 actually this is called Stokes Hypothesis all right. The second way is divergence of V can be if it is 0 then also this is possible. This is possible this is more commonly possible because divergence of V 0 for incompressible flow.

So that is when we deal with water and hydraulics you see I mean all the time we assume incompressible flow. So both the mechanical pressure and the thermodynamic pressure they are same, but in general it is not true. Now coming back to this Stokes Hypothesis it says even in case of I mean compressible fluid is lambda + 2/3 mu = 0 then we can have. However, the experiments indicate that it is rare.

So lambda is generally not = 2/3 – 2/3 mu. So Stokes Hypothesis is not satisfied since lambda is usually and the reason is lambda is usually positive okay and I mean (()) (05:37) we have not heard it is negative so that is why lambda + 2/3 mu = 0 this Stokes Hypothesis is rarely satisfied okay all right.

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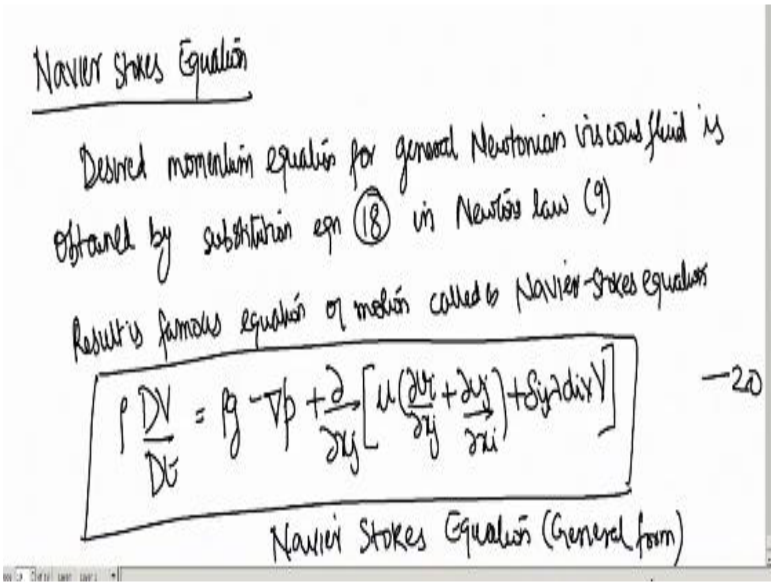

So now going back to the objective of this module the Navier–Stokes equations okay. So desired momentum equation for general Newtonian viscous fluid is obtained by equation 18 that is the deformation law in rewritten Newton's Law which was equation number 9 and the result is famous equation of motion called as Navier–Stokes equation so it is

$$\rho \frac{DV}{Dt} = \rho g - \nabla p + \frac{\partial}{\partial x_j}\left[\mu\left(\frac{\partial u_i}{\partial x_j} + \frac{\partial u_j}{\partial x_i}\right) + \delta_{ij}\lambda divV\right]$$

This is equation number 20 and now we have written the general Navier–Stokes equations all right. So after we have written this Navier–Stokes equations it also makes a little bit sense to see what happens with the incompressible flow, how does this equation modify in case of incompressible flow.

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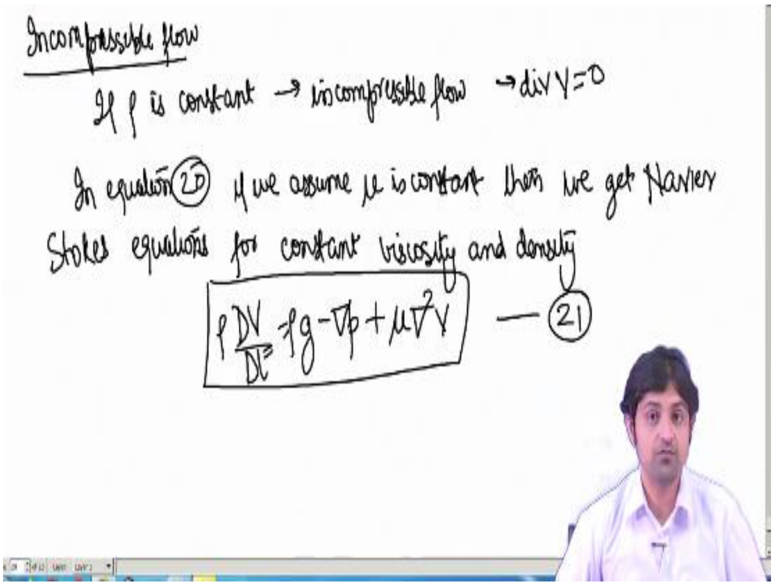

So to do that we go to a fresh page so incompressible flow what is that if rho is that means incompressible flow that means divergence of V = 0. So if you look at the Navier–Stokes equations above this will be 0 this the last term divergence of V all right. Therefore, in equation 20 if we assume mu is constant okay as well because in most of the hydraulic purposes we assume mu is constant.

Then we get Navier–Stokes equations for that is

$$\rho \frac{DV}{Dt} = \rho g - \nabla p + \mu \nabla^2 V$$

. This is another equation of significance because when it comes to Navier–Stokes equations for practical purposes this is the equation that we are going to use.

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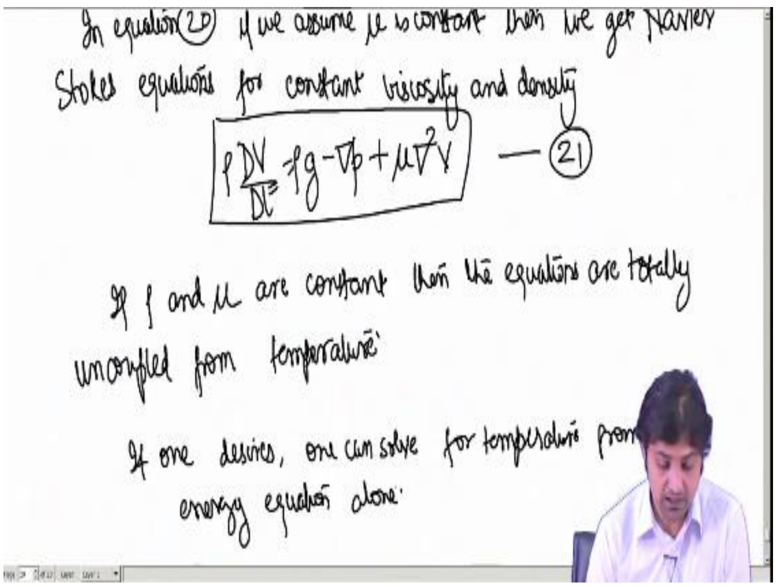

So one important thing to note is I mean we know from before if rho and mu are constant than the equations are totally are uncoupled from temperature correct right. I mean that is why rho and mu will be constant because if we consume constant then there is no effect of temperature right or if there is no effect of temperature we assume at a one fixed temperature just in case.

Because if you remember in the beginning we said that the thermodynamics properties I mean the properties are both pressure, temperature including the velocity that we need to find during the conservation loss. We have not discussed about temperature because this is not relevant to us so outside the scope by this particular course, but if one desires one can solve for temperature from energy equation alone okay, but we have not seen what that equation is.

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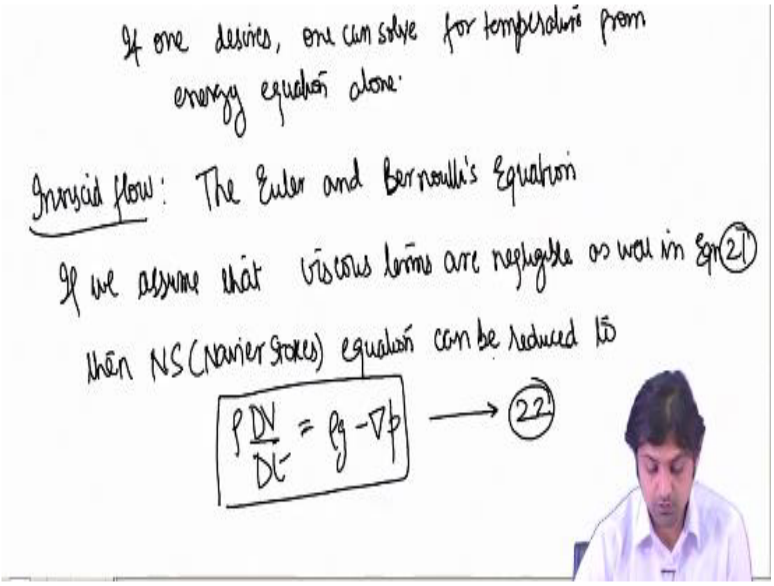

And now moving to the last part again that is called Inviscid flow. So actually we are going to study wave mechanics as part of Inviscid flow because there we will have potential theory and other things, but here in Inviscid the Euler it (()) (13:14) to discuss about mention Euler and the Bernoulli theorem how it has its origin okay. So if we assume that viscous terms are negligible as well in equation 21 then Navier–Stokes NS can be reduced to

$$\rho \frac{DV}{Dt} \approx \rho g - \nabla p$$

and this is equation number 22.

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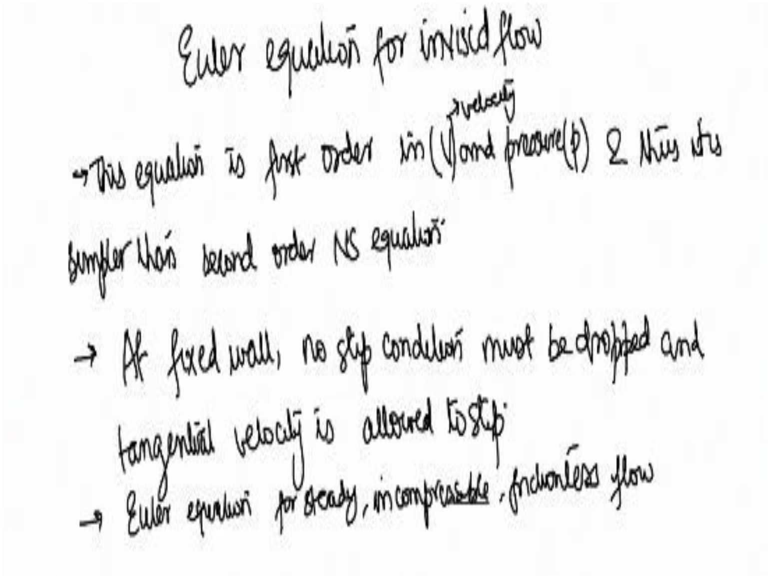

Very famously called Euler equation for Inviscid flow. So now you see how these equations have come into existence the base going to the origin of these equations okay. A couple of sentences about this equation so this equation is first order right in velocity and pressure all right. Thus, it is simpler than another change is in viscous fluid flow we have already been assuming no slip condition that you have seen also in the boundary layer theory and turbulent and laminar flows.

So at fixed wall no slip condition must be dropped and tangential velocity is allowed to slip because of the absence of viscous forces. Now the last point before we close our lecture is Euler equation for steady incompressible frictionless flow.

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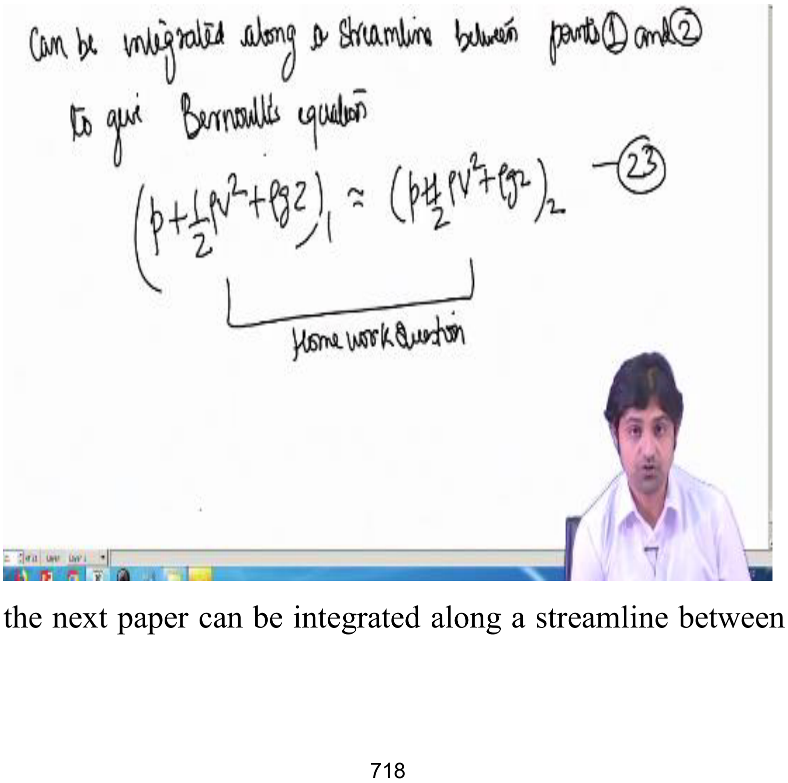

I think I will go the next paper can be integrated along a streamline between points 1 and 2, points 1 and 2 means any two points to give Bernoulli's equation which is

$$\left(p + \frac{1}{2}\rho V^2 + \rho g z\right)_1 \approx \left(p + \frac{1}{2}\rho V^2 + \rho g z\right)_2$$

sorry at 2 okay this is equation number 23 and this is a homework question. Please try to do this at home so if I enable we can actually (()) (18:44) the solution in the forum.

So from the beginning of this module we have seen how we have went ahead and tried to from basics from material derivative and the geometrical properties talked about the strain rates the shear strain rates then we went into the equation of continuity using the material derivative then equation of momentum we then we saw the deformation laws in the fluids, we derived the Navier–Stokes equation.

We also saw the difference between the thermal and the mechanical pressure and the condition in which both can be the same then we simplified our general Navier–Stokes equations which we have derived that was the purpose of this module then we simplified to obtain the Euler equation you know and also how the Bernoulli equation got. So you also know the origin of the Bernoulli equation in its purest form.

So with this I would like to close down today’s lecture. Next week we are going to study a topic that is called computational fluid dynamics and is a very well continuation of these equations that we have read this week. So thank you so much for listening to me for this particular module. I will see you next week.