We will write delta Na by delta y, delta Na by delta z, terms also in this case so that will become so there if I am only doing for one x, I get this equation. If I do the others, I will get this extended equation, we have we have dispersion in all the three directions. This is where we stopped last class.

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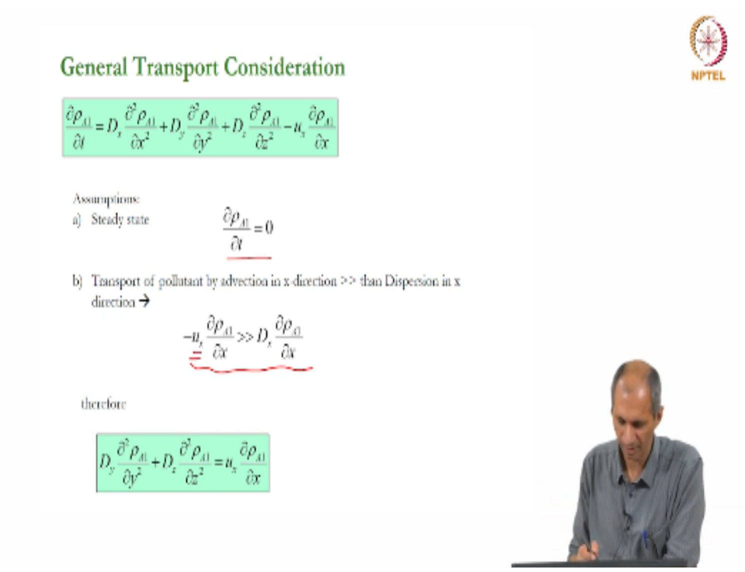

Now here we make two assumptions, one is a steady state assumption you don't have to do this but for that Gaussian dispersion model that we generally present we use the steady state assumptions where we say $\frac{\partial \rho_{A1}}{\partial t} = 0$, which means that at any point in time the concentration is the same, at any location you measure it concentration will not change with time. It will be different with space but it will not change with time, ok. So, if you are looking at it in a plume nothing is going to change so which means for this to be true, everything else has to be true, the emission has to be constant; the properties have to be constant. Nothing should change with the time. If something if any of the parameters in this model changes with time, then this is not true. You cannot use a steady state assumption. So, which means it is an assumption that in an environment nothing is constant everything is changing. Which is why we use average values and then we use standard variation and then we see if you use average and you use the variation what is going to be the range in which this con values are going to be fluctuating from that therefore we make our decisions based on it.

The second thing that we do is since we already have ux bulk flowing in the x direction. We neglect this dx term, so this entire thing reduces to this simpler equation this is where we stopped last class.

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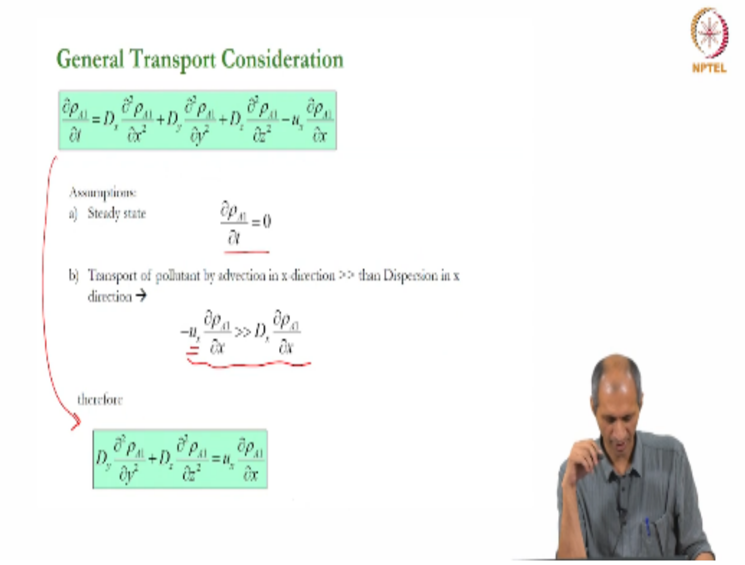

Now, the general solution for this, this is an equation there are solutions are already there. This form, you do separation of variables and do all that.

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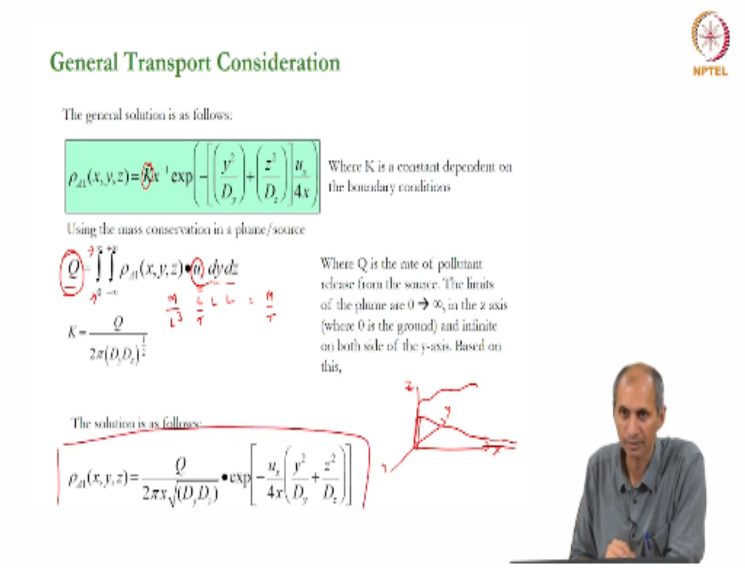

You will get an equation of this form so this there are multiple constants that will come out, there are c1, c2, c3 because there are three dimensions here x is there, y is there, z is there, there are multiple things so we club all of them into this constant, everything comes into one constant and then people use this single equation as the combination of all the boundary conditions, ok. So normally when you say boundary condition will say x equals to x 1 value of rho A is this much or something happens to no flux, there is a wall boundary condition and all that. We fix the values or we give some indication as to what it is. Here, we are doing all of that in one shot what we are saying is there using a mass conservation in the plume if you take the entire volume of the plume, the entire volume is coming from the source so we are saying Q is the rate of pollutant release equals u multiplied by dy and dx, ux into dy into dz will be if you look at the dimensions L by T into L into L into M by L cubed this is M by L cubed equals M by T, yeah. So, this Q and so we are integrating y from minus infinity to plus infinity, which means this is the y axis this is the this is the x axis is, this is the z axis and this is the y axis. So, this there is a plume that is originating let us say here and it is going here. It is expanding in the y axis, it can it is free to expand wherever it wants y axis. In the z axis, it is not free to expand wherever it wants there is a limit it will it has to stop at 0 because that is a ground can't go beyond that.

So, that is why the limits are 0 to infinity. Above the ground, it can go however long it wants. So the plume boundary is fixed on the in the y direction y minus infinity to plus infinity when x dire in the z direction in the x it is anyway moving and it is said by this term ux. So, it is the rate at which it is moving and that the rate at which so the plume boundary is expanding in the x direction and in y direction and z direction, so, overall boundaries is now the total amount of mass present in this plume equals Q, ok. If you use this the constant now is determined as the Q divided by 2 pi Dy Dz raised to half, you substitute it back into this equation, this is the general solution. So this this you get is the general solution, ok.

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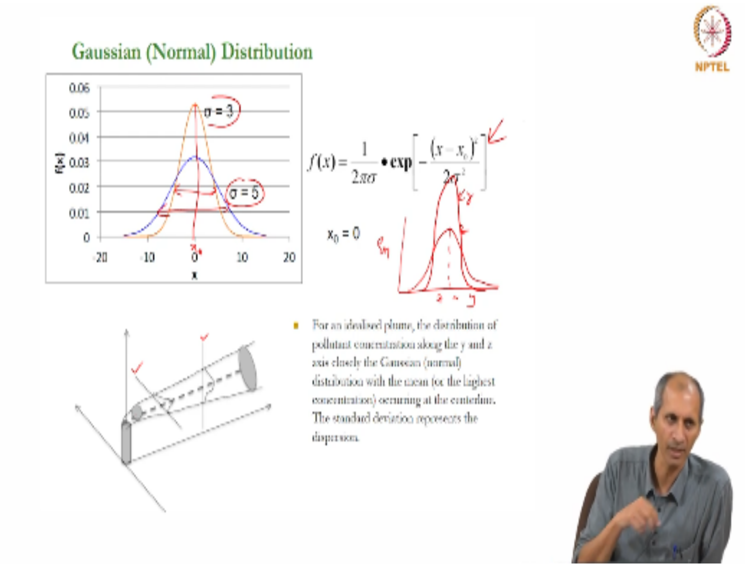

Now here this general solution is you know, you need this term Dy and Dz and all that. So, this is all important. So, you can use this equation as it is. But there is something that people have gone ahead and manipulated this particular equation with the ideal assumption that if you look at this equation, this is very similar to another equation which is called as a Gaussian distribution or a normal distribution.

So, where the equation for normal distribution is this

$$f(x) = \frac{1}{2\pi\sigma}.exp\left[\frac{(x - x_0)^2}{2\sigma^2}\right]$$

so in this this is a distribution this is x0 mean value at this value, what is the the value of x0 is the value of f of x is the maximum. So it is a symmetric bell curve, ok and this spread sigma is a spread the spread and the value of the spread at 60 percent some value some percentage of the the the main f of x.

The magnitude of the sigma indicates how how much of spread there is so if more spread, because of conservation we are assuming that it spreads more the highest concentration is going to be smaller. If it spreads less the highest concentration is going to be higher which is consistant with our this thing that your plume is confined to a smaller volume concentration is likely to be higher plume spreads more concentration is the highest concentration is likely to be lower.

So, where do you find the highest concentration? Where will you find it? So the for which we look at an ideal plume. So an ideal plume is like this it is nicely going in this cone kind of fashion. The cross section is looks like an ellipse or even a circle some form of ellipse or a circle. So in this if you look at the distribution of pollutant concentrations so concentration distribution, so if I am plotting instead of this if I plot instead of, f of y what I am plotting this rho A1 as the function of z or y either.

I am likely to find that this concentration will look like this, which means that some value the highest concentration occurs at some value of z or y, yeah. So let us say this is z and this is this is y, this y dispersion occurs like this, z dispersion occurs like this depending on the the the how much of dispersion is occurring in the y and z directions it's different. So this this this curves here show the dispersion the spread of the distribution of the concentration.

So it goes to 0 outside the plume at the plume boundary it stops, the concentration goes to 0 somewhere in the middle of the plume right in the center point it is the highest concentration. This is an idealized curve; you have to understand that this is an ideal curve. This does not usually happen a lot of times this this this is not like this, it is a bit skewed and all that but this is a good starting point for for lot of these kinds of things. So, what people did is to fit this equation into the format that we have.

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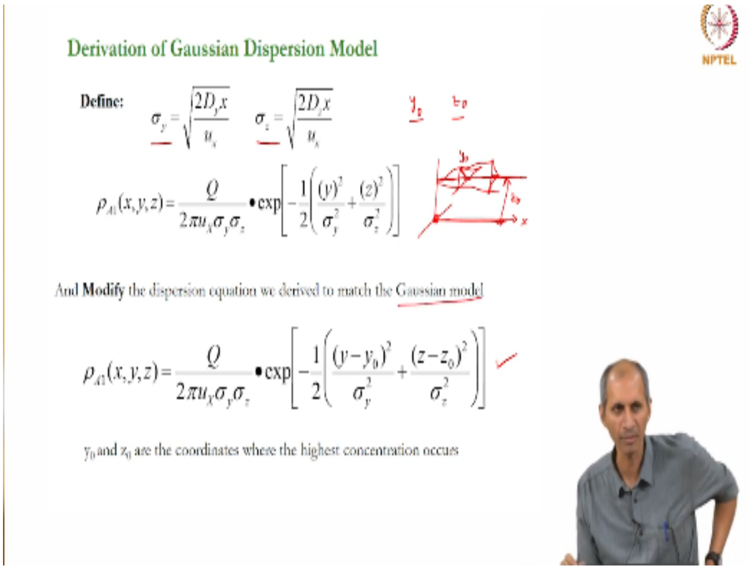

So this this equation here doesn't look it looks almost the same but is not in the same format. So we we are fitting this into the same format. So which means we have to define some new parameters to fit this equation into a format So, this is the Gaussian equation. If you look at the bottom equation here the Gaussian dispersion model looks like the other equation, except that there are few other additional terms here.

Now to make it look like this you have to do some transformation so one of the transformation is this one this one we do sigma y and sigma z into 2 Dyx divided by ux and so on. So you make these transformations and we we enforce that the concentration that at y0 and z0 these are the points at which the highest concentration occurs which means this is the center of the plume wherever so this is the value of y at which the highest concentration occurs this is the value of z that is the highest concentration will occur.

So this depends on the source itself. This depends on the shape of the plume and where is the source and all that. So if you have an idealized this thing, so the plume is going like this. This point, this line in the along the x axis or top of the x axis, we will determine what is z 0, because this is the center point when we are looking at the concentration the distribution of along this will be like this.

So at this at this height you're you are going to get the highest concentration. Similarly, if you if you look at this from axis from this side, ok, if I am looking at it from the y axis y direction, this is the y axis. So I have, right, but again along the x axis because this is a plume this is called as the at x axis this is called as the center line where the y is zero there some height at height z equals zero is the highest concentration but also at y equals to 0, the highest concentration occurs here, ok.

So this is y y0. So in this particular case, y0 is on top of the x axis. If you look at it by flip the axis this way if I am looking along the x axis towards the source it is along the x axis it is on top of the x axis, which means y is 0 that is center point. It could be anything so it depends on how you are defining the coordinates and all that so we will come to that later why when that will become important.

So, this is not a there is no there is no there is no general reference fixed you have to take a reference point if you take the source as the origin from there everything else is defined. That is why it is the Lagrangian concept of this model is that we are there is no fixed reference point. There is no universal this thing it is with reference to some the plume itself and the stack itself. So so this final equation looks like this. So the Gaussian dispersion model in its preliminary form looks like the; this equation here.

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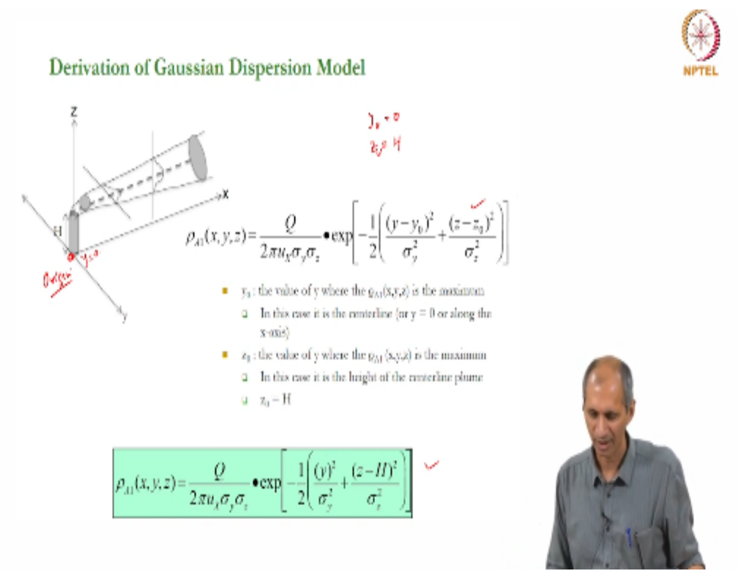

Now, when we put input this y - y0, x - x0. So if the plume the plume if the stack source is that the origin. This is x equals to y x, this is y equals to 0 and there is the height of this emission the stack. So let us say there is a chimney and this goes to a certain height and that is where the emission is occurring. So you expect the highest concentration to occur at that point. So therefore here in this case, y0 equals 0 and z0 is the height.

So therefore this equation modifies itself and becomes this z minus h where h is the height of your source. Now, sometimes the source may not be in a chimney it could be on the ground. So that time h will be 0 so which will become then that equation will modify further, so you can modify this equation in whichever way you want so this is a general equation. Since we have now I would recommend starting here because there are lot of cases in which you will not have a stack you will not even have y you will not be defined as x axis it may be separate.

But most often y is the central line that is central line so do not worry about that. The second thing that is important here is the x axis itself is defined as the direction of the wind speed x axis can change wherever it wants it can point north it can point east west south doesn’t matter. In the reference for the Gaussian dispersion model x axis is the direction of the wind so the wind speed keeps changing over if you look at Chennai for example, wind speed is in the morning in one direction the afternoon it is in different direction, it keeps changing with season and all that so you cannot have the wind speed you cannot have fixed frame of reference, the frame of reference is with the wind speed. So you have to find out what is the wind speed and then start there and then the rest of the axis is defined y axis is defined based on this so there is no fixed x axis, ok, this is very important so the first thing you need to start all of this you need to know what is the wind direction.