1.19 Tacheometry

In tacheometry, horizontal and vertical distances are determined by angular observations with a techeometer. Tacheometry is more accurate than the chaining/taping, and more rapid in rough and difficult terrain where levelling is tedious and measuring distance by chaining/taping is not only inaccurate but slow and laborious. It is a best suited method when taking observations for steep and undulating/broken ground, river, water or swampy areas. Tacheometry is preferably used for traversing, but it is also used for contouring.

1.19.1 Instruments used

The main instruments used in tacheometry are a tacheometer, and a levelling rod. A tacheometer is a transit theodolite where telescope is fitted with a special diaphragm, called stadia hairs, i.e., a diaphragm fitted with three horizontal hairs; one at the top, another in the middle and third at the bottom of diaphragm. These horizontal hairs are equidistant from the central one. The types of stadia diaphragm commonly used in tacheometers are shown in Figure 1.46. The term tacheometer is restricted to a transit theodolite which is provided with an anallactic lens in the telescope. The essential characteristics of a tacheometer are that the value of the multiplying constant ($K= f / I$) should be 100 and additive constant ($C= f + d$) should be zero. Levelling rod used is similar to as used in levelling work. To make the value of additive constant zero, an additional convex lens, known as anallatic lens, is provided in the telescope. By having K=100 and C=0, the calculation work is considerably reduced.

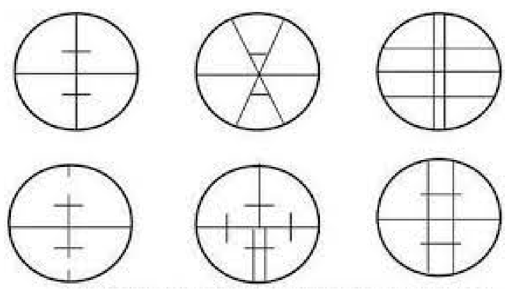


Figure 1.46 Stadia diaphragm commonly used in tacheometers

1.19.2 Methods of tacheometry

The principle used in tacheometry is that the horizontal distance between an instrument-station and a point where levelling rod is kept can be determined with the staff intercept (difference of top reading and bottom reading). For determining the horizontal distance between two points, tacheometer is kept at one point and levelling staff is kept at another, and stadia readings on the levelling staff are read.

Case I- When the line of sight is horizontal

The staff intercept reading is multiplied by the instrument constant (K) and added with an additive constant (C) to get the horizontal distance between tacheometer and levelling staff. The horizontal distance (D) is computed as;

$$D = K\,S + C \tag{1.16}$$

Where K is the multiplying constant (usually 100), S is the staff intercept and C is the additive constant (usually zero). In a simplified from, the above equation can be written as-

$$K = 100S \tag{1.17}$$

There could be situations when the ground is undulating and the levelling staff is either above or below the line of sight. In such cases, a vertical angle subtended by the point at instrument station is also measured, in addition to stadia readings, to determine the horizontal distance using trigonometrical relationship. This is a fast method to determine the horizontal distance.

The vertical distance (elevation difference) between instrument and levelling staff can also be determined, if we take one more levelling staff observation at the BM.

RL of levelling staff point = RL of instrument axis– Central hair staff reading (1.18)

Case II- When the line of sight is inclined

In case the ground is undulating and horizontal sights are not possible, inclined sights are taken. In this case, the staff may be held either vertical or normal to the line of sight. In general, most commonly adopted method is when the staff is held vertical as it is simpler in calculation.

When the staff is held vertically.

In Figures 1.47 and 1.48, the staff is held vertical; in one case the point is at higher elevation and in other case, the staff is at lower elevation than the tacheometer. From tacheometer, all the three stadia readings and vertical angle θ subtended by middle wire reading at C is observed. From C, if we draw a perpendicular line to line of sight O'C will cut O'A line at A' and extended O'B line at B'. Let A'O'C be angle α, then by geometry B'O'C will also be angle α. In two Δs, AA'C and CB'B, angles AA'C and CB'B are equal to $(90^0+\alpha)$ and $(90^0-\alpha)$, respectively. The angle α being very small, angles AA'C and CB'B may be considered practically equal to 90^0.

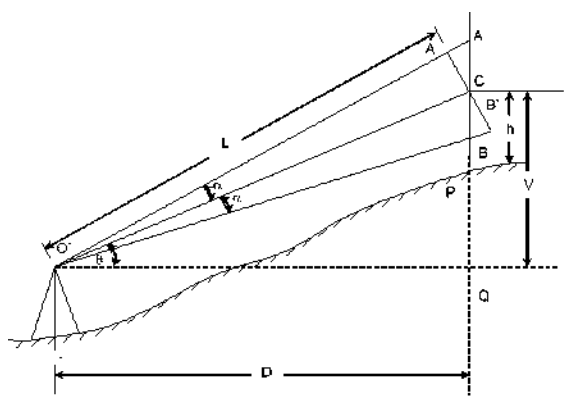


Figure 1.47 Staff held vertical at higher elevation

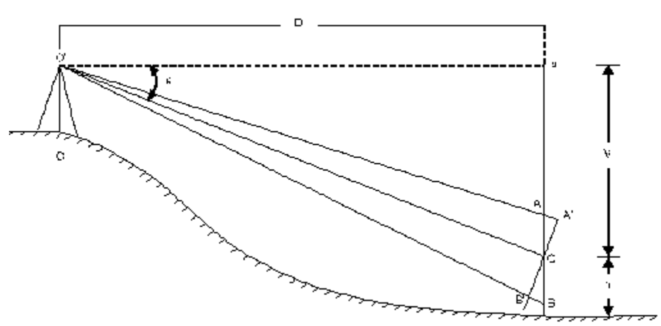


Figure 1.48 Staff held vertical at lower elevation

A'B' = AB cos θ = S cos θ

$$D = L\cos\theta \qquad (1.18)$$

$$= \frac{f}{i}(A'B')\cos\theta + (f+d)\cos\theta$$

$$= \frac{f}{i}\times S\cos^2\theta + (f+d)\cos\theta$$

And $$V = L\sin\theta \qquad (1.20)$$

$$= \frac{f}{i}\times S\sin\theta\cos\theta + (f+d)\sin\theta$$

$$= \frac{f}{i}S\frac{\sin 2\theta}{2} + (f+d)\sin\theta$$

Also V = D tan θ (1.21)

Knowing the value of V, the RL of the staff point is calculated as-

When θ is an angle of elevation (Figure 1.48)

RL of staff station P = RL of instrument axis + V – h (1.22)

When θ is an angle of description (Figure 1.49).

RL of staff station P = RL of instrument axis – V – h (1.23)

1.20 Trigonometrical Levelling

It is an indirect method of levelling in which the elevation of the point is determined from the observed vertical angles and the measured distances. It is commonly used in topographical work to find out the elevations of the top of buildings, chimneys, churches etc., from a distance. This is a faster method to get the elevations of top of structures and objects. Elevation of a BM in the area must be known.

1.20.1 Finding height of an object which is accessible

Let PP' is a tower whose elevation of the top is to be determined (Figure 1.49). Set up the theodolite at a convenient ground point A so that the top of tower and a staff kept on the BM can be bisected. Measure vertical angle α of the top of tower as well as take the staff reading at the BM. Measure D, the horizontal distance between theodolite station and tower.

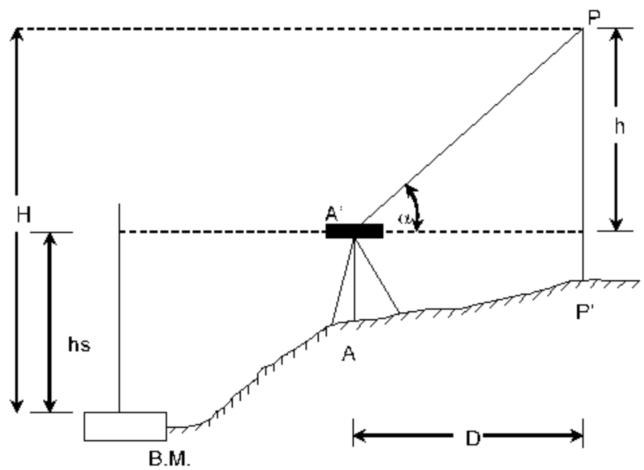


Figure 1.49 Measurement when the object is accessible

To find the height of the object above a BM:

Let H = height of the object above the BM

h = height of the tower above the instrument axis

h_s = height of the instrument axis above the BM

α = vertical angle of the top of tower at the instrument station

D = horizontal distance from the instrument station to the base of the tower.

Then, $h = D \tan \alpha$

$$H = h + h_s = D \tan \alpha + h_s \qquad (1.24)$$

If the distance D is large, correction for curvature and refraction, i.e., $\left\{0.00673\left(\frac{D}{1000}\right)^2\right\}$ is to be applied.

1.20.2 Finding height of an object which is inaccessible

To find the height of the tower PP' above a BM, select two stations A and B suitably on a fairly level ground so that these points lie in a vertical plane with the tower, and measure the distance AB with a tape (Figure 1.50). Set up the theodolite at station A to take a staff reading kept on the BM. Read the vertical angle α. Shift the theodolite at B point and take similar observations as taken at A point. Read the vertical angle β.

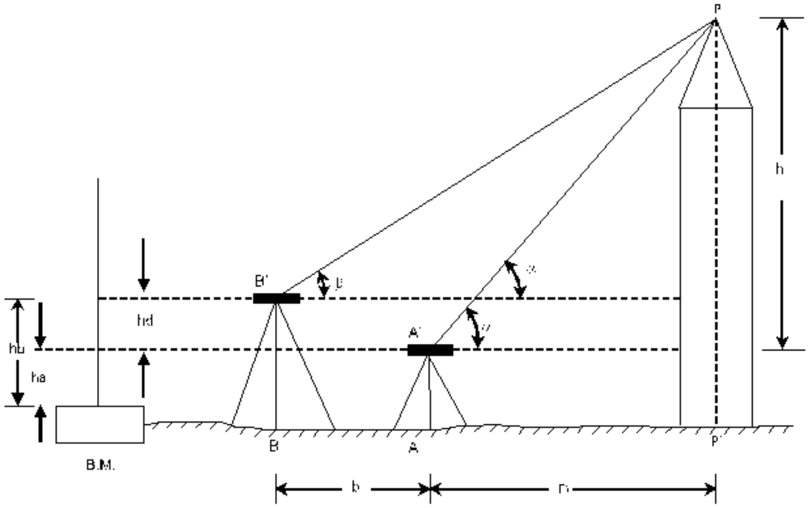


Figure 1.50 Measurement when object is inaccessible

Let

b = horizontal distance between A and B.

D = distance of the object from A point.

h = height of the tower P above instrument axis at A'.

h_a = staff reading at the BM when the instrument is at A.

h_b = staff reading at the BM when the instrument is at B.

h_d = the level difference between A and B of the instrument axes = $h_a \sim h_b$.

When the instrument at farther station B is higher than that at the near station A (Figure 1.51).

$h = D \tan \alpha$

$h - h_d = (D + b) \tan \beta$

Putting the value of h from (i) and (ii),

$D \tan \alpha - h_d = (D + b) \tan \beta$

or $D (\tan \alpha - \tan \beta) = h_d + b \tan \beta$

$$\text{or } D = \frac{b \tan \beta + h_d}{\tan \alpha - \tan \beta} \qquad (1.25)$$

Put this value of D in $h = D \tan \alpha$:

$$h = \frac{b \tan \beta + h_d}{\tan \alpha - \tan \beta} . \tan \alpha \qquad (1.26)$$

Height of the tower above the BM,

$H = h + h_d$

When the base of tower is inaccessible and instrument can't be kept in same vertical plane

Let A and B be the two instrument stations not in the same vertical plane as that of a tower P (Figure 1.51). Select two stations A and B on a level ground and measure the horizontal distance b between them. Set the instrument at A and level it. Use it as a level and take a backsight h_s on the staff and kept at BM. Now, measure the angle of elevation α_1 to P, and horizontal angle BAP (θ_1).

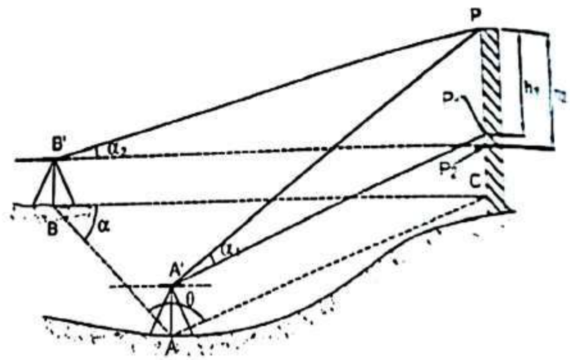


Figure 1.51 The base of tower is inaccessible and instrument is kept in different vertical planes

Now, shift the instrument to point B, and measure the angle of elevation α_2 to P. Also measure the horizontal angle ABP at B as θ_2.

Let

α_1 = angle of elevation from A to P

α_2 = angle of elevation from B to P

θ_1 = Horizontal angle BAC (or BAP) at station A

θ_2 = Horizontal angle ABC (or ABP) at station B

$h_1 = PP_1$ = height of the object P from instrument axis of A

h = PP = height of the object P from instrument h_2 PP_2 axis of B

In triangle ABC

Angle ACB = $180^0 - (\theta_1 + \theta_2)$

AB = b

BC= $b \sin \theta_1 / [\sin (180^0 - (\theta_1 + \theta_2))]$

AC = $b \sin \theta_2 / [\sin (180^0 - (\theta_1 + \theta_2))]$

By knowing the AC and BC from equation, we get-

$h_1 = AC \tan \alpha_1$

$h_2 = BC \tan \alpha_2$

RL of P = height of the instrument axis at A + h_1 (1.27)

or

R.L of P = height of the instrument axis at B + h_2 (1.28)

Height of the instrument axis at A = RL of BM + BS

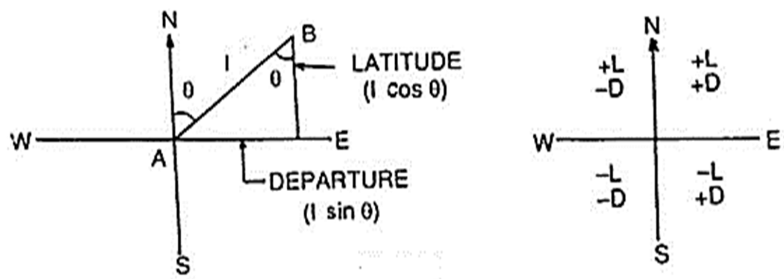
Height of the instrument axis at B = RL of BM + BS

1.21 Traverse Computations

One the field observations are completed for a traverse; the next task is to compute the coordinates of traverse stations. These coordinates are required to be plotted to carry out the detailed mapping of the area. For the computation of coordinates (x, y and z), following observations are to be taken in the field:

(i) Magnetic bearing of at least one traverse line
(ii) Length of at least one traverse line
(iii) Elevation of at least one traverse station
(iv) Included angles between traverse lines
(v) Vertical angles of traverse lines
(vi) Real-world coordinates of one traverse station

Once the coordinates of traverse points are computed, these are plotted on a plan with reference to x-axis and y-axis. If the length and bearing of a line are known, its projections on the y-axis and x-axis may be done, called *latitude* and *departure* of the line, respectively. Latitude is measured northward, and is also known as *northing*, and departure, if measured eastward is known as *easting*. The latitude of a line is determined by multiplying the length of the line with the cosine of its reduced bearing; and departure is computed by multiplying the length with the sine of its reduced bearing (Figure 1.52). If *l* is the length of the line and θ is its reduced bearing, then latitude and departure are calculated as-



The latitude and departure of lines are also expressed in the following ways:

Northing = latitude towards north = + *L*
Southing = latitude towards south = − *L*
Easting = departure towards east = + *D*
Westing = departure towards west = − *D*

Figure 1.52 Comutation of latitude and departure of a line

The reduced bearing of a line will determine the sign of its latitude and departure, the first letter N or S of bearing defines the sign of the latitude and the last letter of bearing E or W defines the sign of the departure. If the WCB of a line is known, it can be converted into RB to determine the sign of latitude and departure. By knowing the bearing of one line and the included horizontal angle, the bearings of remaining lines can be computed. The latitude and departure of any point with reference to the proceeding traverse station are called *consecutive co-ordinates of the station*. The coordinates of a traverse station with reference to a common origin are called *independent coordinates*.

1.21.1 Adjustment of a closed traverse

Due to errors present in the observations, the coordinates of a closed traverse stations when plotted may not close itself, but will have a small difference. The errors in the linear and angular observations therefore are to be adjusted before using them for computational purpose. It is also called *Balancing a Traverse*. These errors include:

(a) Adjustment of angular errors
(b) Adjustment of bearings.
(c) Adjustment of closing error of traverse

(a) Adjustment of angular error

In a closed traverse, the sum of all interior angles should be equal to $(2n–4) \times 90^0$, and that of the exterior angles should equal $(2n + 4) \times 90^0$, where '*n*' is the number of sides in a closed traverse. The difference between this sum and the sum of the measured angles in a closed traverse is called the *angular error of closure*. The angular error of closure should not exceed the least count of theodolite (x) used, i.e., x √n. If it exceeds, observations are to be repeated. These permissible errors are shown in Table 1.6. To distribute the error, one of the approaches is to distribute it equally among all the angles, if all the angles are measured with equal precision and under similar conditions, this error. The other approach is to distribute the error in each angle according to its magnitude. This approach is considered to be more accurate and requires computation, however, the first approach is simple and fast and may not be as accurate.

Table 1.6 Permissible errors in Theodolite traversing :

Traversing for	Permissible Angular Error	Permissible Linear error
Land surveys and location of roads, railways, etc	$1'\sqrt{N}$	1 in 3000
Survey work for cities and important boundaries	$30''\sqrt{N}$	1 in 5000
Important Surveys	$15''\sqrt{N}$	1 in 10, 000

Where N = Number of angles

(b) Adjustment of bearings

Many times, bearings of a traverse are measured, instead of angles. In such cases, the closing error in bearings may be determined by comparing the fore bearing of a line and back bearing of that line of a closed traverse, as they should differ by 180^0. The difference is the error which has to be adjusted in the bearings. Alternatively, we compare the known bearing of the traverse line with the measured bearing, and difference, if found, is adjusted in the bearings. At the end, we must ensure that the back bearing and fore bearing of all the lines differ by 180^0.

If we know the RB of a traverse line, the RBs of remaining lines are computed using the adjusted (corrected) included angles.

(c) Adjustment of closing error

For all the sides of traverse, latitude and departure are computed using the adjusted RB of lines, and proper sign is used as per the quadrant of traverse line. Ideally, the sum of all latitudes and sum of all departures must be zero in a closed traverse. But due to errors in the field measurements (e.g., bearings, distances, etc., the sum of all latitudes and sum of all departures, individually, may not come out to be zero). That means, the traverse will not close at the starting point. The distance by which the end point of a survey fails to meet with the starting point is called the *closing error* or *error of closure.* Figure 1.53 shows the plotting of an anticlockwise closed traverse ABCDEA, where A and A_1 are the starting and end points, respectively, and AA_1 represents the closing error.

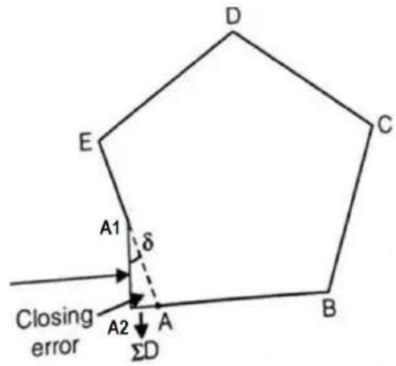


Figure 1.53 Representation of closing error

The magnitudes of two components of this error (A_1A_2 and AA_2) perpendicular to each other may be determined by finding the algebraic sum of the latitude (ΔL), as well as departures (ΔD). Since the triangle A_1A_2A is right-angled at A_2, the linear closing error (AA_1) is computed as:

$$\text{Closing error} = AA_1 = \sqrt{(A_1A_2)^2 + (AA_2)^2} = \sqrt{(\Sigma L)^2 + (\Sigma D)^2} \qquad (1.29)$$

The direction of the closing error is given by the relation,

$$\tan\theta = \frac{\Sigma D}{\Sigma L}, \qquad (1.30)$$

where θ is the reduced bearing. The signs of ΔL and ΔD will define the quadrant of the closing error.

The latitudes and departures are now adjusted by applying the correction to them in such a way that the algebraic sum of the latitudes and departures should be equal to zero. Any one of the two rules (Bowditch Rules and Transit Rules) may be used for finding the corrections to balance the survey:

(1) *Bowditch Rule:* It is also known as the *Compass rule*. It is used to adjust the traverse when the angular and linear measurements are equally precise. By this rule, the correction in each latitude or departure of line is computed as:

Correction to latitude or departure of any side
= Total error in latitude or departure

$$\times \left(\frac{\textit{length of that side}}{\textit{perimeter of traverse}} \right) \qquad (1.31)$$

(2) *Transit Rule:* The Transit rule is used to adjust the traverse when the angular measurements are more precise than the linear measurements.

(i) Correction to latitude of any side
= total error in latitude

$$\times \left(\frac{\textit{latitude of that side}}{\textit{arithmetical sum of the all latitudes}} \right) \qquad (1.32)$$

(ii) Correction to departure of any side

$$= \text{total error in departure} \times \left(\frac{\textit{departure of that side}}{\textit{arithmetical sum of all departures}} \right) \quad (1.33)$$

1.21.2 Computation of coordinates

Very popular method of showing the adjustment of closing error as well as computation of coordinates is systematically done using Gale's Traverse Table.

Gale's Traverse Table:

The computations for a closed traverse may be made in the following steps and entered in a tabular form known as Gale's Traverse, as shown in Table 1.8.

(i) Adjust the included angles, if the sum is not equal to (2n-4) x 90^0, as explained above

(ii) Calculate the adjusted RB of all traverse liens, as explained above.

(iii) Compute the latitudes and departures (consecutive coordinates) of traverse lines, as explained above.

(iv) Apply the necessary correction to the latitude to the latitudes, and departures so that the sum of the corrected latitudes is zero and sum of corrected departures is equal to zero.

(v) By knowing the exact coordinates of a traverse point, find the independent coordinates of the remaining traverse points from the consecutive coordinates.

(vi) Plot all the traverse stations on a plan for map preparation.

Table 1.8 Sample Gales traverse table for computation of corrected coordinates

										Consecutive Co-ordinates				Corrections				Corrected Consecutive Co-ordinates				Independent Co-ordinates		
										Lat.		Dep.												
Inst. Stn	**Angles**	**Correction**	**Corrected Angles**	**W.C. bearings**	**Reduced Bearings**	**Quadrant**	**Line**	**Length**	**Point**	**+Northi**	**-**	**+easting**	**-**	**Northin**	**Southin**	**Easting**	**Westing**	**Northin**	**Southin**	**Easting**	**Westing**	**Northin g**	**Easting**	**Remark s**