

Hydraulic Engineering
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Lecture - 43
Pipe Networks

Welcome back student to this new week, where we are still continuing the pipe flow.

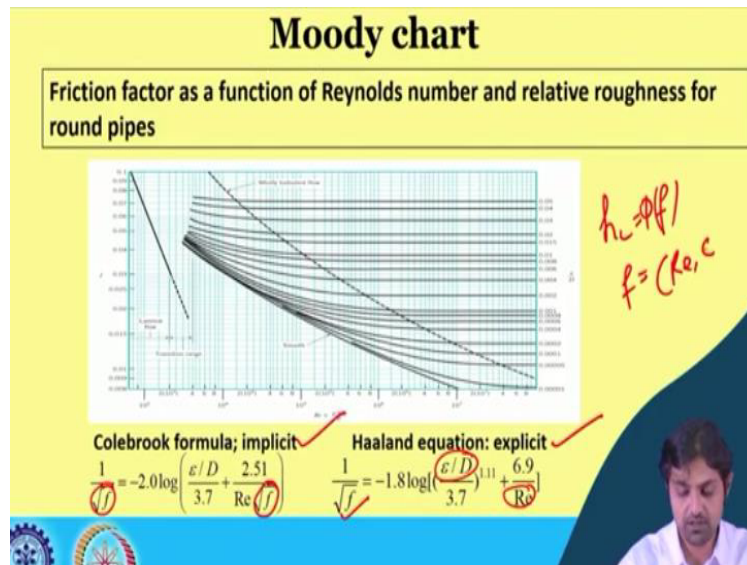
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Equivalent roughness for pipes		
Equivalent Roughness for New Pipes [From Moody (Ref. 7) and Colebrook (Ref. 8)]		
Pipe	Equivalent Roughness, ϵ	
	Feet	Millimeters
Riveted steel	0.003–0.03	0.9–9.0
Concrete	0.001–0.01	0.3–3.0
Wood stave	0.0006–0.003	0.18–0.9
Cast iron	0.00085	0.26
Galvanized iron	0.0005	0.15
Commercial steel or wrought iron	0.00015	0.045
Drawn tubing	0.000005	0.0015
Plastic, glass	0.0 (smooth)	0.0 (smooth)

Last week, we finished the lecture at finding equivalent roughness of pipes and said that we are going to solve one particular problem, but I will think it is better to take that problem after I complete this particular topic. So until this point in time, what are the things that we know? We need to find to an f , f is the Darcy Weisbach friction factor. Darcy's friction factor and that is f is equal to ϕ of function of Reynolds number and epsilon by D .

So Reynolds number we can calculate if the flow is given, D is the pipe diameter, so epsilon, we saw that we can find through these tables. Most of the cases, for your significance in practice for the numerical, you will be given the value of epsilon by D . So if you are able to calculate the value of epsilon by D and Reynolds number, then there is a dependence of these two parameters, Re and D on the friction factor f and if we know the friction factor, we can calculate by using these values and find out the head loss, the major head loss.

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So do that there is something called a Moody chart. So friction factor is a function of Reynolds number and relative roughness for round pipes is a chart like this, okay, where this is Reynolds number on x-axis and f you can find out and ϵ/D is plotted in the right. So this is the oldest method of finding these friction factors, Darcy friction factor. However, for this course, of course, if you are given this Moody chart, you should be able to find a corresponding Reynolds number and a corresponding ϵ/D line here and then go ahead and find out the respective f , friction factor.

But for your convenience, I am going to provide you two formulas, one is a Colebrook formula, which relates this friction factor f to ϵ/D and Reynolds number. Can you see there is one trick in this formula? If you note, you will see f is also in the left hand side and f is also on the right hand side, that makes it implicit in nature, alright. But I still expect you to remember this formula. The solution for this formula can be done through trial and error, alright.

Or you can use another formula, which is totally explicit in nature, which is called Haaland equation. So here you see, there is friction factor unknown is only on the left hand side, so if you know ϵ/D and you know Reynolds number, you will be able to find out the value of f . So this equation you must remember and also this, because some of the questions could be based on Colebrook formula as well, alright.

With this thing in mind, we can solve the problems now. Head loss was a function of friction factor, right? I mean, it was dependent on friction factor and f was a function of Reynolds number and epsilon by D , so we can find f using these two formulas or Moody chart and therefore, we will be easily able to calculate the head loss.

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Class Problem

- A badly corroded concrete pipe of diameter 1.5 m has an equivalent sand roughness of $\epsilon_s = 15\text{ mm}$. A 10 mm thick lining is proposed to reduce the roughness value to $\epsilon_s = 0.2\text{ mm}$. For a discharge of $4.0\text{ m}^3/\text{s}$ in the pipe calculate the power saved per kilometer of pipe. Take $\nu = 1 \times 10^{-6}\text{ m}^2/\text{s}$

$P_s = 162.5\text{ kW}$

To demonstrate that, we have a problem question here, that we are going to solve now. So the question is this, a badly corroded concrete pipe of diameter 1.5 m has an equivalent sand roughness of epsilon S 15 mm. So we have already been given epsilon S. At 10 mm thick lining is proposed to reduce the roughness value to epsilon S of 0.2 mm. So a thick lining would be put, which is 10 mm long and this will bring down the roughness to 0.2 mm.

The question is, for a discharge of 4 meter cube per second, calculate the power saved per kilometer of the pipe. If you see, because of this epsilon, there is going to be a head loss. So head loss is related with energy loss. If you reduce that energy loss, you will save some power. So that is the idea of this particular question. So like always we are going to start the solution on the white screen.

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Soln 7

Before the lining:

$$V_1 = \frac{Q}{A} = \frac{4.0}{\frac{\pi (1.5)^2}{4}} = 2.264 \text{ m/s}$$

Reynold's number

$$Re_1 = \frac{V_1 D_1}{\nu} = \frac{2.264 \times 1.5}{1 \times 10^{-6}} = 3.395 \times 10^6$$

$$\frac{C_{f1}}{D_1} = \frac{15 \times 10^{-3}}{1.5} = 10^{-2}$$

if we use the empirical equation of Haaland (Explicit formula)

$$\frac{1}{\sqrt{f_1}} = \dots$$

$$\Rightarrow f_1 = 0.0379$$

After the lining was put

$$D_2 = 1.50 - 2 \times 0.01 = 1.48 \text{ m}$$

$$V_2 = \frac{Q}{A} = \frac{4}{\frac{\pi (1.48)^2}{4}} = 2.325 \text{ m/s}$$

Reynold's number

$$Re_2 = \frac{V_2 D_2}{\nu} = \frac{2.325 \times 1.48}{1 \times 10^{-6}} = 3.44 \times 10^6$$

$$\frac{C_{f2}}{D} = \frac{0.2 \times 10^{-3}}{1.48} = 1.35 \times 10^{-4}$$

$$f_2 = 0.0132 \text{ (Haaland formula)}$$

So we see, before the measure of reducing the head loss, before the lining was put, V_1 is going to be Q/A , so Q was 4 meter cube per second, area is π by 4 into 1.5 whole square. So this is going to be 2.264 meter per second, alright. For this particular case, Reynold's number is going to be, we call it Re_1 as $V_1 D_1$ by ν . So V_1 we have already found 2.264, diameter we know, it was 1.5 meter and ν is 1 into 10 to the power -6.

So Reynolds number comes out to be 3.395 into 10 to the power 6. Similarly, ϵS_1 by D_1 , the value we have already been given in the question is 15 into 10 to the power -3 meter divided by 1.5. So it comes to 10 to the power -2 alright. So if the empirical equation of Haaland, okay, then we can find under root 1 by f_1 is equal to the things on the right hand side and on calculating, we can get because this is an explicit formula, alright.

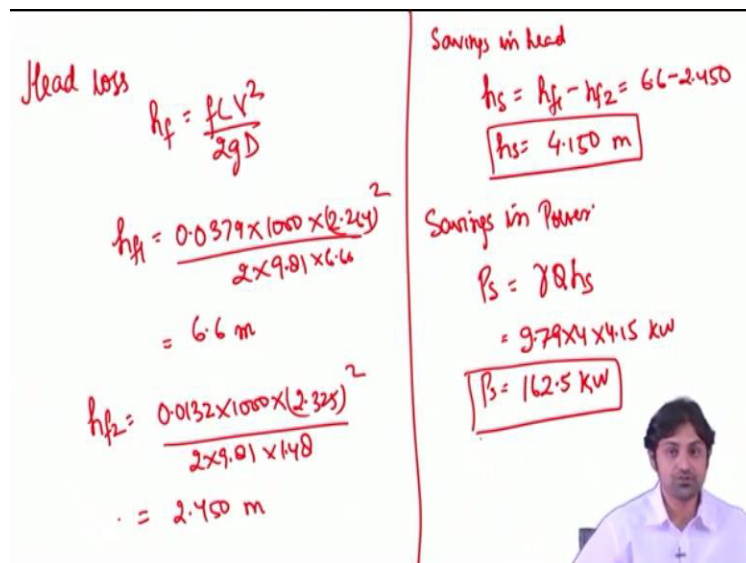
So f_1 we can get 0.0379. Now we have talked about the friction factor we have calculated before the lining was there. Similarly, let us talk after the lining was put. So after the lining is put, diameter will be reduced, how much? 1.50 was the diameter and lining of this, so the diameter becomes 1.48 meters. Therefore, the velocity will be $4Q$ by A π by 4 into 1.48 to the whole square, that means 2.325 meters per second, alright.

Corresponding Reynolds number Re_2 as we call it is $V_2 D_2$ by ν . So 2.325 into 1.48 divided by 1 into 10 to the power -6. So it comes out to be 3.44 into 10 to the power 6. Similarly, friction

epsilon S2 by D, we said it has been reduced to 0.2, so it has become 0.2 into 10 to the power -3 meter divided by 1.48, so this value come out to be 1.35 into 10 to the power -4, alright. So similarly using the Haaland formula, you can calculate that f will come out to be 0.0132.

So now we have calculated f1 and f2, f1 is the friction factor before the lining was put and f2 is the friction factor after the lining was put. So to continue further, alright.

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The image shows handwritten calculations on a whiteboard, divided into two columns. The left column is titled 'Head loss' and the right column is titled 'Savings in head' and 'Savings in Power'.

Head loss:

$$h_f = \frac{fLV^2}{2gD}$$

$$h_{f1} = \frac{0.0379 \times 1000 \times (2.264)^2}{2 \times 9.81 \times 6.6} = 6.6 \text{ m}$$

$$h_{f2} = \frac{0.0132 \times 1000 \times (2.325)^2}{2 \times 9.81 \times 1.48} = 2.450 \text{ m}$$

Savings in head:

$$h_s = h_{f1} - h_{f2} = 6.6 - 2.450$$

$$h_s = 4.150 \text{ m}$$

Savings in Power:

$$P_s = \gamma Q h_s$$

$$= 9.79 \times 4 \times 4.15 \text{ kW}$$

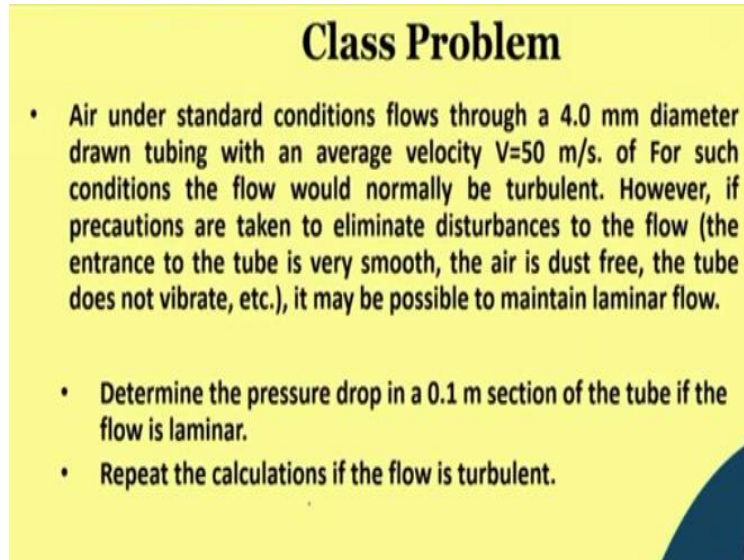
$$P_s = 162.5 \text{ kW}$$

We are going to calculate head loss h_f is equal to fLV square by $2gD$. So before the lining was put, h_{f1} would be 0.0379 that was the friction factor 1, length is 1000 and velocity was 2.264 whole square divided by 2 into 9.81 into 6.60. So h_{f1} comes out to be 6.6 meter. Similarly, we will calculate h_{f2} , same formula, f here now the new f was 0.0132 into 1000 into 2.325, the velocity had changed divided by 2 into 9.81 into 1.48 and this comes out to be 2.450 meter head loss.

So savings in head, you see the frictional loss before the lining was 6.6 meter and after the lining is put as 2.450. So saving in head, h_s is $h_{f1} - h_{f2}$ and this will come out to be 6.6 – 2.450, so h_s is 4.150 meter. This is saving in the head. Now for power, savings in power would be nothing but $\gamma Q h_s$, right. So γ is 9.79 into 4 into 4.15 kilowatt, because we have taken this 9.79 into 10 to the power 979 into g.

So 9790 instead we did 9.79 therefore we are telling in kilowatt and this will come to be 162.5 kilowatt. So this is the saving in power. So you see how we were able to find the major loss and the savings in major loss, if we put the lining, which will reduce the friction factor in a pipe. This was a quite illustrative example, alright. So just writing it down the final result here that the power saving is 162.5 kilowatt, alright.

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A yellow rectangular slide with the title "Class Problem" in bold black text at the top center. Below the title, there are three bullet points in black text. The first bullet point describes a flow scenario of air through a tube, mentioning standard conditions, a 4.0 mm diameter, an average velocity of 50 m/s, and the possibility of maintaining laminar flow by eliminating disturbances. The second bullet point asks to determine the pressure drop in a 0.1 m section of the tube if the flow is laminar. The third bullet point asks to repeat the calculations if the flow is turbulent. The slide has a blue curved shape in the bottom right corner.

Class Problem

- Air under standard conditions flows through a 4.0 mm diameter drawn tubing with an average velocity $V=50$ m/s. For such conditions the flow would normally be turbulent. However, if precautions are taken to eliminate disturbances to the flow (the entrance to the tube is very smooth, the air is dust free, the tube does not vibrate, etc.), it may be possible to maintain laminar flow.
- Determine the pressure drop in a 0.1 m section of the tube if the flow is laminar.
- Repeat the calculations if the flow is turbulent.

Moving ahead, there is another question that says air under standard condition flows through a 4 mm diameter drawn tubing with an average velocity of 50 meter per second. For such conditions, the flow would normally be turbulent, however the precautions are taken to eliminate disturbances to the flow. It may be possible to maintain laminar flow. So the first part is determine the pressure drop in 0.1 meter section of the tube, if the flow is laminar.

And the second part is repeat the calculations if the flow is turbulent, alright. So we will solve this question again on the white screen.

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Solution 8:
 Under standard temperature and pressure conditions $\rho = 1.23 \text{ kg/m}^3$ & $\mu = 1.79 \times 10^{-5} \text{ N/m}^2$

$$Re = \frac{\rho V D}{\mu} = \frac{1.23 \times 50 \times 0.004}{1.79 \times 10^{-5}} = 13743 \text{ which would normally indicate turbulent flow.}$$

Unlabeled flow:

a) If the flow were laminar $f = \frac{64}{Re} = \frac{64}{13743} = 0.00467$

drop in 0.1 m long horizontal section of the pipe would be

$$\Delta p = f \times \frac{L}{D} \times \frac{\rho V^2}{2}$$

$$\Delta p = 0.00467 \times \frac{0.01}{0.004} \times \frac{1}{2} \times 1.23 \times 50^2 = 179 \text{ N/m}^2$$

So solution number 8, so under standard temperature and pressure conditions, rho is 1.23 kilogram per meter cube and mu is 1.79 into 10 to the power -5 Newton second per meter square, because we are talking about air, alright. Therefore, the Reynolds number will be rho VD by mu. So if you put 1.23 into 50, because velocity of 50 meters per second is given, diameter is 4 mm, so 0.004 meter divided by 1.79 into 10 to the power -5 and this will come to be almost 13,743, which would normally indicate turbulent flow, alright.

However, since we have been told, assume this as a laminar flow, we say, if the flow were laminar, friction factor will be given by 64/Re. Therefore, in this particular case, although it is turbulent, but we still assume that it is laminar, so we will find the friction factor, which will come almost to 0.00467. Therefore, drop in 0.1 meter long horizontal section of the pipe would be delta p is equal to f into L by D, our formula into rho V square by 2, alright.

So delta p is going to be f 0.00467, length is 0.01, diameter you already know 0.004 into half into rho is 1.23 and V square is 50 and this is going to be 179 Newton per meter square, alright. So we can calculate everything here. So 179 Newton per meter square, alright. So we could actually have reduced directly the formula for delta p.

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b) If the flow were turbulent $f = \phi\left(Re, \frac{\epsilon}{D}\right)$ from table $\epsilon = 0.0015 \text{ mm}$ (drawn tubing)

$$\frac{\epsilon}{D} = \frac{0.0015}{4} = 0.000375, Re = 1.37 \times 10^4$$

if we use either Moody chart or Haaland equation corresponding to $Re = 1.37 \times 10^4$ and $\frac{\epsilon}{D} = 0.000375 \Rightarrow f = 0.028$

$$\Delta p = f \times \frac{L}{D} \times \frac{\rho V^2}{2}$$

$$= 0.028 \times \frac{0.1}{0.004} \times \frac{1}{2} \times 1.23 \times 50^2$$

$$\Delta p = 1.076 \text{ KPa}$$

But now before we go to the next part, it says if the flow were turbulent, then f is a function of Reynolds number to epsilon by D . So if we look at the, I told you if we know what type of pipe it is, we can look down the epsilon value. So of course, I have not given it here written in the question, but if you look at the table, from table, epsilon will come to be 0.0015 mm, okay, because it is a drawn tubing, alright. So epsilon is 0.0015 for drawn tubing, okay.

So we calculate epsilon by D , which is 0.0015 by diameter is 4 mm, so we get 0.000375, alright and Reynolds number came out to be earlier 1.37 into 10 to the power 4, alright. So if use either Moody chart or Haaland equation corresponding to Reynolds number is equal to 1.37 into 10 to the power 4 and epsilon by D of 0.000375, we will get f is 0.028, alright. Therefore, the pressure drop will be f into L by D into half rho V square.

And after putting 0.028 into length will still be the same, diameter will still be the same, rho is 1.23 into 50 square, so delta p is going to be 0.076 kilopascal. So the pressure drop as you see is much, much more when the flow is turbulent, alright. In the first case, it was only 0.179 kilopascal, alright.

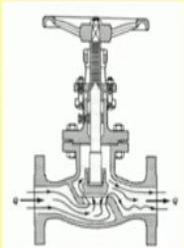
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Minor losses

It is due to the change of the velocity of the flowing fluid in the magnitude or in direction [turbulence within bulk flow as it moves through and fitting]

The minor losses occurs due to

- Valves ✓
- Tees ✓
- Bends ✓
- Reducers ✓
- And other appurtenances ✓



Flow pattern through a valve

swayam

So we talk about the minor losses. We have talked much about the major losses, but now we must put our focus a little bit on the minor losses. So minor losses is due to the change of the velocity of the flowing fluid either in the magnitude or in the direction, okay. If the magnitude or the direction, any of them would change or both will change, the loss is associated with them is called minor loss in pipes. So this is a typical flow pattern through a valve, that is shown here.

It will comprise of both major and minor losses. The minor losses occur due to, if there are walls present, if there are Ts, if there are bends in the pipes, if there are reduces and there are other appurtenances, alright.

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Minor losses

• It has the common form

$$h_m = k \frac{V^2}{2g} = k_L \frac{Q^2}{2gA^2}$$

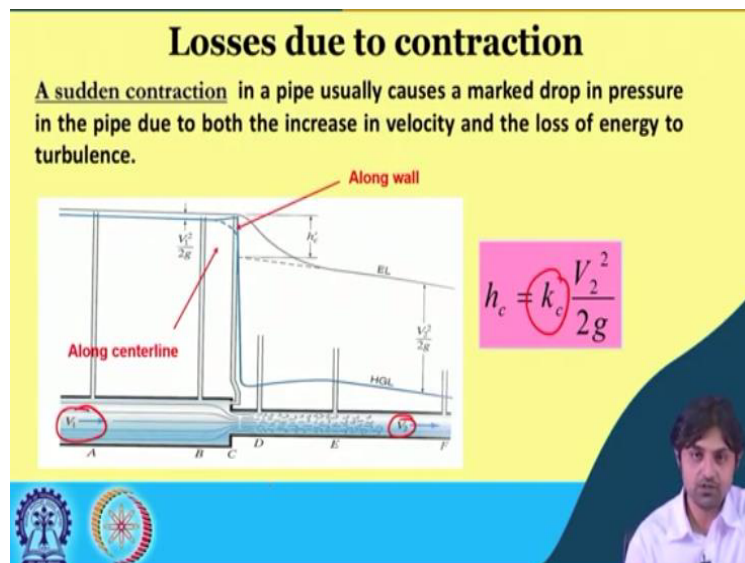
$h_f = f \times \frac{L}{D} \times \frac{V^2}{2g}$

“minor” compared to friction losses in long pipelines but, can be the dominant cause of head loss in shorter pipelines

So minor losses also have a common form of this. So it is k_L multiplied by V square by $2g$, okay and this k_L is the minor loss coefficient. In terms of Q , we can also write Q square by $2ga$ square. So this is minor compared to friction losses in long pipes, but can be dominant cause of head loss in shorter pipes. So if the pipe is long, because the major loss is given by f into L by D , remember $2V$ square by $2g$, alright. So if the pipe is long, the major losses would be too much.

But minor losses, this is called minor because the frictional loss is due to these phenomenon like bending, contraction and expansion are less in case of long pipes. Therefore, it is called minor losses, but it is not always minor. If the pipes are of shorter length, the term minor losses actually will be dominant.

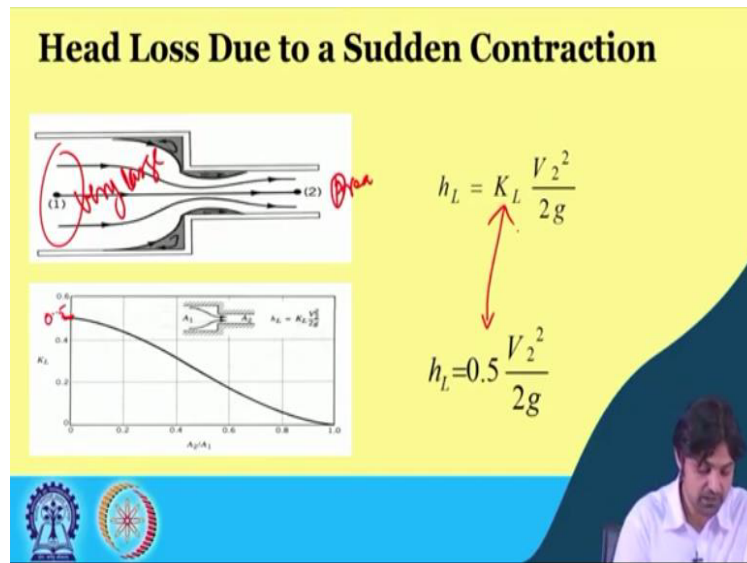
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Now, let us see the loss due to contraction. A sudden contraction in a pipe usually causes a marked drop in pressure in the pipe due to both increase in the velocity and loss of energy due to turbulence, that is a well established fact, correct. So you see there is a fluid that is coming with velocity V and after the sudden contraction, the velocity changes to V_2 . So in this case, for a contraction, the minor head loss will be k_c that we do not know yet into V_2 square by $2g$, not V_1 .

So V_2 you should be able to find using the continuity equation and k_c either you can derive it or but most of the cases in many general scenarios, it is given, okay.

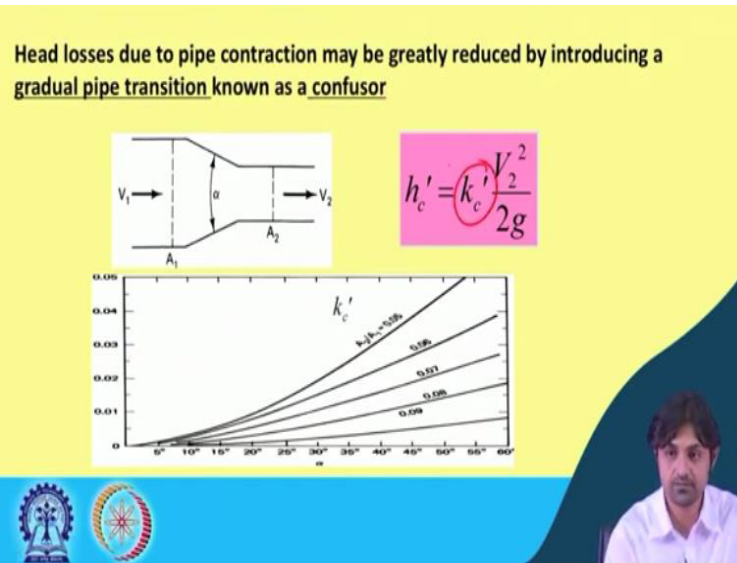
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So if this is the contraction depicted by 0.2, it is a fluid at 0.1 goes a contraction and is at 0.2, we write head loss is K_L into V_2^2 square by $2g$ and what is this K_L , K_L can be found out as a function of A_2 by A_1 . What the area 2 and area 1 is, okay, ratio of the areas, so A_2 by A_1 . So most of the time, you would be given these values directly or a curve like this will appear, but for certain standard conditions, you are expected to know this.

So in case of sudden contraction, okay, you see, you can assume safely K_L as 0.5. This means A_2 was 0, okay. Basically, A_2 is not 0, but A_1 is so large this area is very large, like reservoir compared to this area. In that case, you see according to the curve, this comes at around 0.5. so sudden contraction when it means, it will always be implied that you have to assume K_L is equal to 0.5. Please remember this. In case of a sudden contraction, K_L is 0.5 and V_2^2 square by $2g$ when multiplied with this will give the minor losses.

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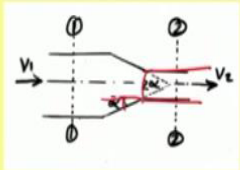


So head losses due to pipe contraction may be greatly introduced, so you see due to sudden contraction, there is going to be head losses. Now our aim as an engineer is to reduce those head losses, correct and that can be done by introducing a gradual pipe transition called as confusor. So confusors are used to introduce a gradual pipe transition, something like this. You see there is an angle α .

In that case also, the head loss is going to be k_c dash, another coefficient k into V^2 square by $2g$ in case of contraction. But what that k_c is, is given by this graph. So this is k_c on this side, for every α , we go and look at what is A_2 by A_1 ratio or A_2 by A_1 is and we pick a point for example and try to find out what that k_c is. In such cases, either you will be given this graph or you will be exactly told what the k_c is, alright.

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Head Loss Due to Gradual Contraction (reducer or nozzle)



$$h_L = K_L \frac{(V_2^2 - V_1^2)}{2g}$$

$A_1 V_1 = A_2 V_2$

α	10°	20°	30°	40°
K_L	0.2	0.28	0.32	0.35

Now, in this particular case, head loss due to gradual contraction, you see, we had one equation and we can also write this in form of another thing where both V_2 and V_1 could be used. So K_L into V_2 square minus V_1 square by $2g$, okay in case of contraction like this, you see. This is 2α and this is α and here also we are going to use for different α , we have different K_L s, in that case, you do not have to worry about area of 1 and area of 2.

Because it will already be taken care by the velocity continuity equation $A_1 V_1$ is equal to $A_2 V_2$, just a different way of writing. So this will simply give you, suppose if you have 15 degree, you can assume like if there is a 15 degree α is there, then you just take the linear interpolation as 0.24 will be the K_L . So one advantage here is we do not have to worry about the ratio of A_2 by A_1 , just looking at the table is pretty simple. I do not expect you to remember this.


But you can be given this type of table in your assignments and exams and could be asked to find out the value of K_L and corresponding head losses. So in that case, it will be V_2 square – V_1 square by $2g$, alright.

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Head Loss Due to Gradual Contraction (reducer or nozzle)

A different set of data is :

Table Loss Coefficients (K) for Gradual Contractions. ✓

	Included Angle, θ , Degrees						
	A_2/A_1	10	15-40	50-60	90	120	150
0.50	0.05	0.05	0.06	0.12	0.18	0.24	0.26
0.25	0.05	0.04	0.07	0.17	0.27	0.35	0.41
0.10	0.05	0.05	0.08	0.19	0.29	0.37	0.43

Note: Coefficients are based on $h_{L_c} = K(V_1^2/2)$. ✓

So a different set of data, see many people have done a lot of experiments and different set of data gives this for different angle, gives ratio of, I mean the value of k as a function of A_2 by A_1 . So for example, first you calculate A_2 by A_1 , if it comes 0.50 and look at the angle. Let us say it was between 50 to 60 degrees, so this k can be assumed as 0.06, alright and head loss can be calculated as k into V_2 square by 2. So whole thing has already been given here. Now I think, I should finish this lecture here with the gradual contraction.

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Losses due to Enlargement

When we meet in the next lecture, we will talk about enlargement. When there is a pipe enlargement, what is going to be the minor loss. So this is all for this lecture and I will see you next. Thank you so much.