Chapter 4

TRUSSES

4.1 Introduction

4.1.1 Assumptions

1 Cables and trusses are 2D or 3D structures composed of an assemblage of simple one dimensional components which transfer only **axial** forces along their axis.

2 Cables can carry only tensile forces, trusses can carry tensile and compressive forces.

3 Cables tend to be **flexible**, and hence, they tend to oscillate and therefore must be stiffened.

4 Trusses are extensively used for bridges, long span roofs, electric tower, space structures.

5 For trusses, it is assumed that

1. Bars are **pin-connected**
2. Joints are frictionless hinges[1].
3. Loads are applied at the **joints only**.

6 A truss would typically be composed of triangular elements with the bars on the **upper chord** under compression and those along the **lower chord** under tension. Depending on the **orientation of the diagonals**, they can be under either tension or compression.

7 Fig. 4.1 illustrates some of the most common types of trusses.

8 It can be easily determined that in a Pratt truss, the diagonal members are under tension, while in a Howe truss, they are in compression. Thus, the Pratt design is an excellent choice for steel whose members are slender and long diagonal member being in tension are not prone to buckling. The vertical members are less likely to buckle because they are shorter. On the other hand the Howe truss is often preferred for for heavy timber trusses.

9 In a truss analysis or design, we seek to determine the internal force along each member, Fig. 4.2

[1] In practice the bars are riveted, bolted, or welded directly to each other or to gusset plates, thus the bars are not free to rotate and so-called **secondary bending moments** are developed at the bars. Another source of secondary moments is the dead weight of the element.

4.1.2 Basic Relations

Sign Convention: Tension positive, compression negative. On a truss the axial forces are indicated as forces acting on the joints.

Stress-Force: $\sigma = \frac{P}{A}$

Stress-Strain: $\sigma = E\varepsilon$

Force-Displacement: $\varepsilon = \frac{\Delta L}{L}$

Equilibrium: $\Sigma F = 0$

4.2 Trusses

4.2.1 Determinacy and Stability

10 Trusses are **statically determinate** when all the bar forces can be determined from the equations of **statics** alone. Otherwise the truss is **statically indeterminate**.

11 A truss may be statically/externally determinate or indeterminate with respect to the reactions (more than 3 or 6 reactions in 2D or 3D problems respectively).

12 A truss may be internally determinate or indeterminate, Table 4.1.

13 If we refer to j as the number of joints, R the number of reactions and m the number of members, then we would have a total of $m + R$ unknowns and $2j$ (or $3j$) equations of statics (2D or 3D at each joint). If we do not have enough equations of statics then the problem is indeterminate, if we have too many equations then the truss is unstable, Table 4.1.

	2D	3D
Static Indeterminacy		
External	$R > 3$	$R > 6$
Internal	$m + R > 2j$	$m + R > 3j$
Unstable	$m + R < 2j$	$m + R < 3j$

Table 4.1: Static Determinacy and Stability of Trusses

14 If $m < 2j - 3$ (in 2D) the truss is not internally stable, and it will not remain a rigid body when it is detached from its supports. However, when attached to the supports, the truss will be rigid.

15 Since each joint is pin-connected, we can apply $\Sigma M = 0$ at each one of them. Furthermore, summation of forces applied on a joint must be equal to zero.

16 For 2D trusses the external equations of equilibrium which can be used to determine the reactions are $\Sigma F_X = 0$, $\Sigma F_Y = 0$ and $\Sigma M_Z = 0$. For 3D trusses the available equations are $\Sigma F_X = 0$, $\Sigma F_Y = 0$, $\Sigma F_Z = 0$ and $\Sigma M_X = 0$, $\Sigma M_Y = 0$, $\Sigma M_Z = 0$.

17 For a 2D truss we have 2 equations of equilibrium $\Sigma F_X = 0$ and $\Sigma F_Y = 0$ which can be applied at each joint. For 3D trusses we would have three equations: $\Sigma F_X = 0$, $\Sigma F_Y = 0$ and $\Sigma F_Z = 0$.

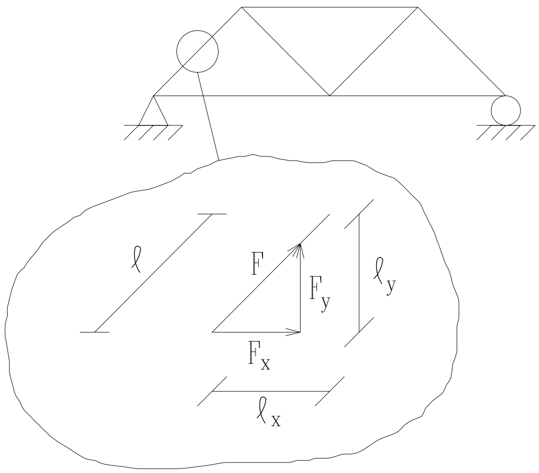


Figure 4.4: X and Y Components of Truss Forces

23 In truss analysis, there is **no sign convention**. A member is **assumed** to be under tension (or compression). If after analysis, the force is found to be negative, then this would imply that the wrong assumption was made, and that the member should have been under compression (or tension).

24 On a **free body diagram**, the internal forces are represented by arrow acting **on the joints** and not as end forces on the element itself. That is for tension, the arrow is pointing away from the joint, and for compression toward the joint, Fig. 4.5.

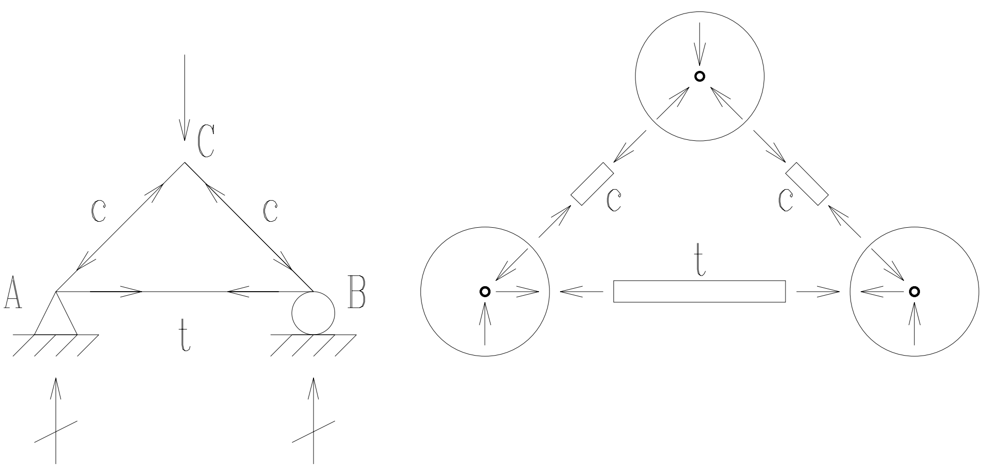
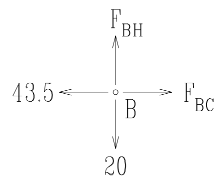


Figure 4.5: Sign Convention for Truss Element Forces

■ **Example 4-1: Truss, Method of Joints**

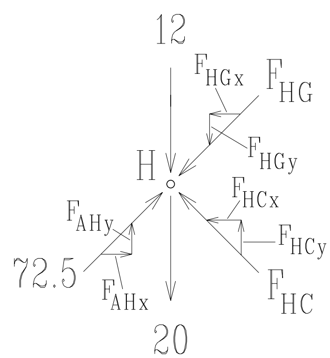
Using the method of joints, analyze the following truss

Node B:



$$
\begin{array}{rcl}
(+\rightarrow)\ \Sigma F_x = 0; \quad \Rightarrow \quad F_{BC} & = & \boxed{43.5 \text{ k Tension}} \\
(+\uparrow)\ \Sigma F_y = 0; \quad \Rightarrow \quad F_{BH} & = & \boxed{20 \text{ k Tension}}
\end{array}
$$

Node H:



$$
\begin{array}{rcl}
(+\rightarrow)\ \Sigma F_x = 0; & \Rightarrow & F_{AH_x} - F_{HC_x} - F_{HG_x} = 0 \\
 & & 43.5 - \frac{24}{\sqrt{24^2+32^2}}(F_{HC}) - \frac{24}{\sqrt{24^2+10^2}}(F_{HG}) = 0 \\
(+\uparrow)\ \Sigma F_y = 0; & \Rightarrow & F_{AH_y} + F_{HC_y} - 12 - F_{HG_y} - 20 = 0 \\
 & & 58 + \frac{32}{\sqrt{24^2+32^2}}(F_{HC}) - 12 - \frac{10}{\sqrt{24^2+10^2}}(F_{HG}) - 20 = 0
\end{array}
$$

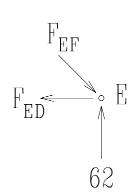
This can be most conveniently written as

$$
\begin{bmatrix} 0.6 & 0.921 \\ -0.8 & 0.385 \end{bmatrix} \left\{ \begin{array}{c} F_{HC} \\ F_{HG} \end{array} \right\} = \left\{ \begin{array}{c} -7.5 \\ 52 \end{array} \right\} \tag{4.2}
$$

Solving we obtain

$$
\begin{array}{rcl}
F_{HC} & = & \boxed{-7.5 \text{ k Tension}} \\
F_{HG} & = & \boxed{52 \text{ k Compression}}
\end{array}
$$

Node E:



$$
\begin{array}{llll}
\Sigma F_y = 0; \quad \Rightarrow \quad F_{EF_y} = 62 & \Rightarrow F_{EF} = \frac{\sqrt{24^2+32^2}}{32}(62) & = & \boxed{77.5 \text{ k}} \\
\Sigma F_x = 0; \quad \Rightarrow \quad F_{ED} = F_{EF_x} & \Rightarrow F_{ED} = \frac{24}{32}(F_{EF_y}) = \frac{24}{32}(62) & = & \boxed{46.5 \text{ k}}
\end{array}
$$