

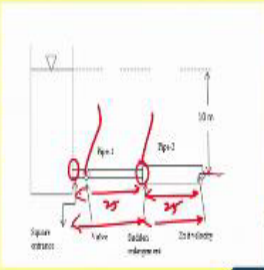
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Lecture - 46
Pipe Networks (Contd.,)

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Class Problem

- A horizontal pipeline, 50 m long, is connected to a reservoir at one end and discharges freely in to the atmosphere at the other end. For the first 25 m length from the reservoir the pipe has a diameter of 15 cm and it has a square entrance at the reservoir. The remaining 25 m length of pipe has a diameter of 30 cm. The junction of the two pipes is in the form of a sudden expansion. The 15 cm pipe has a gate valve ($k=0.2$) in fully open condition. If the height of the water surface in the tank is 10 m above the centreline of the pipe, estimate the discharge in the pipe by considering the Darcy Weisbach friction factor $f = 0.02$ for both the pipes. [Include all minor losses in the calculations].







Welcome back students. In the last class we finished the lecture by introducing a problem that is supposed to be done in the class. So there is a reservoir it is connected to a pipe, pipe is in two different areas sorry, in two different diameters. The length is the total length is 50 meter long, but this is 25 centimeter having a different diameter and this is having a different diameter. There is a sudden expansion, there is a valve here, there is going to be major losses here.

And there is going to be major losses here, minor losses will be at this particular point here, here and there will also be going to be a minor loss at the square entrance. So this is a problem where you will get to understand and practice all the major and minor head losses again. So that is why I thought it to include it as the solved problem, normally I give this question in the class test or exam for my B.Tech second year students.

So I am going to solve this now it might be a little lengthy problem, but it is important for you to follow what is going on. So you should try to remember this figure that a pipe is connected to the reservoir, this is the square entrance, this is the valve, this is the sudden

enlargement and there is an exit. So there is also going to be an exit thing, exit loss and the water height, I mean the total height is 10 meters in the reservoir.

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Solution 14

The pipe system has following

(a) Minor losses

(i) square entrance $= 0.5 \frac{V_1^2}{2g}$

(ii) valve $= K \frac{V_1^2}{2g} = 0.2 \frac{V_1^2}{2g}$

(iii) sudden expansion $h_{L2} = \frac{(V_1 - V_2)^2}{2g}$

(b) Next there exist friction loss h_f in pipe 1 & 2

given by

$$h_f = \frac{fLV^2}{2gD}$$

(c) further there exists an exit velocity head at the end of pipe 2 of magnitude $= \frac{V_2^2}{2g}$

$$H = \left[\frac{0.5V_1^2}{2g} + \frac{0.2V_1^2}{2g} + \frac{fLV_1^2}{2gD_1} \right] + \left[\frac{(V_1 - V_2)^2}{2g} + \frac{fL_2V_2^2}{2gD_2} + \frac{V_2^2}{2g} \right]$$

Here $H = 10\text{m}$
 $D_1 = 0.15\text{m}$
 $D_2 = 0.30\text{m}$
 $h_{L2} = \frac{(V_1 - V_2)^2}{2g} = \frac{V_1^2}{2g} (1 - \frac{V_2}{V_1})^2$

So solution 14, so the pipe system that we have in question, first we should be able to write what all things are there. A it will have minor losses. So what are the minor losses? The first one is the square entrance and the head loss here will be equal to $0.5 v_1^2 / 2g$. We have learned this formula, since there is a valve, there is a minor loss that is equal to K into $V_1^2 / 2g$, K is already given in the question as I told you, it is $0.2 V_1^2 / 2g$.

And the third one is sudden expansion, so h_L , so between 1 and 2, so I call it h_{L2} as $V_1 - V_2$ whole square/ $2g$ and in the end, then we have next there exists friction loss h_f in pipe 1 and 2, which is given by fL , general formula is $fL V^2$ by $2 gD$ and also there exist an exit velocity, exit velocity at the end of pipe 2. So this loss of magnitude, so entire velocity is loss, so $V_2^2 / 2g$.

Now we have listed all the type of losses here. So we have to rewrite the total head loss H as, so starting from here it will be $0.5 V_1^2 / 2g + 0.2 V_1^2 / 2g +$ so in the pipe 1 there will be one major loss as well $fL v_1^2 / 2gD_1 +$ in pipe 2. First of all, it will be $V_1 -$ sudden expansion $V_1 - V_2$ square/ $2g + fL_2 V_2^2 / 2gD_2$. This is the major loss due to the flow and in the end there is an exit loss $V_2^2 / 2g$.

So we have listed down the energy loss. So we start calculating the different you know, so what we can do is, we can you know, so we can start writing here the total head is 10 meters

that is and D1 is because that is what is going to lose, entire head is going to lose 0.15 meter and then there is D2 as 0.30 meter that we know. So h12 this one sudden expansion is $V_1^2 - V_2^2$ whole square/2g or V_2^2 square/2g/ $V_1^2 - 1$ whole square.

So we have written, now there are two velocities V_1 and V_2 and you know what the easiest method to find V_1 and V_2 is the equation of continuity and that is what we are going to do next.

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Handwritten derivations:

Left side:

$$A_1 V_1 = A_2 V_2$$

$$A_1 D_1^2 = A_2 D_2^2$$

$$h_{12} = \frac{V_2^2}{2g} \left(\left(\frac{D_2}{D_1} \right)^2 - 1 \right)$$

$$h_{12} = \frac{V_2^2}{2g} \left(\left(\frac{0.3}{0.15} \right)^2 - 1 \right)$$

$$h_{12} = \frac{9 \cdot V_2^2}{2g}$$

$$H = 10 = \frac{V_1^2}{2g} \left[0.5 + 0.2 + \frac{0.02 \times 25}{0.15} \right] + \frac{V_2^2}{2g} \left[9 + \frac{0.02 \times 25}{0.30} + 1 \right]$$

Right side:

$$10 = \frac{4.03 V_1^2}{2g} + \frac{11.67 V_2^2}{2g}$$

$$V_1 D_1^2 = V_2 D_2^2$$

$$\Rightarrow V_1 = V_2 \left(\frac{D_2}{D_1} \right)^2 \Rightarrow V_1 = 4 V_2$$

$$10 = \left[\frac{4.03 \times 4^2 V_2^2}{2g} + \frac{11.67 V_2^2}{2g} \right] = \frac{76.14 V_2^2}{2g}$$

$$\Rightarrow V_2 = 1.605 \text{ m/s}$$

$$Q = \frac{\pi \times (0.30)^2 \times 1.605}{4}$$

$$Q = 0.1135 \text{ m}^3/\text{s}$$

So we also know that $A_1 V_1 = A_2 V_2$, so $A_1 D_1^2 = A_2 D_2^2$. So using this we can write $h_{12} = V_2^2$ square/2g, instead of V_1 sorry $A_1 V_1 = A_2 V_2$ sorry, so this not correct. So we can transform V_1 and V_2 in form of D_2 and D_1 . So V_2^2 square/2g, this will become D_2^2/D_1^2 whole square – 1 whole square. So h_{12} is going to be V_2^2 square/2g and D_2^2/D_1^2 we all know it is 2, so $0.3/0.15$ whole square – 1 whole square.

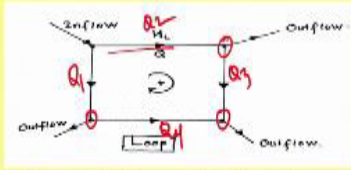
So h_{12} we can say, so this is 2 square is 4, $4 - 1$ is 3, 3 square 9. So it will be 9 into V_2^2 square/2g. So we can simply now write down the value of H as = 10 as $V_1^2/2g$ $0.5 + 0.2 + 0.02$ into $25/0.15 + V_2^2$ square/2g $9 + 0.02$ into $25/0.30 + 1$. So $10 = 4.03 V_1^2$ square/2g + 11.667 into V_2^2 square/2g and we also know that $V_1 D_1^2 = V_2 D_2^2$ square. So this implies V_1 will be V_2 into D_2^2/D_1^2 whole square implies $V_1 = 4V_2$.

So putting this here we can get $10 = 4.03$ into 4 square + 11.67 into V_2^2 square/2g that is 76.14 into V_2^2 square / 2g and on solution, this is going to give V_2 as 1.605 meters per second and if we know V_2 we can simply find Q as $\pi/4$ into 0.30 whole square 1.605 that will give

us 0.1135 meter cube per second. So you see how we have applied the principles of major and minor losses and the equation of continuity to solve such a complex problem like this.

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Hardy Cross Method



• Assigning clockwise flows and their associated head losses are positive, the procedure is as follows:

- Assume values of Q to satisfy $\sum Q = 0$.
- Calculate H_L from Q using $H_L = KQ^2$
- If $\sum H_L = 0$, then the solution is correct.
- If $\sum H_L \neq 0$, then apply a correction factor, ΔQ , to all Q and repeat from step (2).

Handwritten notes on the slide:

$H_L = KQ^2$

Node	Q	H_L	H_L/A
AB	"	"	14/8
BC	"	"	"
CD	"	"	"
DE	"	"	"

Handwritten correction factor: $\Delta Q = -0.01$

So we go back and we are going to start what we promised is like, the Hardy Cross Method. So what does the Hardy Cross Method say? So if there is a flow like this, there is an inflow and there is an outflow you see, and there is a loop that is formed. So what we do? We assign the clockwise flows and their associated head losses as positive. The clockwise flows and their associated head losses are positive.

So what we do is at each nodes we distributes cubes and write that delta Q = 0, at each node and we calculate head loss from Q using $H_L = K Q^2$ because you remember head loss was $K \text{ into } V^2 / 2g$ or we can simply write $K Q^2 / 2g A^2$. So basically H can be written as $K \text{ dash } Q^2$, if you bring $K/2gA^2$. So our head loss is written like this, $K \text{ into } Q^2$ because head loss is proportional to the discharge square.

If the head loss = 0 then that means the solution is correct. If it is not equal to 0 then we apply a correction factor delta Q and we go to the next step, this delta Q is not arbitrary.

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Hardy Cross Method contd.

- For practical purposes, the calculation is usually terminated when $\sum H_L < 0.01$ m or $\Delta Q < 1$ L/s.
- A reasonably efficient value of ΔQ for rapid convergence is given by;

$$\Delta Q = - \frac{\sum H_L}{2 \sum (H_L / Q)}$$

Handwritten notes on the slide:

- $\Delta Q = -5$ (written in red)
- A diagram of a rectangular loop with flow rates: 10, 30, 20, and 90.
- A diagram of a rectangular loop with flow rates: 10, 30, 20, and 90, with a note "20 + 10 = 30" and "90 + 20 = 110".

We have a led procedure in Hardy Cross Method. That will tell what delta Q is but in principle getting delta HL, sum of all the head losses exactly 0 is not possible so we terminate the calculations if we get head loss, if it is less than 0.01 meter or delta Q is less than 1 liters per second we stop the calculations there itself. So instead of 0 we go for a very, very small quantity.

So a reasonable and efficient value of delta Q for rapid convergence is given by this. So this thing you have to remember, delta Q is written as minus of sigma head losses/2 sum of HL/cube. So for each pipe in the loop we are going to calculate head losses, do the summation of it, not only we are going to calculate HL, but also the value of HL/cube because if the sigma head loss is not coming 0 we will need that value later.

So I will just repeat the Hardy Cross Method. So at each node you distribute Q such that at each node delta Q should come to 0. It should be an arbitrary Q, of course you should satisfy the continuity equation. Suppose 100 is coming from there and there are two ways you can say 60-40, 70-30 or 80-20 whatever, but it should satisfy the continuity equation. Second thing you have to do it at all nodes in the loop.

So once Q is defined here Q1, Q2, Q3, Q4 for example, then you start calculating HL. HL is k into Q square. This K either you have to calculate yourself or it will be given also, both can happen. So you will prepare a table like this, you see, pipe you will say AB, BC, CD, there are 4 pipes here DE for example, could be. A circular pipe for example, then you need to whatever Q you have distributed you will write it down here, whatever those are.

You have to find HL based on this Q and thus you also have to calculate HL/Q for each of these pipes and do the summation here and do the summation here. You have to stop the calculation if this is very near to 0. If not, then we will rely on this value of delta HL/Q by this equation, this equation and you will add this delta Q to each of those Qs in each pipes.

Suppose let us say if this pipe was 10, if it was 30, it was 20, it was 90 and our delta Q comes to be - 5, for example. Then what you do, in the next iteration, so you have to repeat this process again and in the next iteration the first pipe will have 10 + of -5. This will have 30 + of - 5. This will have 20 + of - 5 and 90 + of - 5 and then you have to make the table again for each of the pipe you have to write Q, you have to write HL.

And you have to write HL/Q, do the summation here and you have to stop if this delta HL comes less than 1. So this is the broad explanation of Hardy Cross Method. So I think I will stop here now and in our final lecture I will start with the solution of this Hardy Cross problem. So the next lecture is going to be the last one for this module pipe flows. So thank you so much and we will start right from this place. See you in the next lecture.