Design and Analysis of Algorithms, Chennai Mathematical Institute

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Week - 08
Module - 02
Lecture – 51

LP Modeling: Production Planning

Let us look at another example of modeling a problem using Linear Programming, again to do with production.

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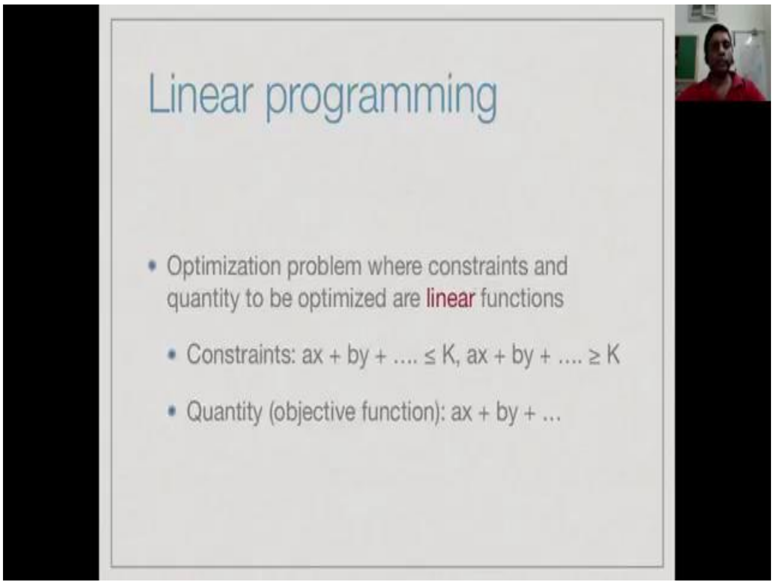

So, recall that a linear program is an optimization problem, where you have some variables which describe the quantities you want to compute. And you now have linear constraints on these variables, as well as a linear function describing what it is that you want to optimize, maximize or minimize, that is called the objective function.

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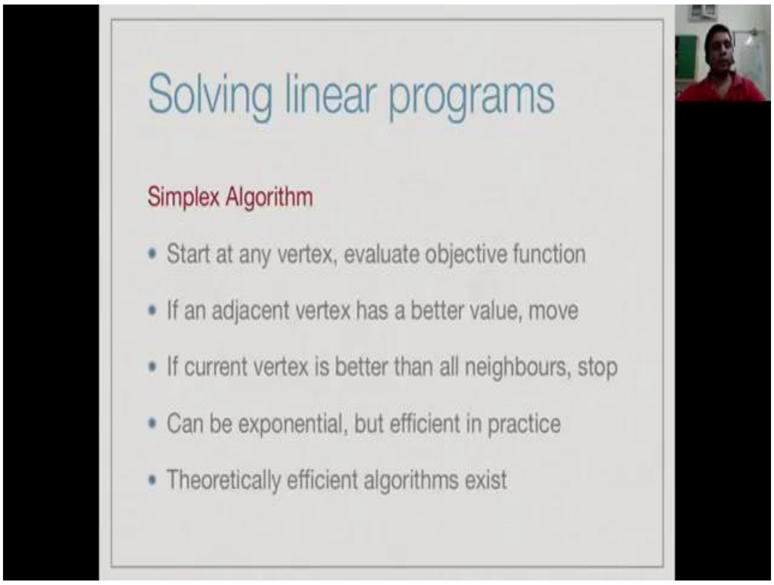

And one way to solve a linear program is to think of it geometrically and uses simplex algorithm and the simplex algorithm exploits the fact that the optimum value of a linear program is always at the vertex of the fusible region. So, it starts at some vertex and it keeps going from one vertex to a neighbor, until it finds a vertex whose value is optimum with respect to it is neighbor and the claim is that vertex actually represents the solution.

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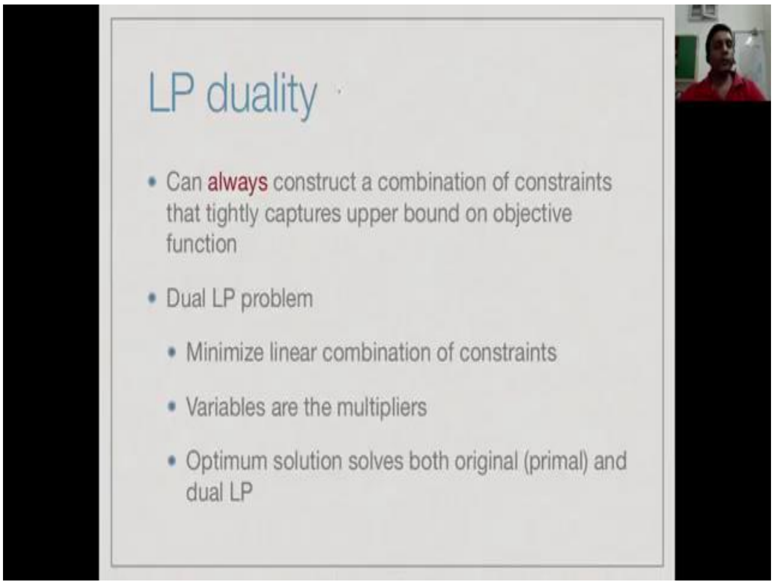

We also said that we can justify that this is the optimum value by constructing the dual, which has how to combine the constraints together to minimize the combination and the solution which solves both the original and the dual is actually an optimum value.

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So, the next example we are going to look at is the carpet manufacturing company. So, we have a company which makes hand woven carpets and we currently employee 30 employees, each employee produces 20 carpets a month and his pay 20,000 rupees a salary. So, if we just look at the cost per carpet, then we are paying the 1000 rupees to manufacture each carpet.

Now, it turns out that our monthly demand is seasonal, so after looking at some task data we are made an estimate of the amount of carpets at we can expect to sell in each month of the year. So, we have a range of demand from 440 to 920, so remember that you have 30 employees and 20 carpets each, I can make 600 carpets in a given month, this is what I can do with current stake. Now, some times the demand is as low as 440.

So, if I make 600 carpets, now if I have 30 employees and each one is going to produce 20 carpets a month, I am going to get that back. So, I am paying a percent to make carpets it not set to ideal, so every employee will produce 20 carpets a month. Therefore, I will always produce 600 carpets, but I will have 160 pairs which I cannot sell that. But, there are some months where I would have a demand for 320 and now the question of whether I have enough carpets to sell or not.

So, I know the demand from January to December and say these are stored as d 1 for January, d 2 for February and so on to d 12 for December. So, I have these 12 quantities which are known net marks, these are how many carpets I can expect to sell and each of these marks.

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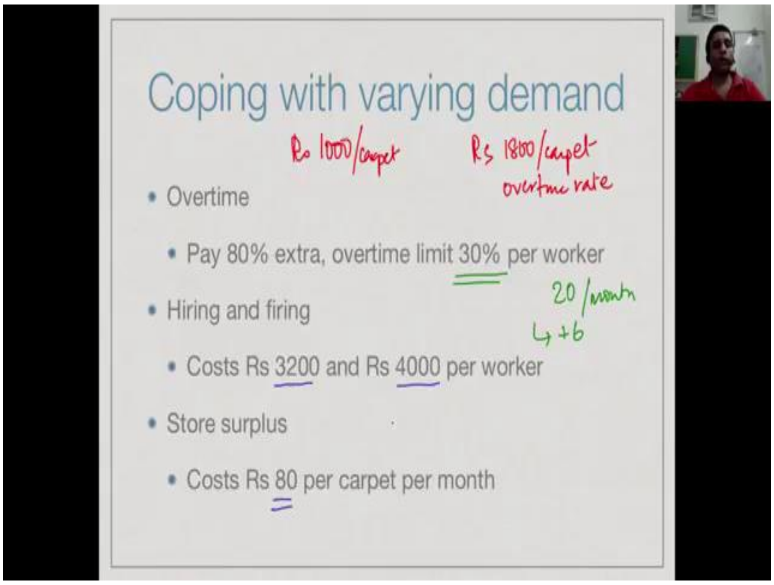

So, now, I have to come up with this strategy to cope with this varying demand, in order to make sure that I lose the least amount of money, because of this coping demand fluctuation. So, one possibility is that whenever I have extra demand I take overtime, so if I pay over time I get workers to work longer hours and make more carpets. But, there are 2 cost to this, the working wage per hour for overtime is typically higher than the regular edge, let us assume it is 80 percent extra.

So, if you remember originally we paid 1000 rupees per carpet as our labor cost, so if I make it in overtime, it is going to cost be instead rupees 1800 per carpet. Because, it is going to take the same amount of time and the person is going to be paid 80 percent more. So, 80 percent of 1800, so it is going to cost 1800 per carpet is the overtime rate, in the other thing is that you cannot of course, expect somebody to work 24 hours a day.

So, there is a limit saying that a worker cannot make more than 30 percent, cannot spend more than 30 percent overtime. So, if they make 20 per month normally, then at most they can go plus 6, a single worker can make at most now including overtime 26 per month. The other option for me is to add or subtract employees, I might want to add new workers, in case I have a higher demand projected a month or I might need to terminate some employees in case my demand probes below 600.

But, these also come to the cost, it goes I have to do some paper work and also I have to give some compensation and so on. So, let us assume that there are some certain cost 3200 is the cost associate in hiring a worker and 4000 rupees is the cost associate in firing a worker. And finally, I can store carpets when I made exist and sell them later when I have a demand, but storage also cost something. So, let us assume that storing a carpet cost rupees 80 per month.

Because, I have to keep it carefully avoid damage from moisture and so on, these are the various options I can pay overtime. But, up to a limit and it cost more for carpet made by a person who repaid overtime. I can either add or subtract from my work force, but this also comes with the cost each time I do it. And finally, I have a surplus storage cost in case I want to keep my production, hand it over from one month to the next.

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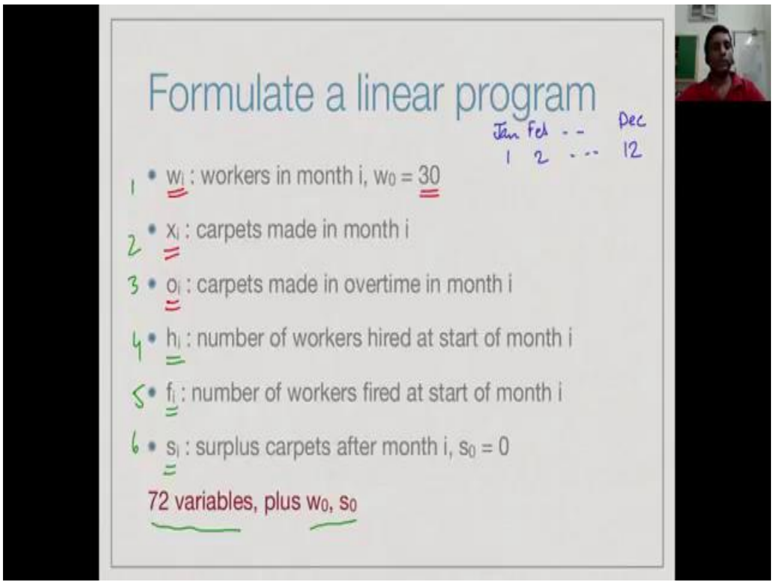

So, we want to make a linear program out of this, so we need some variables. So, we are dealing with these 12 months January, February to December. So, it is natural to think of everything as happening within this scope of one month. So, for month i where i ranges from 1 to 12, let us assume that we have Wi workers, initially we have 30 workers. Then, from this 30 we will either hire or fire some to get the workers in the empty, then if hire or fire some to get February and so on.

So, each month potentially we have more or less workers, but we start with 30, now these 30 workers will make some number of carpets. So, let us assume that the total number of carpets made in a month is X i and O i which is included in X i in terms of the total. But, O i is specifically the number of carpets which are made in overtime, because for these there is an extra cost, this is over in other salary we already pay the worker.

And finally, we said that we have this hiring and firing cost, so let h i be the number of workers hired at the beginning of a month and f i be the number of workers fired at the beginning of a month. And let S i stands for the number of surplus carpets that I have in stock at the end of month i. So, I initially assume that I have an empty variables, so s 0 as 0, but s 1 with say at the end of January how many carpets on my story, s 2 with say at the end of February how many carpets on my story and so on.

So, there are 1, 2, 3, 4, 5, 6 quantities, each of these quantities is there for 12 months. So, as 72 variables plus I have these default initial variables for w0 or s0, so 74 variables. So, this is the lot of things that I have estimate, but it turns out an actually in a linear program this is not a large number, it can be solved quite efficiently by a new of the standard things including simplex.

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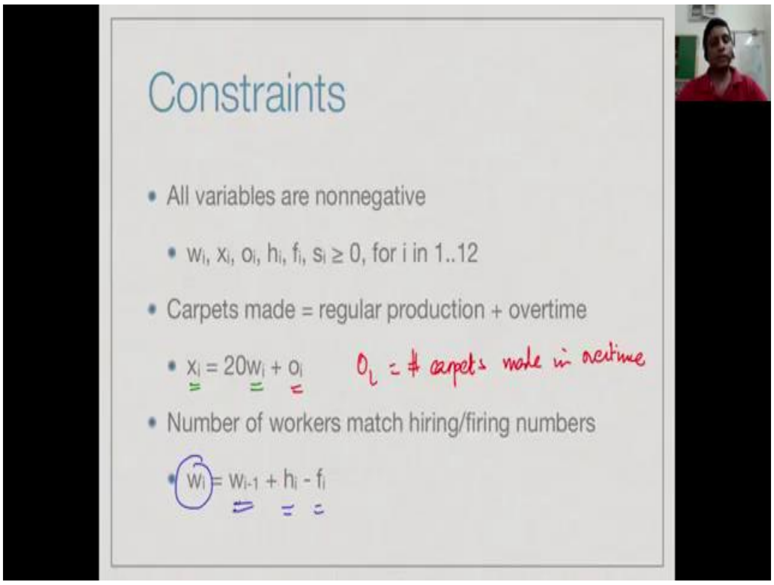

So, let us now thing of the constraints, so the first constraint to the usual one, which is their every quantity that we are talking about is strictly greater than equal to 0. The second constraints talks about, how the number of carpets made break up into the regular production plus the overtime. So, if I make i carpets in a month, then each worker remember W i number of workers in my establishment in month i will make 20. So, 20 time W i what workers in month i produce within the normal time and in addition to that O i is just the number of carpets made.

So, O i is not the number of hours overtime, so O i is the number of carpets. So, each month I make 20 times number of workers plus O i, now if I start if I had W i minus 1 workers last month and I hired a few and fired a few, then the total number I have now is the old number plus the number of hired plus number of hired. So, this is the constraint on how each of the W i is connected to the previous W i.

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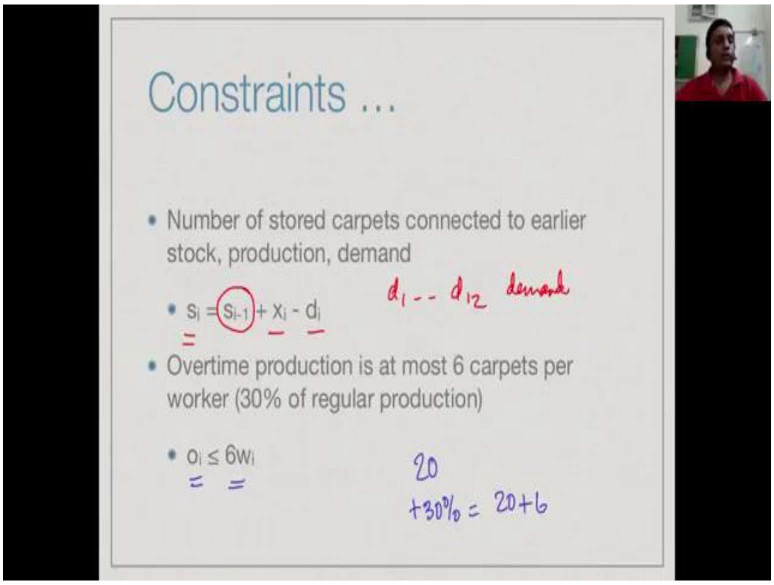

Similarly, the stack is connected to how much I had before I started this month, how much I made this month and how much I sold. So, remember that d 1 to d 12 was the demand, so this is the fixed quantity, either not variables these are known values. So, in this month I would have sold di, but I produce X i. So, X i minus d i is the exits that I produce this month are the deficit as the case may be I started with the surplus Si minus 1.

So, if I add this month's surplus are defied I get the total surplus at the end of the month. Remember, I cannot have negative carpets, Si all those things must always be bigger than equal to 0. And finally, as we said before a worker to produces 20 carpets in regular month, if I add now the fact that you can do 30 percent overtime, this is 20 plus 6. So, the number of carpets made in overtime can be at most 6 per worker. So, O i should be less than or equal to 6 times W i, so these are all constraints now.

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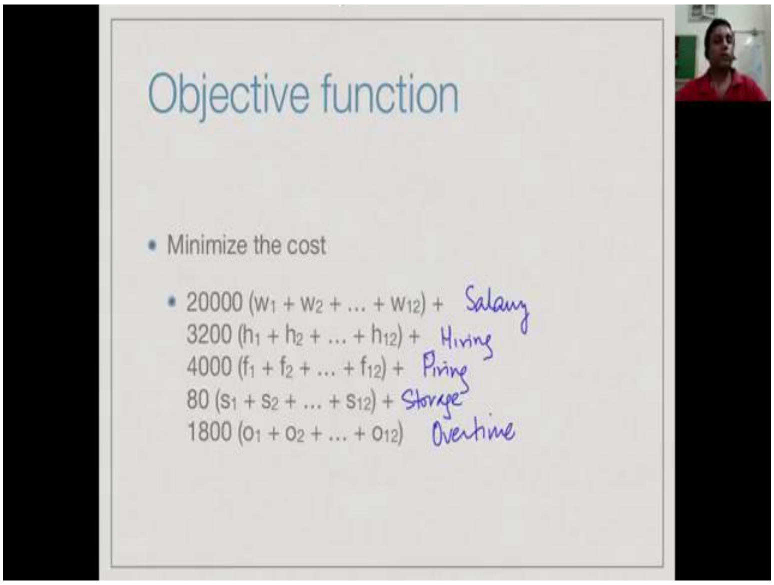

So, now given these constraints what we want to do is minimize the total amount of cost that we are going to put up. So, first this is the regular salary cost, the regular salary cost is 20,000 per worker and we have W 1 workers in month 1, W 2 month 2 and W 2 month 12. So, we add of the total number of workers, each of the mess way 20,000 rupees. So, this is the total salary before the year, then depending on how many people we hire in each month, for each hire we pay 3000, so this is our hiring pick.

Similarly, for each person that we business we have to pay 4000 first that. So, this is are firing pick, then the number of wrong carpet that have been keep in storage in each month in case 80 rupees per carpet cost. So, have to multiply the total storage by 80 and finally, when I pay overtime and paying our number the salary. So, each carpet made in overtime, remember cost 1800 rupees as suppose to 1000 rupees which is already accounted for in salary, so this is my overtime cost.

So, this says that we can take this fairly complicated looking question about production with the demand and hiring and inspiring and productivity all that and set it out with some 72 plus 274 variables and complex cost function and feed it to simplex and get an answer.

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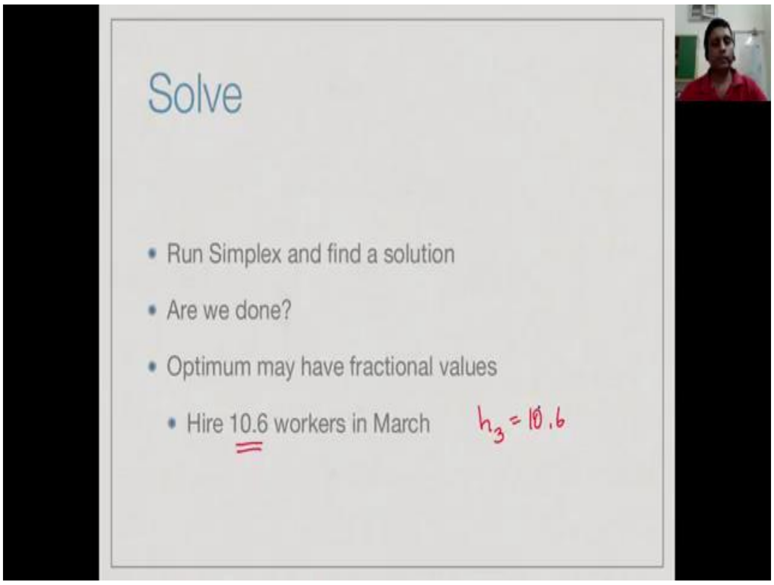

So, we done simplex and we find the solution, so are we done, so turns out that we might with an answer which is not something that we can actually used. For instance, we might it an answer will says h 3 is equal to 10.6. So, this is the, we must hire a fractional number of people in a month. Now, obviously, we cannot do this at the same time we have not any of our constraints express that the values must be integers.

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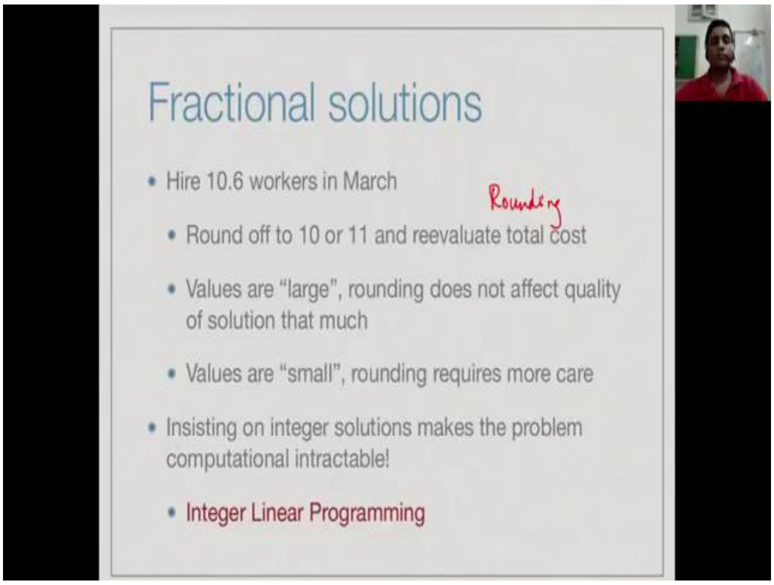

So, what can be do about this, well we can take this 10.6 and we can look for the errors integer 10 or a 11 and reevaluate what happens to the cost, if we make it even integer. So, this is called integer rounding, now if the values are big now direct 10, 11 the two digit value say, then rounding is relatively small displacement and the quality of the optimum will not change must based on it is. But, if the numbers are small, if I am rounding between say I am going to take it from 0.51 hour 0 of 1.52 or 1 then the effect of the rounding is quite can be quite task.

So, this is the general problem with linear programming which is linear programming cannot guarantee that you have the integer solutions, when you want an integer solution you can use rounding to achieve an integer solution, but you have to be careful that the rounding actually gives you one optimum. Now, why not just insist that you want an integer solution. So, can you not set up this same problem and solve it, but require the solutions to be integers, unfortunately this terms out to be quite a hard probe.

So, we have found we have claim rather that when we set up general linear programming problem, we can solve them efficiently. So, simplex is not an efficient solution, but an effective solution, but there are interior point methods and other polynomial time algorithms for linear programming. But, if you change the rules of the game and say I do not want to arbitrary solution; that means, constraints, but I want to one we are all the values that I get an integers, then we have the, so call integer linear programming.

And integer linear programming unfortunately is not solvable efficiently or it is not known divisible efficiently. So, the best we can hope to do for integer linear programming is to actually try it turn into an arbitrary linear program and some of interpret the answers or integers by doing appropriate rounding.