Chapter 6

INTERNAL FORCES IN STRUCTURES

1 This chapter will start as a review of shear and moment diagrams which you have studied in both *Statics* and *Strength of Materials*, and will proceed with the analysis of statically determinate frames, arches and grids.

2 By the end of this lecture, you should be able to draw the shear, moment and torsion (when applicable) diagrams for each member of a structure.

3 Those diagrams will subsequently be used for member design. For instance, for flexural design, we will consider the section subjected to the highest moment, and make sure that the internal moment is equal and opposite to the external one. For the ASD method, the basic beam equation (derived in Strength of Materials) $\sigma = \frac{MC}{I}$, (where M would be the design moment obtained from the moment diagram) would have to be satisfied.

4 Some of the examples first analyzed in chapter 3 (Reactions), will be revisited here. Later on, we will determine the deflections of those same problems.

6.1 Design Sign Conventions

5 Before we (re)derive the Shear-Moment relations, let us *arbitrarily* define a sign convention.

6 The sign convention adopted here, is the one commonly used for design purposes[1].

7 With reference to Fig. 6.1

2D:

Load Positive along the beam's local y axis (assuming a right hand side convention), that is positive upward.

Axial: tension positive.

Flexure A positive moment is one which causes tension in the lower fibers, and compression in the upper ones. Alternatively, moments are drawn on the compression side (useful to keep in mind for frames).

[1] Later on, in more advanced analysis courses we will use a different one.

6.2 Load, Shear, Moment Relations

8 Let us (re)derive the basic relations between load, shear and moment. Considering an infinitesimal length dx of a beam subjected to a positive load[2] $w(x)$, Fig. 6.3. The infinitesimal

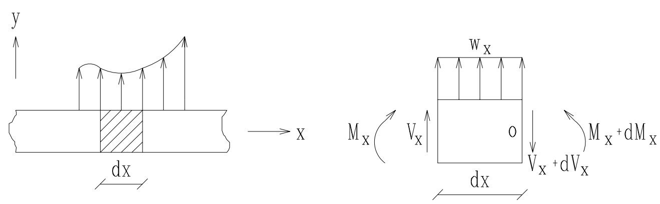


Figure 6.3: Free Body Diagram of an Infinitesimal Beam Segment

section must also be in equilibrium.

9 There are no axial forces, thus we only have two equations of equilibrium to satisfy $\Sigma F_y = 0$ and $\Sigma M_z = 0$.

10 Since dx is infinitesimally small, the small variation in load along it can be neglected, therefore we assume $w(x)$ to be constant along dx.

11 To denote that a small change in shear and moment occurs over the length dx of the element, we add the differential quantities dV_x and dM_x to V_x and M_x on the right face.

12 Next considering the first equation of equilibrium

$$(+\uparrow)\,\Sigma F_y = 0 \Rightarrow V_x + w_x dx - (V_x + dV_x) = 0$$

or

$$\boxed{\frac{dV}{dx} = w(x)} \tag{6.1}$$

> The slope of the shear curve at any point along the axis of a member is given by the load curve at that point.

13 Similarly

$$(+\circlearrowleft)\,\Sigma M_O = 0 \Rightarrow M_x + V_x dx - w_x dx\frac{dx}{2} - (M_x + dM_x) = 0$$

Neglecting the dx^2 term, this simplifies to

$$\boxed{\frac{dM}{dx} = V(x)} \tag{6.2}$$

> The slope of the moment curve at any point along the axis of a member is given by the shear at that point.

[2] In this derivation, as in all other ones we should assume all quantities to be positive.

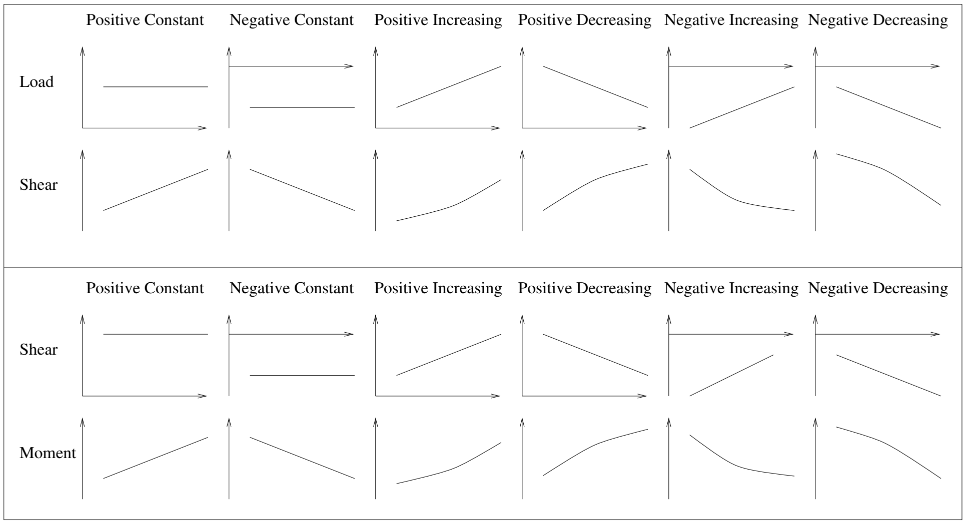


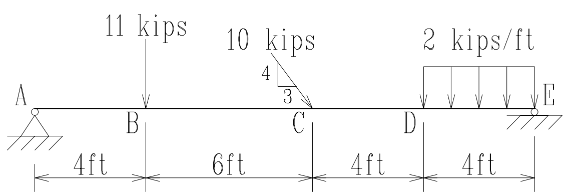
Figure 6.5: Slope Relations Between Load Intensity and Shear, or Between Shear and Moment

6.4 Examples

6.4.1 Beams

■ **Example 6-1: Simple Shear and Moment Diagram**

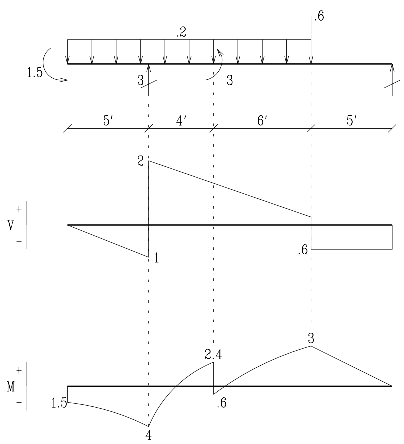
Draw the shear and moment diagram for the beam shown below

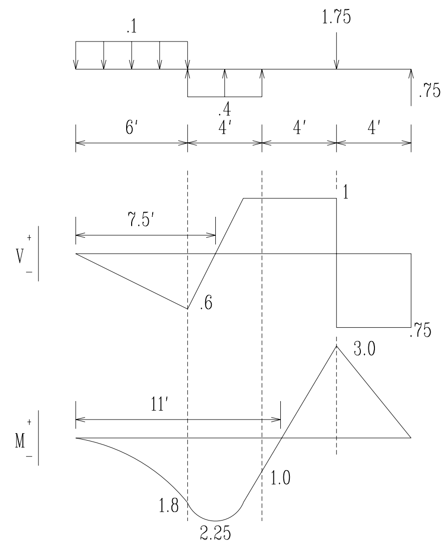


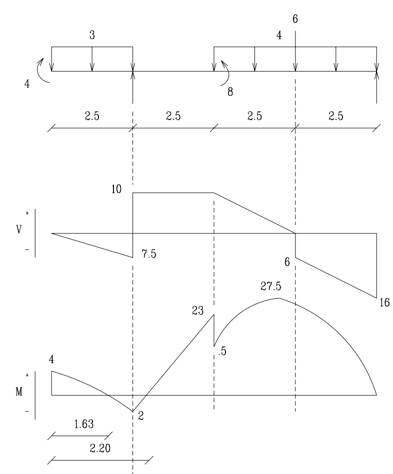
Solution:
The free body diagram is drawn below

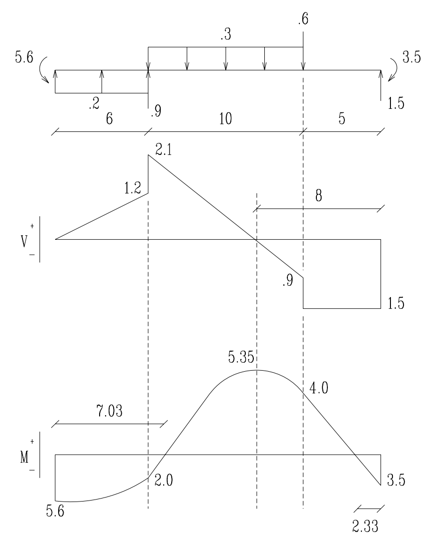
■ Example 6-2: Sketches of Shear and Moment Diagrams

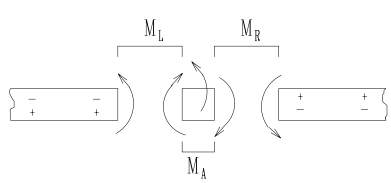
For each of the following examples, sketch the shear and moment diagrams.

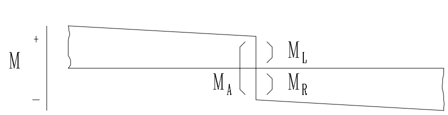






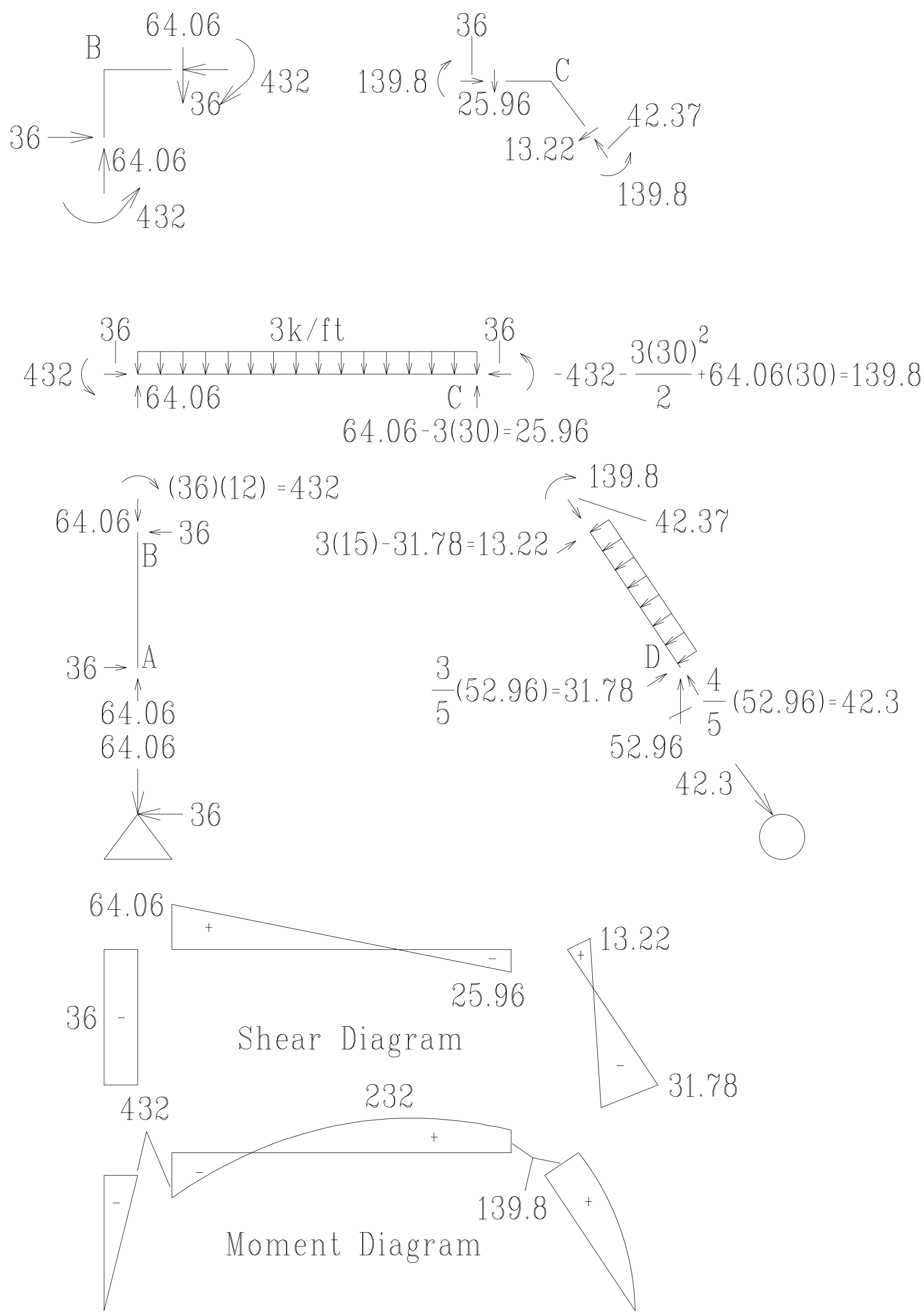






■

Solution:



B
64.06
432
36
36
64.06
432
36
139.8
25.96
C
42.37
13.22
139.8
36
3k/ft
36
432
64.06
C
-432-3(30)^2/2+64.06(30)=139.8
64.06-3(30)=25.96
(36)(12) =432
64.06
36
B
36
A
64.06
64.06
36
139.8
42.37
3(15)-31.78=13.22
D
3/5(52.96)=31.78
4/5(52.96)=42.3
52.96
42.3
64.06
25.96
36
Shear Diagram
13.22
31.78
432
232
139.8
Moment Diagram

B-C

$$
\begin{array}{lllll}
\Sigma F_{x'} = 0 & \Rightarrow & N_{x'}^{B} & = V_{z'}^{C} & & = -60\text{kN} \\
\Sigma F_{y'} = 0 & \Rightarrow & V_{y'}^{B} & = V_{y'}^{C} & & = +40\text{kN} \\
\Sigma M_{y'} = 0 & \Rightarrow & M_{y'}^{B} & = M_{y'}^{C} & & = -120\text{kN.m} \\
\Sigma M_{z'} = 0 & \Rightarrow & M_{z'}^{B} & = {V'}_{y}^{C}(4) & = (40)(4) & = +160\text{kN.m} \\
\Sigma T_{x'} = 0 & \Rightarrow & T_{x'}^{B} & = -M_{z'}^{C} & & = -40\text{kN.m}
\end{array}
$$

A-B

$$
\begin{array}{llllll}
\Sigma F_{x'} = 0 & \Rightarrow & N_{x'}^{A} & = V_{y'}^{B} & = +40\text{kN} \\
\Sigma F_{y'} = 0 & \Rightarrow & V_{y'}^{A} & = N_{x'}^{B} & = +60\text{kN} \\
\Sigma M_{y'} = 0 & \Rightarrow & M_{y'}^{A} & = T_{x'}^{B} & = +40\text{kN.m} \\
\Sigma M_{z'} = 0 & \Rightarrow & M_{z'}^{A} & = M_{z'}^{B} + N_{x'}^{B}(4) = 160 + (60)(4) & = +400\text{kN.m} \\
\Sigma T_{x'} = 0 & \Rightarrow & T_{x'}^{A} & = M_{y'}^{B} & = -120\text{kN.m}
\end{array}
$$

The interaction between axial forces N and shear V as well as between moments M and torsion T is clearly highlighted by this example.