Chapter 26

PRESTRESSED CONCRETE

26.1 Introduction

1 Beams with longer spans are architecturally more appealing than those with short ones. However, for a reinforced concrete beam to span long distances, it would have to have to be relatively deep (and at some point the self weight may become too large relative to the live load), or higher grade steel and concrete must be used.

2 However, if we were to use a steel with f_y much higher than ≈ 60 ksi in reinforced concrete (R/C), then to take full advantage of this higher yield stress while maintaining full bond between concrete and steel, will result in unacceptably wide crack widths. Large crack widths will in turn result in corrosion of the rebars and poor protection against fire.

3 One way to control the concrete cracking and reduce the tensile stresses in a beam is to prestress the beam by applying an initial state of stress which is opposite to the one which will be induced by the load.

4 For a simply supported beam, we would then seek to apply an initial tensile stress at the top and compressive stress at the bottom. In prestressed concrete (P/C) this can be achieved through prestressing of a tendon placed below the elastic neutral axis.

5 Main advantages of P/C: Economy, deflection & crack control, durability, fatigue strength, longer spans.

6 There two type of Prestressed Concrete beams:

Pretensioning: Steel is first stressed, concrete is then poured around the stressed bars. When enough concrete strength has been reached the steel restraints are released, Fig. 26.1.

Postensioning: Concrete is first poured, then when enough strength has been reached a steel cable is passed thru a hollow core inside and stressed, Fig. 26.2.

26.1.1 Materials

7 P/C beams usually have higher compressive strength than R/C. Prestressed beams can have f'_c as high as 8,000 psi.

8 The importance of high yield stress for the steel is illustrated by the following simple example.

If we consider the following:

1. An unstressed steel cable of length l_s
2. A concrete beam of length l_c
3. Prestress the beam with the cable, resulting in a stressed length of concrete and steel equal to $l'_s = l'_c$.
4. Due to shrinkage and creep, there will be a change in length

$$\Delta l_c = (\varepsilon_{sh} + \varepsilon_{cr}) l_c \tag{26.1}$$

   we want to make sure that this amout of deformation is substantially smaller than the stretch of the steel (for prestressing to be effective).
5. Assuming ordinary steel: $f_s = 30$ ksi, $E_s = 29,000$ ksi, $\varepsilon_s = \frac{30}{29,000} = 1.03 \times 10^{-3}$ in/ in
6. The total steel elongation is $\varepsilon_s l_s = 1.03 \times 10^{-3} l_s$
7. The creep and shrinkage strains are about $\varepsilon_{cr} + \varepsilon_{sh} \simeq .9 \times 10^{-3}$
8. The residual stres which is left in the steel after creep and shrinkage took place is thus

$$(1.03 - .90) \times 10^{-3} (29 \times 10^3) = 4 \text{ ksi} \tag{26.2}$$

   Thus the total loss is $\frac{30-4}{30} = 87\%$ which is unacceptably too high.
9. Alternatively if initial stress was 150 ksi after losses we would be left with 124 ksi or a 17% loss.
10. Note that the actual loss is $(.90 \times 10^{-3})(29 \times 10^3) = 26$ ksiin each case

9 Having shown that losses would be too high for low strength steel, we will use

Strands usually composed of 7 wires. Grade 250 or 270 ksi, Fig. 26.3.

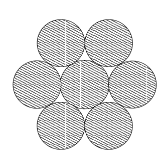

Figure 26.3: 7 Wire Prestressing Tendon

Tendon have diameters ranging from 1/2 to 1 3/8 of an inch. Grade 145 or 160 ksi.

Wires come in bundles of 8 to 52.

Note that yield stress is not well defined for steel used in prestressed concrete, usually we take 1% strain as effective yield.

10 Steel relaxation is the reduction in stress at constant strain (as opposed to creep which is reduction of strain at constant stress) occrs. Relaxation occurs indefinitely and produces

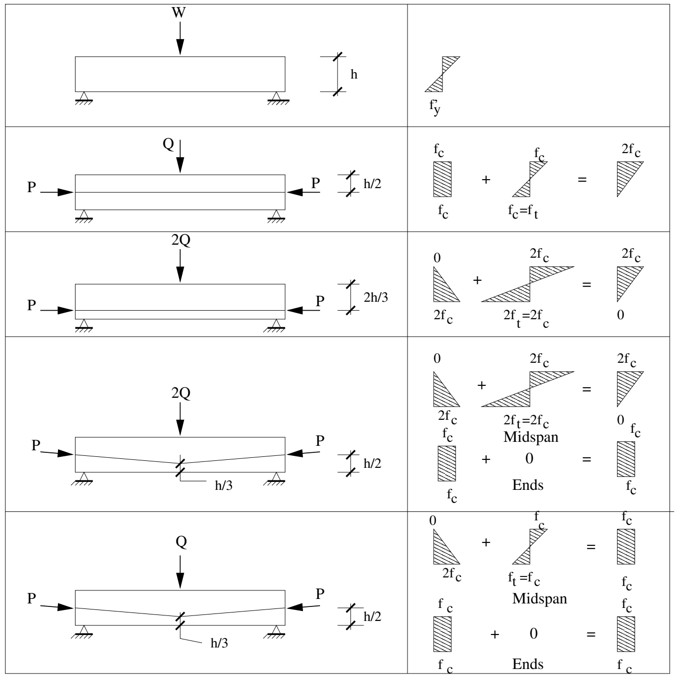

Figure 26.4: Alternative Schemes for Prestressing a Rectangular Concrete Beam, (Nilson 1978)

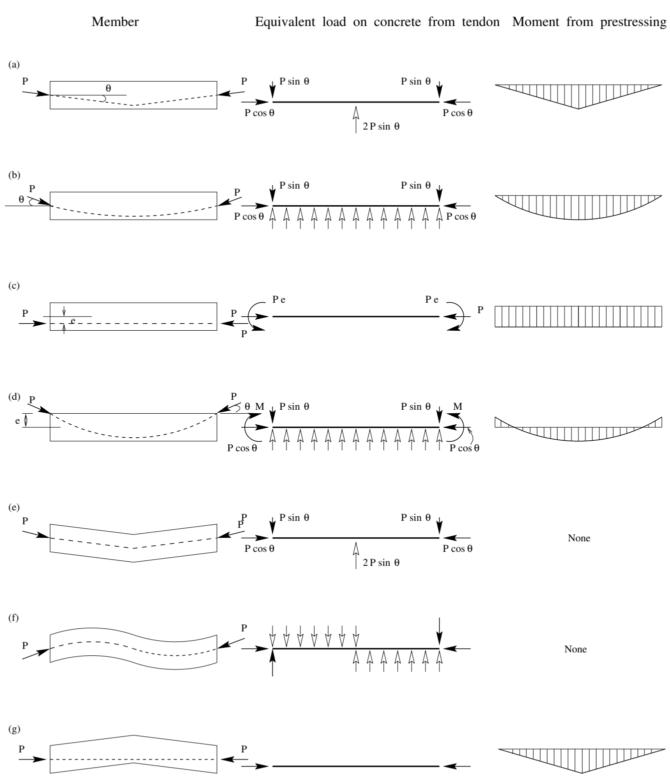

Figure 26.5: Determination of Equivalent Loads

4. P_e and $M_0 + M_{DL} + M_{LL}$

$$\boxed{\begin{array}{lcl} f_1 & = & -\frac{P_e}{A_c}\left(1-\frac{ec_1}{r^2}\right)-\frac{M_0+M_{DL}+M_{LL}}{S_1} \\ f_2 & = & -\frac{P_e}{A_c}\left(1+\frac{ec_2}{r^2}\right)+\frac{M_0+M_{DL}+M_{LL}}{S_2} \end{array}} \tag{26.7}$$

The internal stress distribution at each one of those four stages is illustrated by Fig. 26.7.

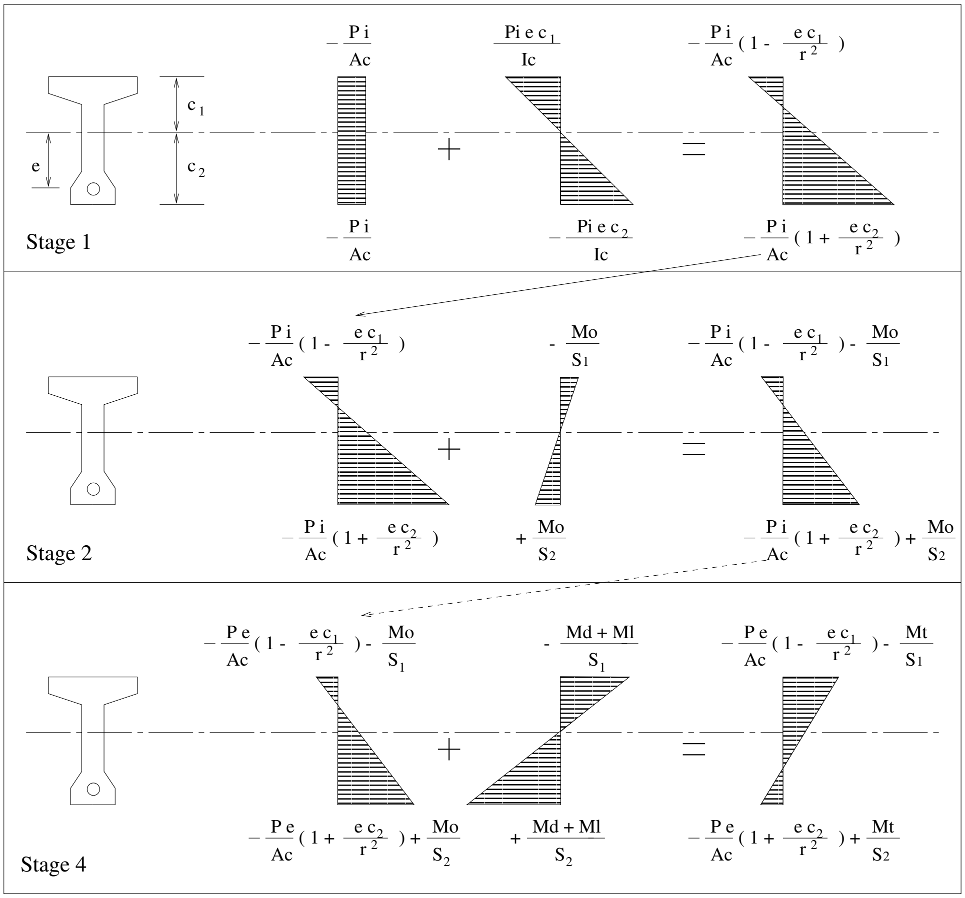

Figure 26.7: Flexural Stress Distribution for a Beam with Variable Eccentricity; Maximum Moment Section and Support Section, (Nilson 1978)

17 Those (service) flexural stresses must be below those specified by the ACI code (where the subscripts c, t, i and s refer to compression, tension, initial and service respectively):

f_{ci}	permitted concrete compression stress at initial stage	$.60f'_{ci}$
f_{ti}	permitted concrete tensile stress at initial stage	$< 3\sqrt{f'_{ci}}$
f_{cs}	permitted concrete compressive stress at service stage	$.45f'_c$
f_{ts}	permitted concrete tensile stress at initial stage	$6\sqrt{f'_c}$ or $12\sqrt{f'_c}$

Note that f_{ts} can reach $12\sqrt{f'_c}$ only if appropriate deflection analysis is done, because section would be cracked.

18 Based on the above, we identify two types of prestressing:

$$M_0 = \frac{(.183)(40)^2}{8} = 36.6 \text{ k.ft} \tag{26.9-b}$$

The flexural stresses will thus be equal to:

$$f_{1,2}^{w_0} = \mp\frac{M_0}{S_{1,2}} = \mp\frac{(36.6)(12,000)}{1,000} = \mp 439 \text{ psi} \tag{26.10}$$

$$\begin{aligned}
f_1 &= -\frac{P_i}{A_c}\left(1 - \frac{ec_1}{r^2}\right) - \frac{M_0}{S_1} && (26.11\text{-a})\\
&= -83 - 439 = \boxed{-522 \text{ psi}} && (26.11\text{-b})\\
f_{ti} &= 3\sqrt{f_c'} = +190\surd && (26.11\text{-c})\\
f_2 &= -\frac{P_i}{A_c}\left(1 + \frac{ec_2}{r^2}\right) + \frac{M_0}{S_2} && (26.11\text{-d})\\
&= -1,837 + 439 = \boxed{-1,398 \text{ psi}} && (26.11\text{-e})\\
f_{ci} &= .6f_c' = -2,400\surd && (26.11\text{-f})
\end{aligned}$$

3. P_e and M_0. If we have 15% losses, then the effective force P_e is equal to $(1-0.15)169 = 144$ k

$$\begin{aligned}
f_1 &= -\frac{P_e}{A_c}\left(1 - \frac{ec_1}{r^2}\right) - \frac{M_0}{S_1} && (26.12\text{-a})\\
&= -\frac{144,000}{176}\left(1 - \frac{(5.19)(12)}{68.2}\right) - 439 && (26.12\text{-b})\\
&= -71 - 439 = \boxed{-510 \text{ psi}} && (26.12\text{-c})\\
f_2 &= -\frac{P_e}{A_c}\left(1 + \frac{ec_2}{r^2}\right) + \frac{M_0}{S_2} && (26.12\text{-d})\\
&= -\frac{144,000}{176}\left(1 + \frac{(5.19)(12)}{68.2}\right) + 439 && (26.12\text{-e})\\
&= -1,561 + 439 = \boxed{-1,122 \text{ psi}} && (26.12\text{-f})
\end{aligned}$$

note that -71 and $-1,561$ are respectively equal to $(0.85)(-83)$ and $(0.85)(-1,837)$ respectively.

4. P_e and $M_0 + M_{DL} + M_{LL}$

$$M_{DL} + M_{LL} = \frac{(0.55)(40)^2}{8} = 110 \text{ k.ft} \tag{26.13}$$

and corresponding stresses

$$f_{1,2} = \mp\frac{(110)(12,000)}{1,000} = \mp 1,320 \text{ psi} \tag{26.14}$$

Thus,

$$f_1 = -\frac{P_e}{A_c}\left(1 - \frac{ec_1}{r^2}\right) - \frac{M_0 + M_{DL} + M_{LL}}{S_1} \tag{26.15-a}$$

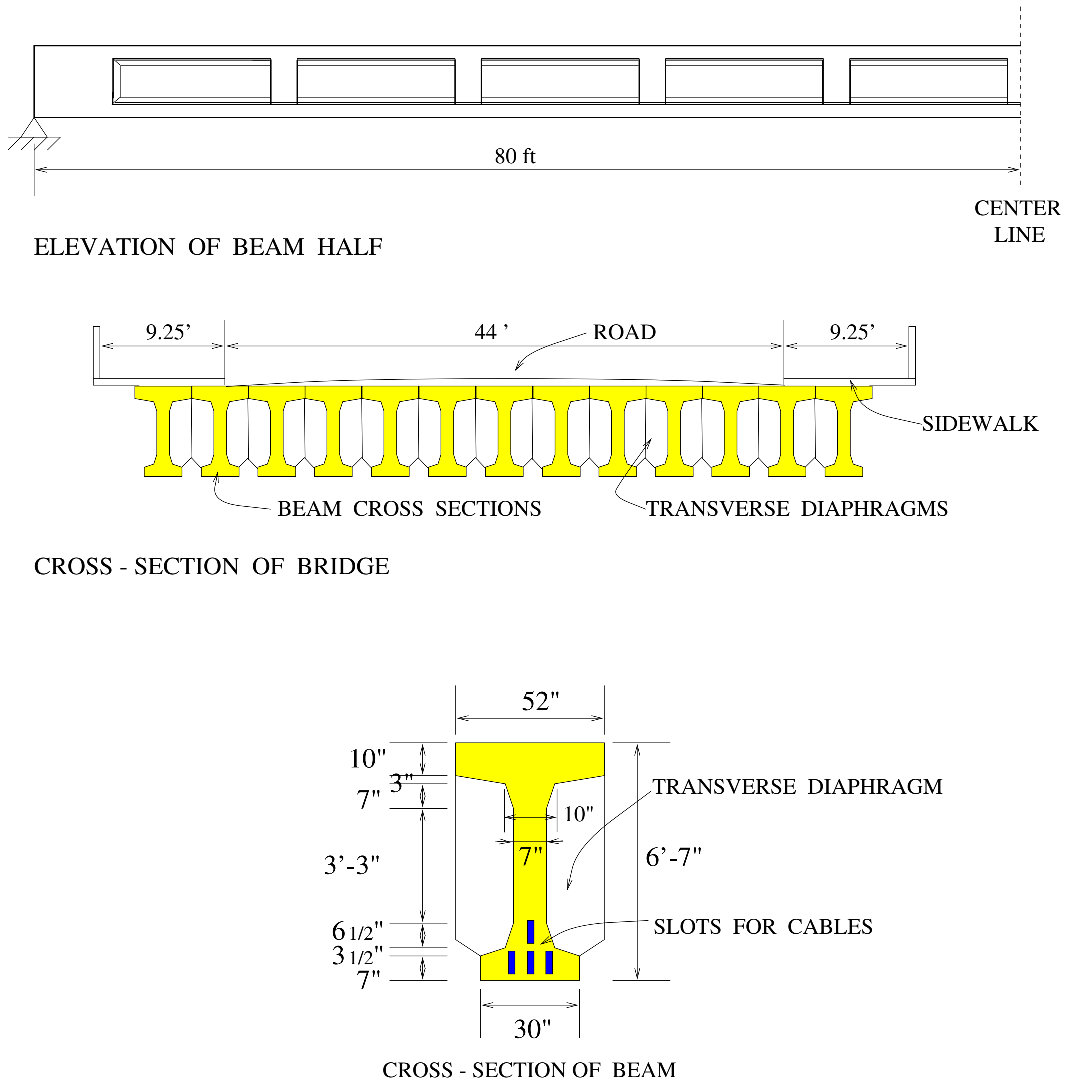

Figure 26.8: Walnut Lane Bridge, Plan View