Chapter 7

ARCHES and CURVED STRUCTURES

1 This chapter will concentrate on the analysis of arches.

2 The concepts used are identical to the ones previously seen, however the major (and only) difference is that equations will be written in polar coordinates.

3 Like cables, arches can be used to reduce the bending moment in long span structures. Essentially, an arch can be considered as an inverted cable, and is transmits the load primarily through axial compression, but can also resist flexure through its flexural rigidity.

4 A parabolic arch uniformly loaded will be loaded in compression only.

5 A semi-circular arch unifirmly loaded will have some flexural stresses in addition to the compressive ones.

7.1 Arches

6 In order to optimize dead-load efficiency, long span structures should have their shapes approximate the coresponding moment diagram, hence an arch, suspended cable, or tendon configuration in a prestressed concrete beam all are nearly parabolic, Fig. 7.1.

7 Long span structures can be built using flat construction such as girders or trusses. However, for spans in excess of 100 ft, it is often more economical to build a curved structure such as an arch, suspended cable or thin shells.

8 Since the dawn of history, mankind has tried to span distances using arch construction. Essentially this was because an arch required materials to resist compression only (such as stone, masonary, bricks), and labour was not an issue.

9 The basic issues of static in arch design are illustrated in Fig. 7.2 where the vertical load is per unit horizontal projection (such as an external load but not a self-weight). Due to symmetry, the vertical reaction is simply $V = \frac{wL}{2}$, and there is no shear across the midspan of the arch (nor a moment). Taking moment about the crown,

$$M = Hh - \frac{wL}{2}\left(\frac{L}{2} - \frac{L}{4}\right) = 0 \qquad (7.1)$$

Solving for H

$$\boxed{H = \frac{wL^2}{8h}} \tag{7.2}$$

We recall that a similar equation was derived for arches., and H is analogous to the $C - T$ forces in a beam, and h is the overall height of the arch, Since h is much larger than d, H will be much smaller than $C - T$ in a beam.

10 Since equilibrium requires H to remain constant across thee arch, a parabolic curve would theoretically result in no moment on the arch section.

11 Three-hinged arches are statically determinate structures which shape can acomodate support settlements and thermal expansion without secondary internal stresses. They are also easy to analyse through statics.

12 An arch carries the vertical load across the span through a combination of axial forces and flexural ones. A well dimensioned arch will have a small to negligible moment, and relatively high normal compressive stresses.

13 An arch is far more efficient than a beam, and possibly more economical and aesthetic than a truss in carrying loads over long spans.

14 If the arch has only two hinges, Fig. 7.3, or if it has no hinges, then bending moments may exist either at the crown or at the supports or at both places.

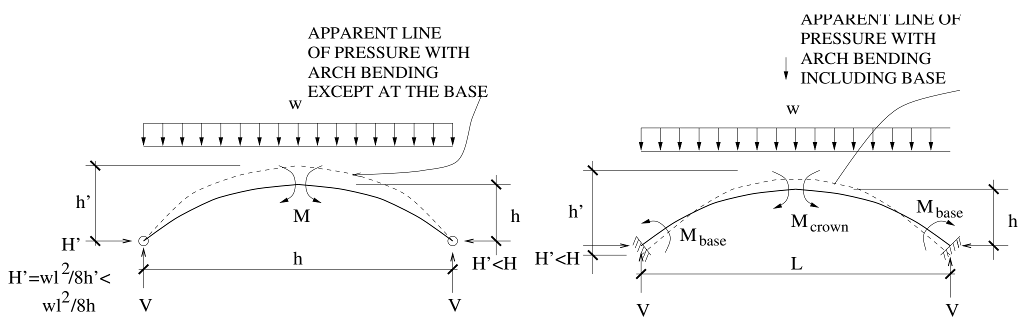

Figure 7.3: Two Hinged Arch, (Lin and Stotesbury 1981)

15 Since H varies inversely to the rise h, it is obvious that one should use as high a rise as possible. For a combination of aesthetic and practical considerations, a span/rise ratio ranging from 5 to 8 or perhaps as much as 12, is frequently used. However, as the ratio goes higher, we may have buckling problems, and the section would then have a higher section depth, and the arch advantage diminishes.

16 In a parabolic arch subjected to a uniform horizontal load there is no moment. However, in practice an arch is not subjected to uniform horizontal load. First, the depth (and thus the weight) of an arch is not usually constant, then due to the inclination of the arch the actual self weight is not constant. Finally, live loads may act on portion of the arch, thus the line of action will not necessarily follow the arch centroid. This last effect can be neglected if the live load is small in comparison with the dead load.

Solving those four equations simultaneously we have:

$$\begin{bmatrix} 140 & 26.25 & 0 & 0 \\ 0 & 1 & 0 & 1 \\ 1 & 0 & 1 & 0 \\ 80 & 60 & 0 & 0 \end{bmatrix} \begin{Bmatrix} R_{Ay} \\ R_{Ax} \\ R_{Cy} \\ R_{Cx} \end{Bmatrix} = \begin{Bmatrix} 2,900 \\ 80 \\ 50 \\ 3,000 \end{Bmatrix} \Rightarrow \begin{Bmatrix} R_{Ay} \\ R_{Ax} \\ R_{Cy} \\ R_{Cx} \end{Bmatrix} = \begin{Bmatrix} 15.1 \text{ k} \\ 29.8 \text{ k} \\ 34.9 \text{ k} \\ 50.2 \text{ k} \end{Bmatrix} \tag{7.4}$$

We can check our results by considering the summation with respect to b from the right:

$$(+\circlearrowleft)\, \Sigma M_z^B = 0; -(20)(20) - (50.2)(33.75) + (34.9)(60) = 0\surd \tag{7.5}$$

■

■ **Example 7-2: Semi-Circular Arch, (Gerstle 1974)**

Determine the reactions of the three hinged statically determined semi-circular arch under its own dead weight w (per unit arc length s, where $ds = rd\theta$). 7.6

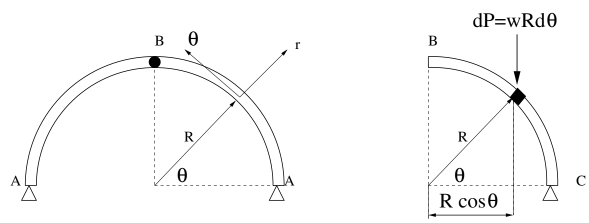

Figure 7.6: Semi-Circular three hinged arch

Solution:

I Reactions The reactions can be determined by **integrating** the load over the entire structure

1. **Vertical Reaction** is determined first:

$$(+\circlearrowright)\, \Sigma M_A = 0; -(C_y)(2R) + \int_{\theta=0}^{\theta=\pi} \underbrace{wRd\theta}_{dP} \underbrace{R(1+\cos\theta)}_{\text{moment arm}} = 0 \tag{7.6-a}$$

$$\begin{aligned} \Rightarrow C_y &= \frac{wR}{2}\int_{\theta=0}^{\theta=\pi}(1+\cos\theta)d\theta = \frac{wR}{2}[\theta - \sin\theta]\,|_{\theta=0}^{\theta=\pi} \\ &= \frac{wR}{2}[(\pi - \sin\pi) - (0 - \sin 0)] \\ &= \boxed{\tfrac{\pi}{2}wR} \end{aligned} \tag{7.6-b}$$

2. **Horizontal Reactions** are determined next

$$(+\circlearrowleft)\, \Sigma M_B = 0; -(C_x)(R) + (C_y)(R) - \int_{\theta=0}^{\theta=\frac{\pi}{2}} \underbrace{wRd\theta}_{dP} \underbrace{R\cos\theta}_{\text{moment arm}} = 0 \tag{7.7-a}$$