Design and Analysis of Algorithms, Chennai Mathematical Institute
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Department of Computer Science and Engineering,

Week - 08
Module - 03
Lecture - 52
LP Modelling: Bandwidth Allocation

For our next example of linear programming, we will look at a problem involving graphs and networks to do with Network Bandwidth.

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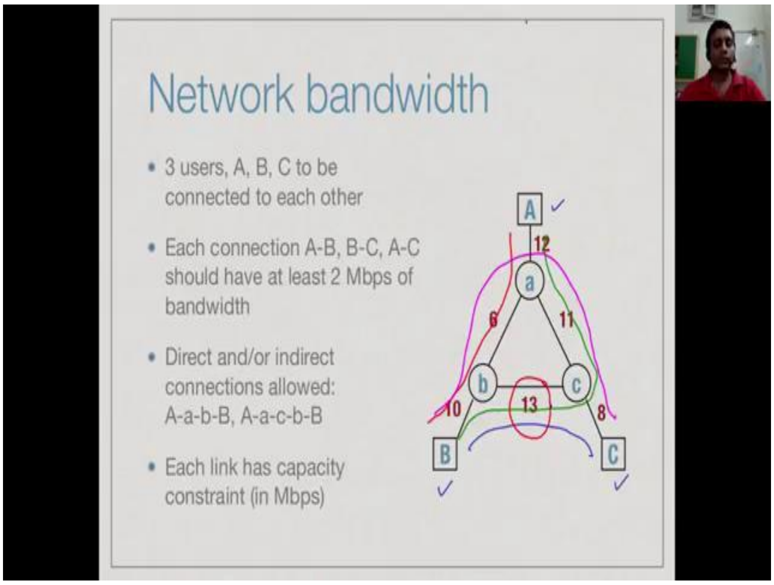

So, suppose we have a small communication network involving three users. So, the users are capital A, capital B and capital C, and each of them is connected through some switches, which we call a, b and c. So, we have a network which this in an internet network which connect these three users and our requirement is to ensure that each pair of users A to B, B to C and A to C gets at least two mega bits per second of connectivity from between that pair of users.

So, we want at least two Mbps connectivity along these three users. Now, notice that from A to B, there are two ways I can sent packets, I can sent that directly via A and B or I can sent them indirectly, I have small seek**.** So, it turns out, it does not really a matter from the point of viewer, we uses whether this 2 Mbps bandwidth comes from the shorter a red route and a longer green route.

So, our aim is to combine the capacity of the red and the green for each pair, so similarly there will be, for example, there will be a direct route from B to C and there will be an indirect route that goes like this. So, for each pair, there is a direct route and an indirect route. So, long as combinations of the capacities of the direct and indirect routes add up to at least 2 Mbps, our customers are satisfied. So, now the constraint that we have is that these links have a capacity. So, if I look at example the link between B and C, it can only transmit 13 mega bits per second total across all the different connections that it is a part off.

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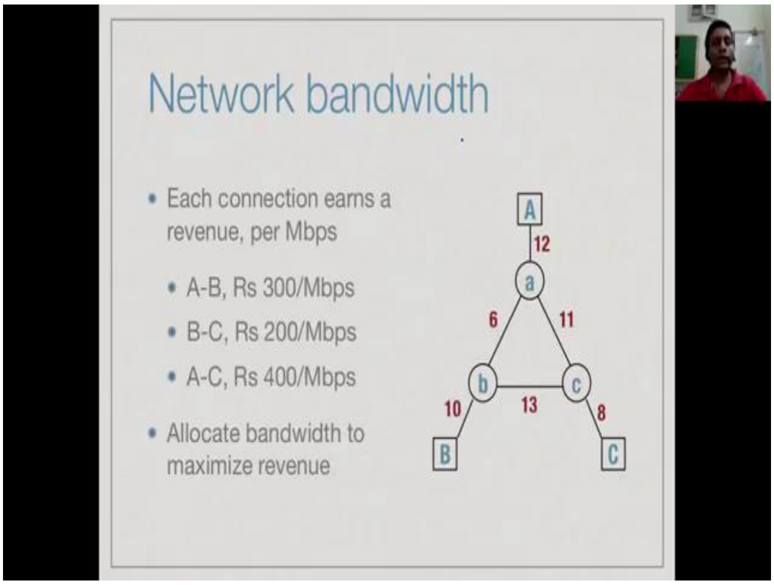

Now, on the other side, we earn some money from this customer which is not uniform. So, for the A, B link, we get 300 rupees for mega bit Mbps per month, but from B to C we get only 200, but from A to C we get 400. So, now, we have to allocate a minimum of 2 mega bits. But customers are will to take as much as we can give them subject to that minimum and we get a certain amount of revenue depending on how we utilizes the capacity. So, our goal is to allocate bandwidth to maximize the revenue given that the customers are willing to take anything above 2 Mbps.

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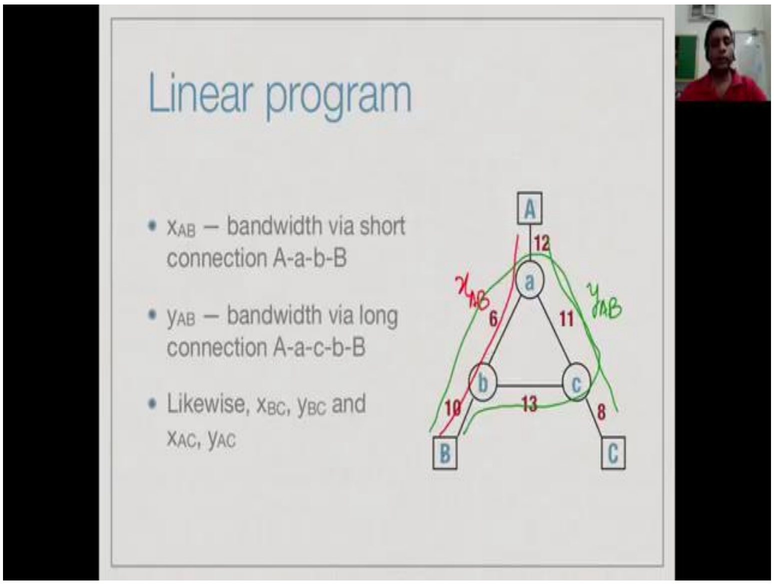

So, as we have been seen will be our aim is to set this up as a linear program. So, in this case, what are the variables that we are going to use? So, recall that we said that every connection has two routes. So, we have A to B coming where the short route and we have A to B coming by either long route. So, what we use is the variable x, from A to B to denote that quantity that is showing on the red route and similarly, y from A to B to denote the quantity, it is flowing on the green route.

So, we have two variables associated with A to B service that becomes provide, how much goes directly x A B, how much goes indirectly y A b. Similarly, between B and C will have x B C which goes to the short route and y B C which goes via the long route and some way for x A C and y A C. So, we have the six variables describing the different ways of connecting pairs of customers.

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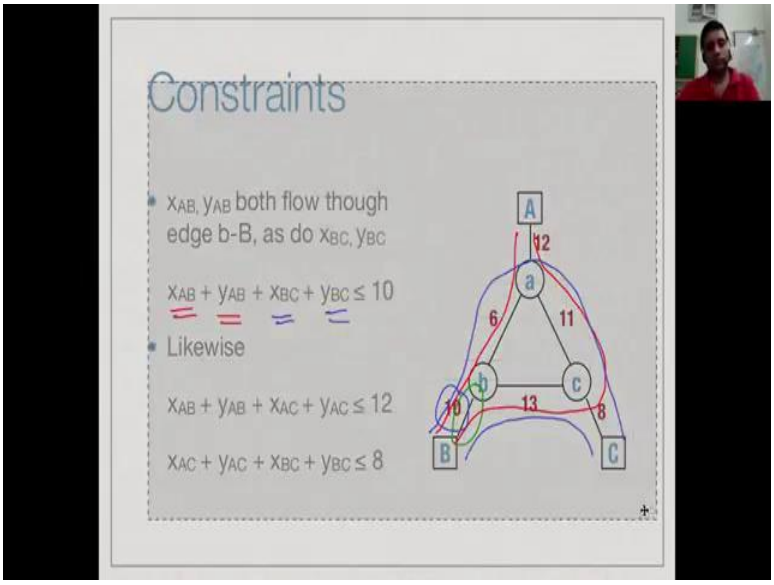

Now, these variables are constraint by the capacities of links. So, supposing we look at this particular link, the link from small b to Capital B, now it has a capacity of 10. Now, what route that it lies, so it certainly lies on the short route A to B. So, it lies on the x A B route. So, if any quantity which is assigned to x A B will reach into these 10, similarly it lies on the y A b route, so that also start ultimately reach capital B to small b.

So, this will also meet into that, it also lies on the B to C route. So, x B C and finally, it lies on the B to C route going to the other way, this is y B C. So, all these four routes put together will add up to whatever capacities flowing through this link and this link has capacity 10, so x A B plus y A b plus x B C plus y B C has to be at most 10.

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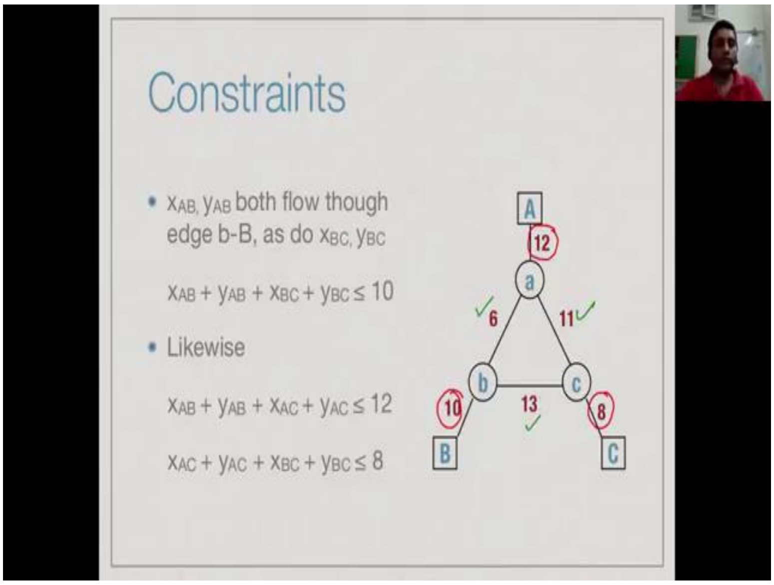

So, likewise, if I look at for instances this link, the same way I have all these different things which are coming here. So, I have four quantities, I have direct link from A to B, the indirect link from A to B, a direct link from A to C and the indirect link from A to C and they all most add up to at most well. And the third same of wholes for this, so these three constrains that we have seen here, account for the capacities of that tail and links. So, we are account for this capacity, this capacity and this capacity, so these three capacities are connected to our flows by these three equations. So, this still leaves us to account for these three constraints.

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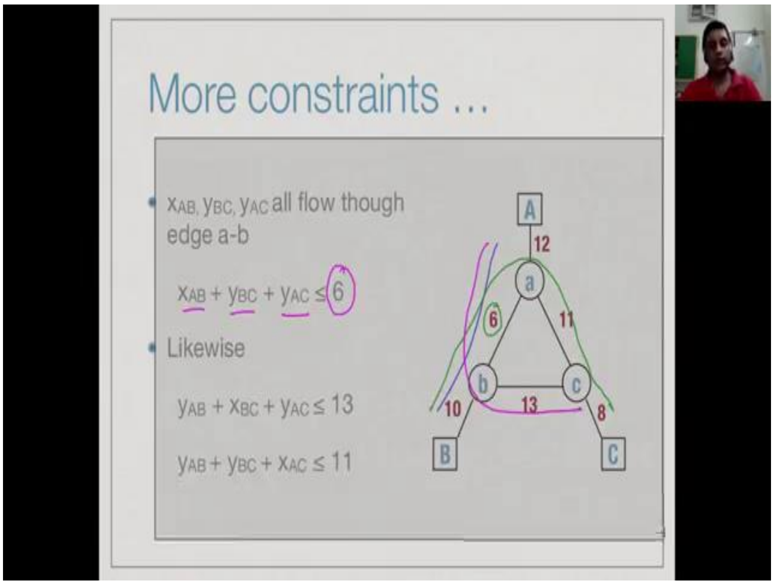

So, now, if we look at this link, then this is a part of three connections, it is the A to B direct connection, it is on the B to C indirect connection, and finally it is on the A to C indirect connection, so there is a direct connection A to B plus the indirect connection B to C plus the indirect connection A to C, all together cannot be more than 6. In this same way, we have similar equation for this link.

So, it is lies on the indirect connection from A to B, it lies on the indirect connection from A to B, it lies on the direct connection from B to C and it lies on the indirect connection from A to C. So, these three things cannot exceed 13 and the third same think whole squared 11. So, for a 11, we have that it lies on the direct connection from A to C and on these two indirect connections, A to B indirect connection and B to C indirect connection. So, in this way now we have covered the six constraints, so we have three constraints corresponding to the tail and links and we have three constraints corresponding to the links, the triangle. So, we have six total constraints.

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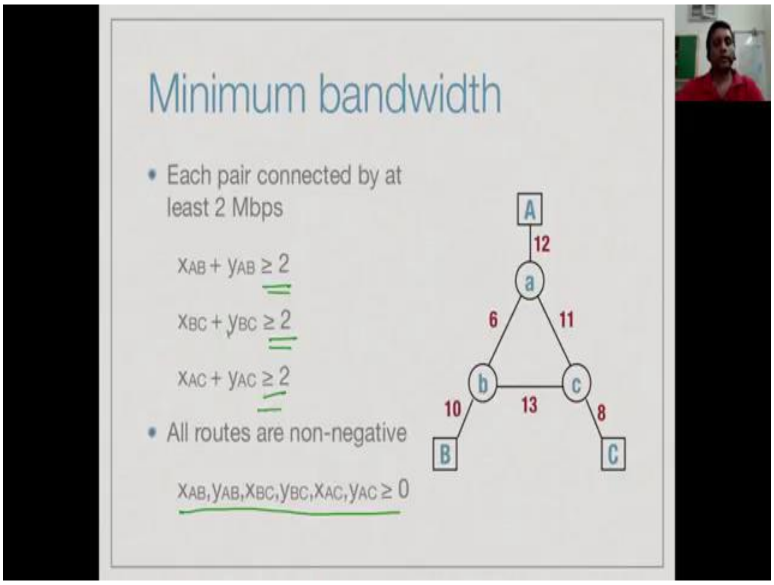

Finally, we have this minimum requirement that between A and B, we must apply at least 2, between B and C we must apply at least 2 and between A and C we must apply at least 2 and these are the sums of the indirect and direct, we do not distinguish between them. And of course, every capacity must be non negative, so this gives us all are constraints.

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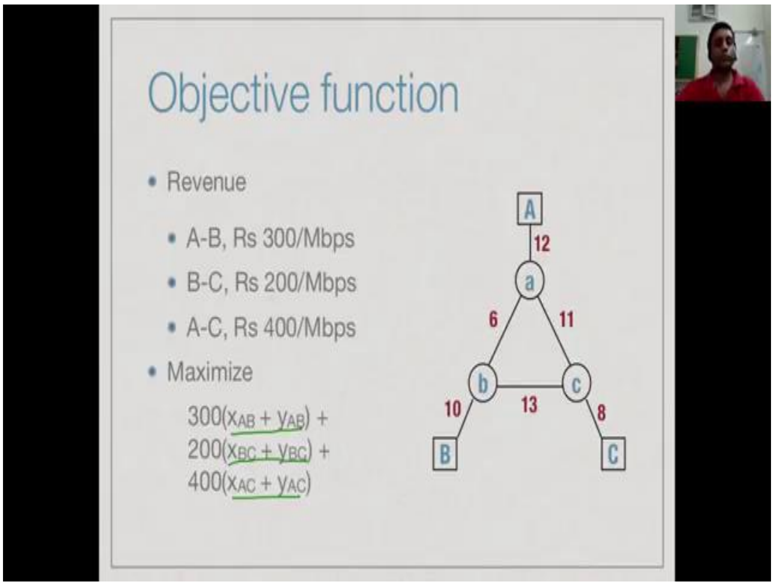

What are the objective functions? The objective function is the revenue that we realize. So, the A to B connection is x A B plus y A B, this is the total volume that gives us 300. Similarly, x B C and y B C gives us 200 and x A C plus y A C gives us 400. So, we multiply 300 into x A B plus y A B 200 into x B C plus y B C and 400 into x A C plus y A C and add it up, this is our total revenue and you want to maximize this revenue**.**

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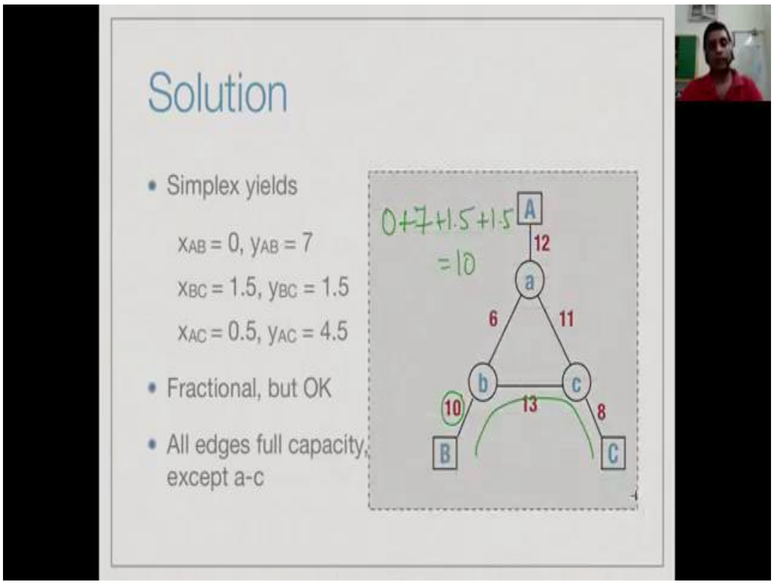

So, for these particular numbers, these are the answer that we get, that we have nothing flowing directly from A to B, we have 7 going from A to B directly and so on. So, if you look for the example at this link. So, this link lies on the direct route from A to B, so that is 0, it lies on the indirect route from A to B, that is 7, it lies in the indirect route from B to C, that is 1.5 and it lies on the indirect route from B to C, that is other 1.5. If you see that this is 10, therefore this link as a total capacity of 10 and all 10 units are utilizes, given the combination that it gives.

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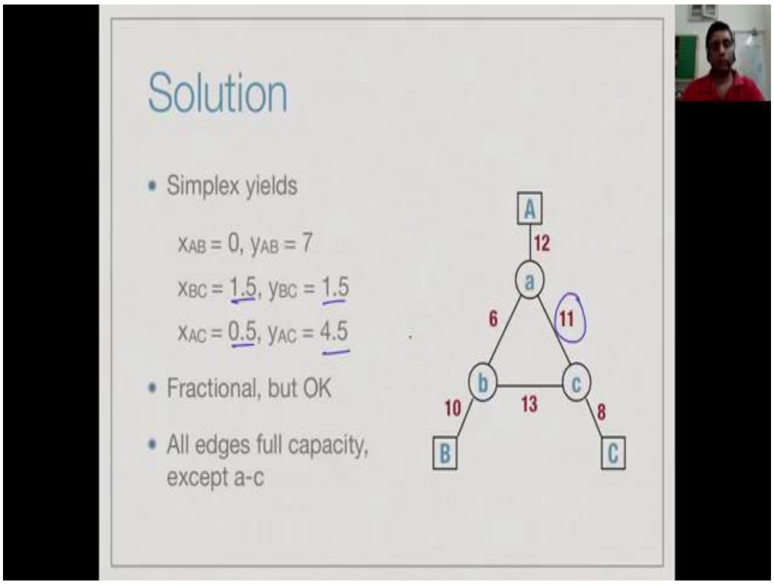

In this way, you can try it for each link and find out that, everything except this link of the 11 is actually saturated by this flow, it also turns out here that you can see that some of the quantities that we get or fraction. But since we are dealing with internet bandwidth, there is no reason it is not like, hiring or firing of person or making offer corporate, we can easily make split our bandwidth in some fractional quantities, so that is not a problem.

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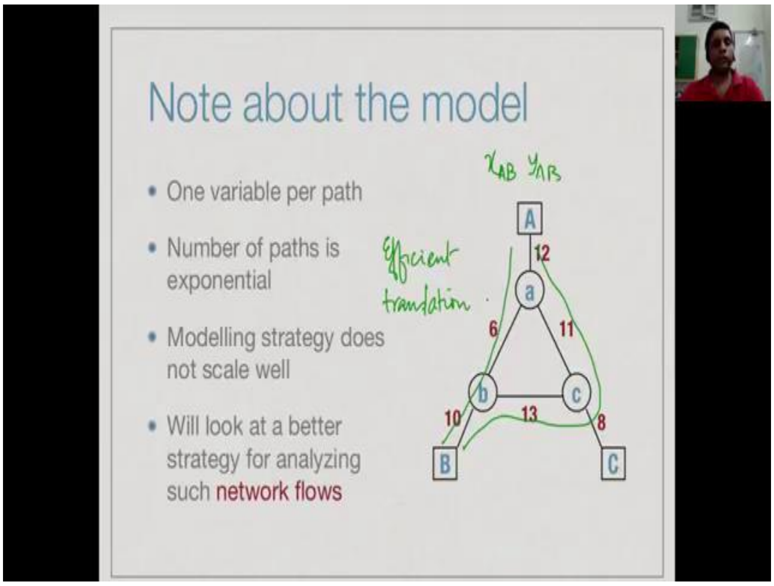

There is however, another problem which is the way we are actually set up the linear program. So, we have the set up the linear program is to take each possible way of outing the traffic. So, we have A to B, we have a link, we have A to B like this, we have another paths and for an each path, we have an variable here, x A B, y A b and so on. So, the every path is represented by the quantity flowing through that path.

So, the problem with is that, the number of paths flowing through graph is going to be exponential. So, this is not good a modeling strategy. So, what we are doing is, we are taking is network bandwidth allocation model and we are implementing it or we are describing it using linear programming. But if we setup a program, linear program, it has a larger number of variables, then the problem is in some sense blowing of in complicity in that translation.

So, we do not want this, we want efficient translation and this is not one, it so happens that for a small problem like this of only three customers, it was fine, but as you grow larger and larger this translation will not scale up. But we will look at another way of looking at the these network flows are the called and see that in general network flows can be easily represented, it terms of linear programs. But it important to note that in general, when we do a translation in to a linear program, we would like the number of variables we get to the small say polynomial in the input problem. So, in the input problem, if we have a certain size, the linear program should not blow up.