



**Environmental Quality: Monitoring and Analysis**  
**Prof. Ravi Krishna**  
**Department of Chemical Engineering**  
**Indian Institute of Technology, Madras**

**Lecture-41**  
**Gaussian Dispersion Model**

(Refer Slide Time: 00:12)




- Critical parameter
- Emission Rate (ER) = Emission Factor (EF) x Activity Rate (AR)
- Emission Factor
  - Listed for different point sources
  - AP-42 emission factors for different species
  - <https://www.epa.gov/air-emissions-factors-and-quantification/ap-42-compilation-air-emissions-factors>
  - Function of the type of activity apparatus
  - Listed as Mass of A / Unit Activity
    - Example: Mass of SO<sub>2</sub> released per Kg of coal burnt using a grate burner
  - For line sources (roads) emission factors for vehicles is available
  - Emission rates for Fugitive emissions (evaporation from different sources) must be estimated using interphase mass transfer equations (to be discussed separately)
- Activity Rate
  - Rate of activity
    - Example: kg of coal burnt per day




So in yesterday's class, we were talking about dispersion modelling parameters, so one of the parameters is the emission rate.

(Refer Slide Time: 00:22)



- Critical parameter
- Emission Rate (ER) = Emission Factor (EF) x Activity Rate (AR)
- Emission Factor
  - Listed for different point sources
  - AP-42 emission factors for different species
  - <https://www.epa.gov/air-emissions-factors-and-quantification/ap-42-compilation-air-emissions-factors>
  - Function of the type of activity apparatus
  - Listed as Mass of A / Unit Activity
    - Example: Mass of SO<sub>2</sub> released per Kg of coal burnt using a grate burner
  - For line sources (roads) emission factors for vehicles is available
  - Emission rates for Fugitive emissions (evaporation from different sources) must be estimated using interphase mass transfer equations (to be discussed separately)
- Activity Rate
  - Rate of activity
    - Example: kg of coal burnt per day



And this emission rate is a combination of emission factor and activity rate. So emission factor is something that needs to be determined for every pollutant for every process.

And this is the source for that. this is a large compilation of this US EPA has this is called as AP-42; its fairly common, this compilation of one set of emission factors, but people are always deriving emission factors, measuring emission factors for various things, for example, there are certain things that are here that are not used in India and in some other countries.

There are certain things in India, which are not used elsewhere and AP-42 will not have some of that. For example, if you want to know the emission factor for burning biomass or burning wood for cooking or something, it is not very common, in US, people use it for different purposes. So here, the lot of people in rural areas still heat water using whatever is available locally and also cook sometimes.

Now but then this LPG and all that now is expanded. So any emission that occurs from any process for whatever parameters is listed here and It is a growing list. Here, we look at this for stationary point and area sources as I mentioned yesterday, so there are fixed things for example, when you say stationary point fixing is for A, the first chapter you say it says is external combustion sources.

And you can look at all the categories external combustion sources of solid waste disposal, stationary internal combustion sources, evaporation loss sources, petroleum industry, organic chemical processor and so on keeps going. Greenhouse gas, biogenic sources so on. So we take a look at the first one external combustion sources. In this there is large set of categories, you can see we keep going through 11 categories.

First one is bituminous and sub-bituminous coal combustion, anthracite coal combustion there are different types of coal, fuel oil combustion, natural gas combustion, LPG combustion. So if you take LPG combustion, for example or wood residue combustion in boilers, lignite combustion. So some of this is applicable in different places like they can be adopted straight away. So it gives you information like residential fireplace is not very commonly used in India.

We do not use indoor fire places, a lot of people use outdoors in winters in northern India, people may use something outdoors and indoors a lot of electrical heaters have replaced many of these things. So let us look at one of them for example say anthracite coal combustion. So it gives the general description of the process what is the combustion process this is very important because the emission factor depends on that.

How you burn the coal is important and what is the coal? What are the firing practices? So this is by itself each one of these chapters can become a large body of work. So people do research, they will study just the emission factor of one particular thing and that is that. So the emissions and everything we have learned in this course comes here because when you do emission factor analysis we need to measure, which means you need to monitor and how do you monitor? And what is the objective of monitoring? Because you need to exactly measure your burning grams of fuel and what is it that is coming out? All of it that is coming over, you need to capture and you need to capture it accurately and so this all comes there. So here we have emissions that are particulate matter and are known emissions. Sometimes there may be unknown emissions you do not know what they are. Unless you close the mass balance, you do not know what is it that you may have to do that. So the known emissions particulate matters, sulphur oxides, nitrogen oxides, carbon monoxide, four major things and controls, so this control is part of the equipment at the practice of doing it. For example, in industries we are burning something and you have an emission control.

Because you know this emission exists then you put a control then you see, what is the emission after the control? The 80% of the emission is removed, this is what is without the control and this is with the control. So that makes a case for having control and those kinds of things. So now for look at NO<sub>x</sub> compound from uncontrolled anthracite coal combustors.

Uncontrolled means this is nothing you have not attached any combustion control equipment for SO<sub>2</sub> ,and NO<sub>x</sub> and SO<sub>x</sub> . So it is again on the first column, you can see stoker fired boiler is some kind of boiler you can go and read what that is back in the section. What is the stoker fire boiler? What is an FBC boiler? It is not important for this discussion, but it gives you first for stoker fire boilers, the emission factor is pounds per ton, lb per ton its 39.

This S and B refer to some weight percent sulphur and all that, so 39 pounds per ton of coal combustor and NO<sub>x</sub> is 9 pounds per ton. So this is a number where you just remember that, based on our quality control things. Here it is a single number and does not have plus minus. something usually it will have there reporting average numbers and this you can use and this as a guideline, for designing so on;

So if you keep going further down says carbon monoxide and carbon dioxide, so even though carbon dioxide is not a pollutant in the classic sense that we are talking about but is still listed now it will become an important emission factor. People want to know how much carbon dioxide is coming out. And this particulate matter PM. Filterable PM, so the definition of filterable PM is given the document.

Sometimes it is listed as PM10, which means you are using a PM10 device to collect what is coming off, so it means PM10, PM 2.5 so on. So you can collect whatever you want and that becomes an emission factor for that particular process. So in the dispersion model you want to get emission rate you, say I am going to find out the emission rate for coal combustion, you need to know the emission factor.

All the information for emission factor, you need to gather and then find the emission factor from a source like this and then find out how long you are going to operate it and what is activity rate for 1000 tons per day and that is a rate at which it is going. If you are going to do dispersion model for 1 hour then what is the activity rate for 1 hour and what is the emission rate in that 1 hour as to be estimated. There are few other things then there is emission factor.

Mass, this is some other things that are listed here and so on. So there are few other compounds. Other than the priority ones Acenaphthene, naphthene, anthracene PAHS a few other things and the people have found out these things. Sometimes people do this under experimental conditions where they will burn a certain amount of collect everything sometimes they want to do it in the real scenario, in which they will go and measure the actual emissions. That is very tricky to do. Measurement of actual emissions in the open exposed to air and difficult to measure that so that there are ways of doing it but it is filled with lot of uncertainties. So there are other things people measure. So for entire things, you get the idea of it how. You can have any number of sections here.

So this is just one set of things, point sources. There is another large set of emission factors which are from vehicles. The vehicles are considered as part of a line source. So there are roads which are full of vehicles. So emission is occurring from a main highway major highway or city road so you need to know how many vehicles are there and each vehicle has a certain emission factor. So again, you can imagine the emission factors for vehicles.

See this stuff, emission factor that you are seeing, this is for CO<sub>2</sub>, you can do this for the other pollutant that we are interested in. The problem here in vehicles is you have large category of vehicles. The category of vehicles, say car, for example. 800CC cars, 1000CC cars, 1400 CC cars big ones 2500CC, 3000CC and so on, keep going and there are fuel gasoline petrol some of them have CNG, some of them have LPG, some of them have diesel.

And there are different kind of engines for each kind of fuel and each kind of fuel composition is different from when it combustion emission is different. So you have to get that, in this particular document they have only given for carbon dioxide, but you can look that benzene, toluene, xylene, hydrocarbons, PM, SO<sub>2</sub>, NO<sub>2</sub>, SO<sub>x</sub>, NO<sub>x</sub> particulate matter everything and ozone that's coming out of that.

**(Video end Time: 09:42)**

**(Video start Time: 09:43)**

So, ARAI is one of the main organizations, so they do a lot of work on this they also measure the emission factors they have a rig where they run the vehicle, you know, it is not done on road. It is done, it is like the instrument detection limit it is done under test conditions because when you go on the road what you measure can include other things from other places.

So we just want to look at what is coming from the vehicle alone under pristine conditions, new conditions when the engine becomes old and all it deteriorates, it does not perform the way it is supposed to. So this also is under conditions like if you have a control device like a catalytic converter without that and with that what is the emission control so on so. ARAI is the organizations in India does that.

Because Indian vehicles are slightly different from the ones that are sold elsewhere, so their fuel is different see lot of it matters, the fuel; fuel that comes is refined in India and a certain fractions of it, they are all not the same, so it varies from batch to batch and varies from company to company all that, so they are not exactly the same. So there is a range in which the emission factors exist in these kind of things.

**(Video ends: 10:58)**

**(Refer Slide Time: 10:59)**

## Parameter Q, Emission Rate

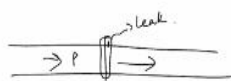
- Critical parameter
- Emission Rate (ER) = Emission Factor (EF) x Activity Rate (AR)
- Emission Factor
  - Listed for different point sources
  - AP-42 emission factors for different species
  - <https://www.epa.gov/air-emissions-factors-and-quantification/ap-42-compilation-air-emissions-factors>
  - Function of the type of activity apparatus
  - Listed as Mass of A / Unit Activity
    - Example: Mass of SO<sub>2</sub> released per Kg of coal burnt using a grate burner
  - For line sources (roads) emission factors for vehicles is available
  - Emission rates for Fugitive emissions (evaporation from different sources) must be estimated using interphase mass transfer equations (to be discussed separately)
- Activity Rate
  - Rate of activity
    - Example: kg of coal burnt per day



So for anything you get emission factors, this is for point sources; we will come to other things which are what we call as fugitive emissions.

(Refer Slide Time: 11:13)

## FUGITIVE EMISSIONS



$$Q = \eta_A \cdot A_{area}$$

SOIL / WATER  
g<sub>air</sub>, W<sub>g</sub>



Fugitive emissions, these are unplanned emissions as the name suggests they are escaping, escaping means, for example, there is a pipeline supposed to carry some gas, some fuel, there is a leak there is a joint and there is a leak here that is a fugitive emission. So these are things like you have a gas tank or a gas pipe there are leaking you always see things are leaking in industry and all.

So these are fugitive emissions and people try to estimate the fugitive emission. Estimation of this is again not easy, depends on several things, so there are people who have developed equations to find out, so the pressure of the gas is this much, ambient temperature is this much,

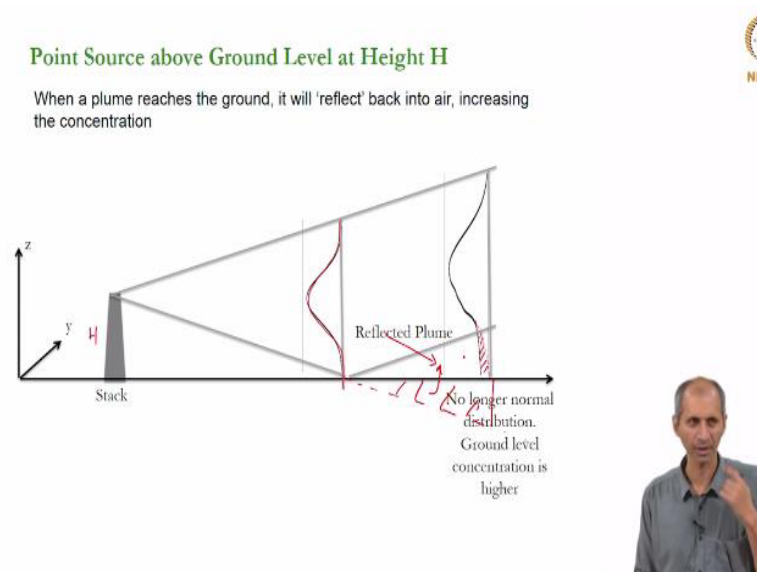
temperature is this much what is going to be estimated release to some of these things. So those are something else. The other class of emissions that are important is things like, there is a soil or there is water some surface some natural surface which contains a chemical. So we have  $\rho A_2$  in water or  $W_{A3}$ . Soil is polluted and water is contaminated and there is going to be a release an emission into air, for example, the soil is contaminated what is the air pollution impact of that? It is not a point source it could be an area source could be what we call as the volume source.

But the way in which we determine the  $q$  for that is by estimating the flux multiplied by the area and the estimation of this flux here requires information of the mechanism by which it happens is the convective or diffusive mass transfer flux. So that entire section will do it, in the next section of this discussion after we finish this portion as interface mass transfer problem.

So we will talk about that separately, there is a large section of problems in that it relates to interface the mass transfer from different phases, not just to air. But it gives you a context in which, why we are doing that since we know this, so if we want to estimate what is the effect of an oil spill in water, air pollution impact? So some some tankers overturn into a river or a sea or a lake, what is the air pollution impact immediately?

So you have a lot of oil sitting there and it is going to evaporate and this is now a source you can apply the dispersion model to this source but to estimate  $Q$ , you need to apply those equations to get to know. So each one of these terms

**(Refer Slide Time: 14:14)**



In dispersion model we have talked about 5 parameters. Each one of these parameters, you have to estimate it independently and then plug it into the dispersion model to the result. So how do we apply this? So there is one more addition to this. So normally when we are looking at this plume we said that the plume is at a height  $h$  from the ground right, which means that at some point it is going to hit the ground if it is spreading like this, so what happens when it hits the ground? The ground is rigid and most of the plumes reflect back into the surface. So what will happen is this, you are expecting a profile like this. You are expecting a Gaussian profile like this. Zero at one end of the plume and the middle it is the highest concentration. But when the plume reflects what you see is that bottom end has some additional concentration.

Because this reflected plume is now adding on, so this plume is now supposed to go here we are nowhere to go so this entire mass adds on to this section. So it is a little more complicated than this, this is a very simplest way of doing. When it reflects it does not go back in the same velocity and all that there is loss of energy and all that and plume itself may some of it can get adsorbed. Whatever pollution is coming can get adsorbed onto the soil surface.

Air may go back but the pollutant may not go out. So not considering all of that, just assuming that all of its bounces back and mixes with rest of the air coming in. So there is a plume that is naturally coming into this direction and this and this adds to that and therefore you see that there it is not symmetric anymore. Now this part is nicely Gaussian normal distribution.

This part is likely more because there is this is what original one is, this is small additional contribution coming from the reflected plume. Okay? So how do you incorporate this into the dispersion model? People have done something very smart about this in simple this thing is that the imagination there is a second store, which is below the ground  $-H$  and it keeps going and is of the same height.

So this plume whatever is being reflected the same volume that is being added here so what we are doing is we are adding the contribution of two sources. One is at a height  $H$  and the other one at height of  $-H$  okay? But the contribution from the second one only comes into play after we cross this point. Till that point we are not even measuring it, how do we measure the concentration of this one?



I have to measure  $\rho_{A1}$ ,  $x$ ,  $y$ ,  $z$ ,  $h$ , but this  $z$  and now has to be - 10 meters. Then I will be able to calculate that. But I am never going to calculate - 10, I am not interested in -10 this is an imaginary plume. So under the ground, I do not care what happens to it. I am only going to be interested in something above the ground with all  $z$  is always greater than 0. So at a point when this reaches this point, it now becomes, it has a concentration, that is positive above  $Z = 0$ .

So at that point we will add to whatever is being contributed from this plume. So, the second plume will add on to the first plume only after this point. So it adds on so essentially what we are doing is, one thing about the Gaussian plume dispersion model is it is additive in the sense that I can add contributions.

**(Refer Slide Time: 18:00)**

For the unreflected plume:

$$\rho_{A1}(x, y, z) = \frac{Q}{2\pi u_x \sigma_y \sigma_z} \cdot \exp \left[ -\frac{1}{2} \left( \frac{y^2}{\sigma_y^2} + \frac{(z-H)^2}{\sigma_z^2} \right) \right]$$

$$+ \frac{Q}{2\pi u_x \sigma_y \sigma_z} \cdot \exp \left[ -\frac{1}{2} \left( \frac{y^2}{\sigma_y^2} + \frac{(z+H)^2}{\sigma_z^2} \right) \right]$$

For the plume with reflection (general solution):

$$\rho_{A1}(x, y, z, H) = \frac{Q}{2\pi u_x \sigma_y \sigma_z} \left[ \exp \left( -\frac{y^2}{2\sigma_y^2} \right) \cdot \left( \exp \left( -\frac{(z-H)^2}{2\sigma_z^2} \right) + \exp \left( -\frac{(z+H)^2}{2\sigma_z^2} \right) \right) \right]$$



I have 10 sources. I can add them at a particular location. I can just simply add contributions from all of the sources. So this is unreflected plume, this is one just from the top when you do plume with reflection you add one more term here. This  $z$  component, where one more term here in the equation.

You are essentially adding two things

$$\rho_{A1}(x, y, z) = \frac{Q}{2\pi u_x \sigma_y \sigma_z} \cdot \exp \left[ -\frac{1}{2} \left( \frac{y^2}{\sigma_y^2} + \frac{(z-H)^2}{\sigma_z^2} \right) \right] + \frac{Q}{2\pi u_x \sigma_y \sigma_z} \cdot \exp \left[ -\frac{1}{2} \left( \frac{(z+H)^2}{\sigma_z^2} \right) \right]$$

you are adding this another source. This is how you do source addition. If you have multiple sources, you just add keep on adding this equations, here when you add this two equations, it becomes this. So the second source adds up here is general equation with reflection.

## Dispersion Equation – Different Forms

## General Equation – Plume with Reflection for Stack Height H

$$\rho_{pl}(x, y, z; H) = \frac{Q}{2\pi u_x \sigma_y \sigma_z} \left[ \exp\left(-\frac{y^2}{2\sigma_y^2}\right) \cdot \left\{ \exp\left(-\frac{(z-H)^2}{2\sigma_z^2}\right) + \exp\left(-\frac{(z+H)^2}{2\sigma_z^2}\right) \right\} \right] \quad \text{--- (1)}$$


Ground Level Concentration – Stack at Height H  $\rightarrow z=0$ 

$$\rho_{pl}(x, y, 0; H) = \frac{Q}{\pi u_x \sigma_y \sigma_z} \left[ \exp\left(-\frac{y^2}{2\sigma_y^2}\right) \cdot \exp\left(-\frac{H^2}{2\sigma_z^2}\right) \right]$$

## Ground Level Center Line Concentration – Stack at Height H

$$\rho_{pl}(x, 0, 0; H) = \frac{Q}{\pi u_x \sigma_y \sigma_z} \left[ \exp\left(-\frac{H^2}{2\sigma_z^2}\right) \right]$$

## Ground Level Center Line – Ground Point Source

$$\rho_{pl}(x, 0, 0; 0) = \frac{Q}{\pi u_x \sigma_y \sigma_z}$$




So these are different forms,

- General Equation – Plume with Reflection for Stack Height H:

$$\rho_{pl}(x, y, z; H) = \frac{Q}{2\pi u_x \sigma_y \sigma_z} \left[ \exp\left(-\frac{y^2}{2\sigma_y^2}\right) \cdot \left\{ \exp\left(-\frac{(z-H)^2}{2\sigma_z^2}\right) + \exp\left(-\frac{(z+H)^2}{2\sigma_z^2}\right) \right\} \right]$$

this equation here is general equation for any dispersion problem including reflection and one should take reflection because it is there and so there are different adjustments and simplification of this big equation for different scenarios.

- Ground level concentration stack at height H:

$$\rho_{pl}(x, y, 0; H) = \frac{Q}{\pi u_x \sigma_y \sigma_z} \left[ \exp\left(-\frac{y^2}{2\sigma_y^2}\right) \cdot \exp\left(-\frac{H^2}{2\sigma_z^2}\right) \right]$$

which means that I am going to calculate the concentration at height 0 at ground level which means that I am trying to calculate, what is the concentration at a certain x, at a certain y and at ground level. This z equals to 0. Usually this is what we are interested in, we are interested in ground level concentration. Because people are on the ground, we are interested in computing ambient concentrations, we will do z equals 0 typically.

Which means that, this equation now will reduce to this, simplify to this particular form. Now we also say ground level centre line, what is centre line mean ground level means z equals 0 central line means y also equals to 0, stack at height H,

$$\rho_{pl}(x, 0, 0; H) = \frac{Q}{\pi u_x \sigma_y \sigma_z} \left[ \exp\left(-\frac{H^2}{2\sigma_z^2}\right) \right]$$

then ground level centreline and ground sources which means, y is 0, z is 0 and H is also 0.

$$\rho_{A1}(x,0,0;0) = \frac{Q}{\pi \mu_x \sigma_y \sigma_z}$$

Something like whatever you are burning something on the ground, you are burning wood or some such thing on ground. The equation becomes very simple. This becomes straight, but it is derived from this first equation, so it is consistent with that.