Hydraulic Engineering
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Lecture-16
Laminar and turbulent flow (Contd.)

Welcome back to this last lecture of turbulent flow and laminar flows where we are going to talk about turbulent flow in smooth pipes. Last time in the last lecture we had seen what a smooth and rough bed is based on the Reynolds particle Reynolds number Re*. Now we are going to continue over the turbulent flow in smooth pipes. So, if we refer to equation 18.

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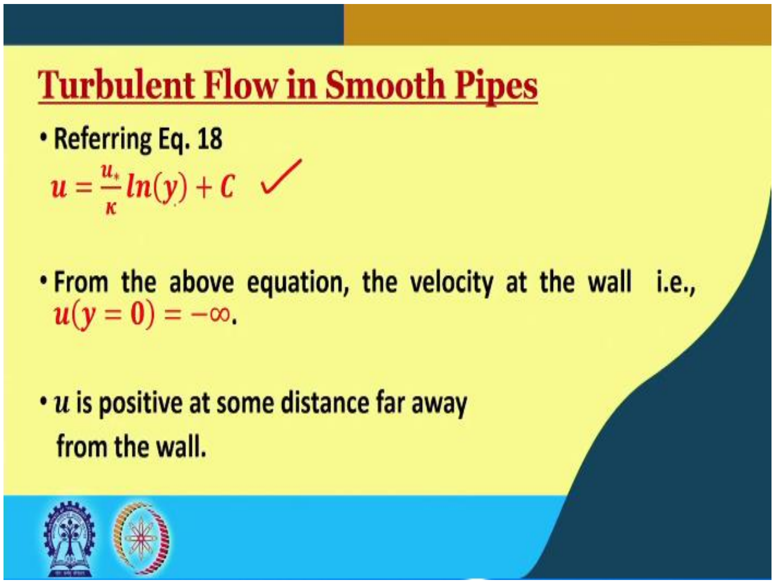

So, what we had found out was

$$u = \frac{u_*}{\kappa} ln(y) + C$$

. The equation 18, in this particular lecture, I mean, in this particular week's lecture. From the above equation, the velocity at the wall u at y is equal to 0 will be minus infinity, correct. If we put y is equal to 0. So, if we put ln 0, it will be minus infinity. u is positive at some distance far away from the wall.

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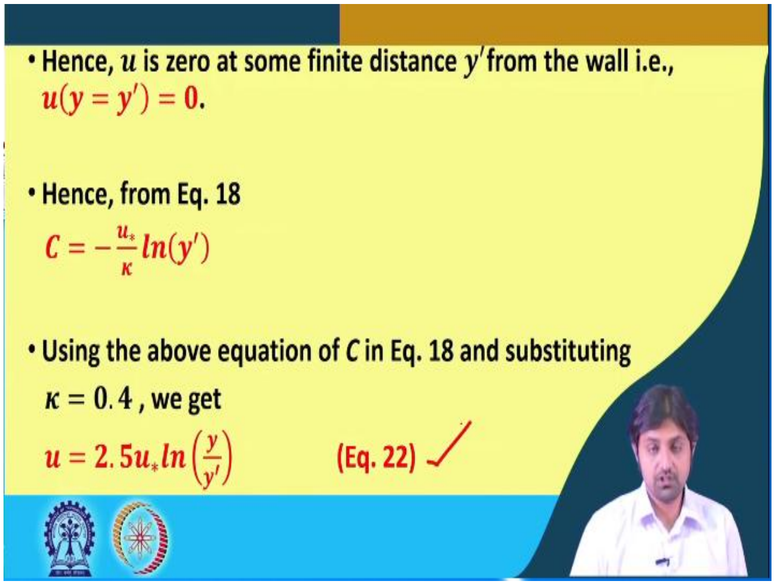

Hence, u is 0 at some finite distance y prime from the wall, that is, y at y prime is equal to 0. Hence, from equation 18, we can say that at distance y prime C will be minus u star by Kappa ln y prime. Because if we put y is equal to 0 it is going to be infinity. So, this logarithmic profile does not fit this. Therefore, we assume that u is 0 at some finite distance y dash y prime from the wall. And if you say the equation was this one, we assume there was u is 0 u star by Kappa ln and we say at this y prime it is 0 plus C. Therefore, C will be minus u star by Kappa 1n y prime.

Therefore, this is what we get, the same thing. Now, if we use this above equation in equation 18 and substitute Kappa is equal to 0.4 we will get

$$u = 2.5u_* ln\left(\frac{y}{y'}\right)$$

, where y dash is the point above the bed where the velocity is 0 and this is equation number 22.

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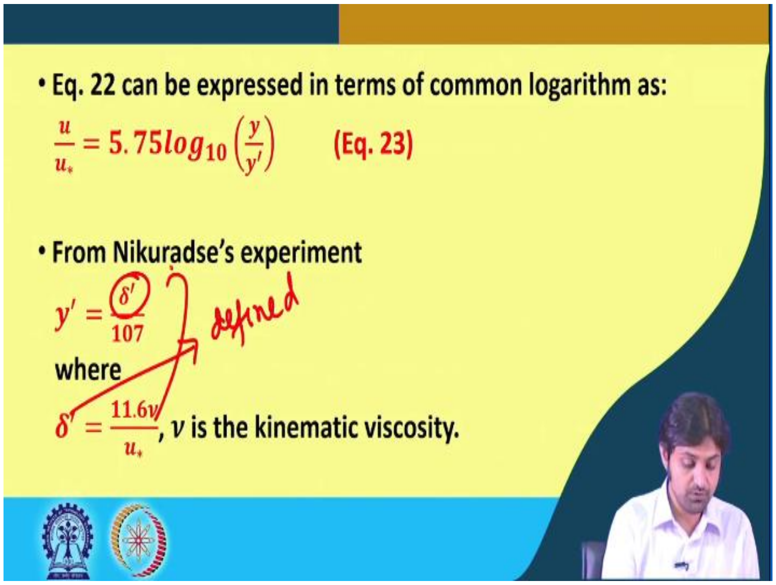

Now, this equation number 22, can be expressed in terms of common logarithm as, so, what we have done? We have done, converted ln into log, because the previous equation was in terms of natural log. We have converted into 10 log base 2 to the power 10 and we get

$$\frac{u}{u_*} = 5.75log_{10}\left(\frac{y}{y'}\right).$$

Now, again from the Nikuradse's experiment, y prime actually has been found out to be delta prime, where this is a thickness of the laminar sublayer viscous sublayer by hundred and seven.

Therefore, delta bar can be written as, because this, I mean, this has been defined, not in this course but we know that delta prime is 11.6 nu by u star.

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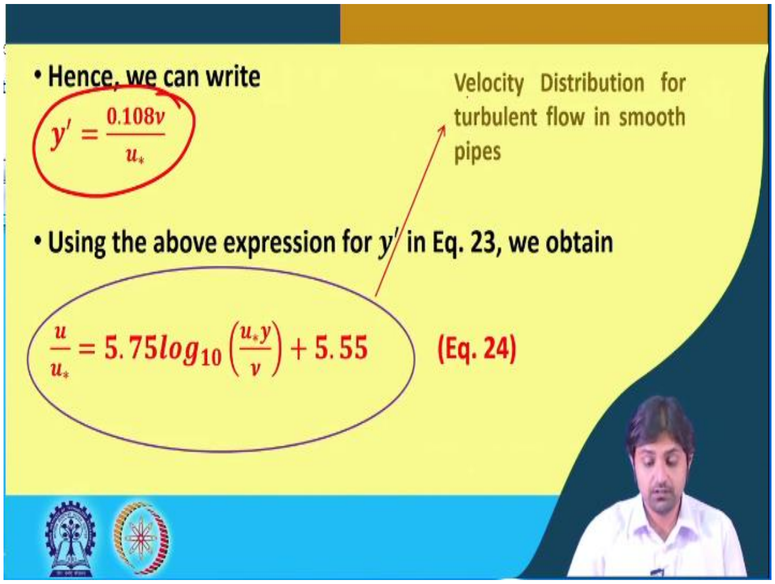

Therefore, we can write, so, if we use this and put it into this equation; we can write

$$y' = \frac{0.108\nu}{u_*}$$

, just simple substitution. And now, we can simply obtain, if we use this in equation number 18 or this particular equation, equation number 23, then we obtain

$$\frac{u}{u_*} = 5.75log_{10}\left(\frac{u_* y}{\nu}\right) + 5.55$$

, very simple. Now, this as you see, the velocity distribution for turbulent flow in smooth pipe. This is the velocity distribution for turbulent flow in a smooth pipe, where there is no irregularity.

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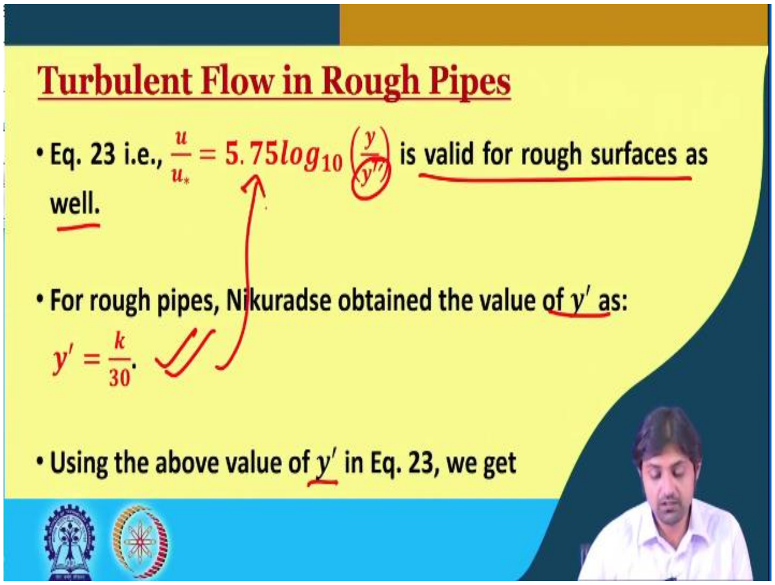

Now, what about the turbulent flow? So, it is better to note down this equation. Now, we have to see equation 23. So, actually this

$$\frac{u}{u_*} = 5.75 log_{10}\left(\frac{y}{y'}\right)$$

is valid for rough surface as well because all the approximation that we did was on this y dash. For rough pipes Nikuradse obtained the value of y prime as k/30. This is obtained by Nikuradse and if you substitute this y prime in equation number 23 this one,

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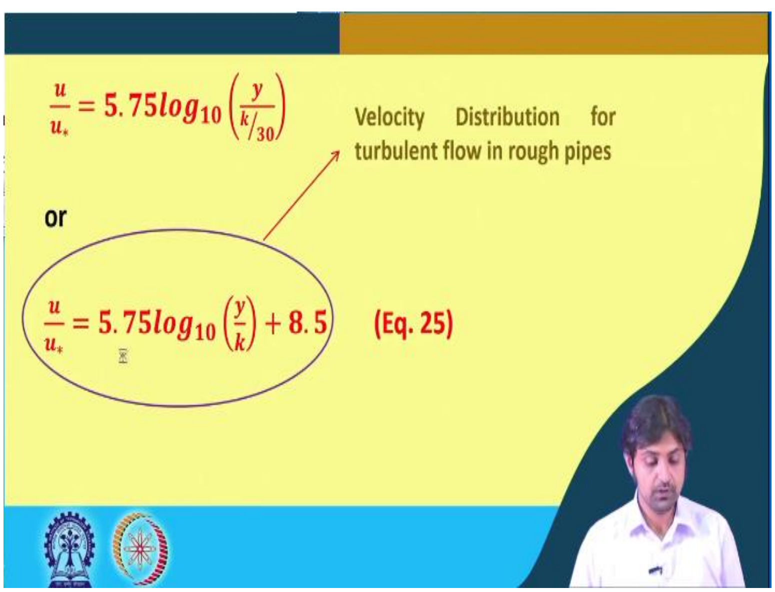

We can obtain,

$$\frac{u}{u_*} = 5.75 log_{10}\left(\frac{y}{k/_{30}}\right)$$

or if you take this 30 out, it is

$$\frac{u}{u_*} = 5.75 log_{10}\left(\frac{y}{k}\right) + 8.5$$

. Now, you see, this is the velocity distribution of turbulent flow in rough pipes. These coefficients are little different this k is different, more importantly, it also has some sort of logarithmic form, both smooth and the rough.

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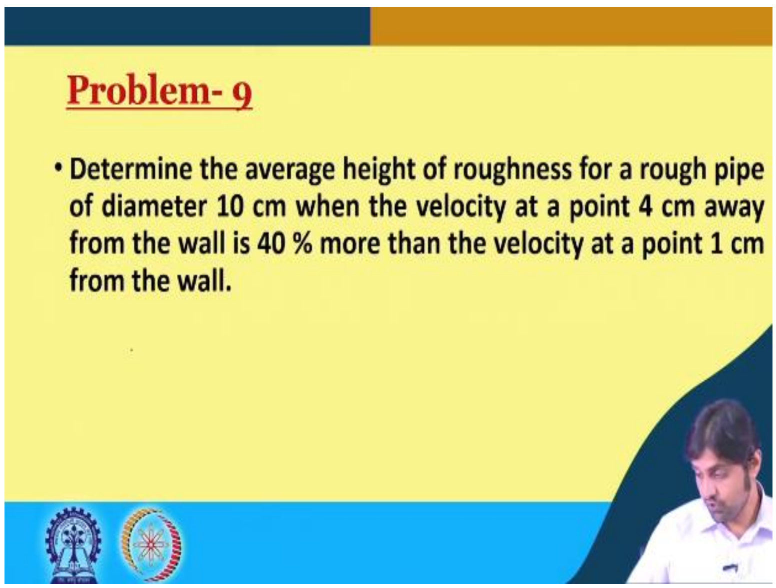

Now, it is a good idea to solve one problem here, on this particular concept, and the solving the problem will give more understanding now. You know, the question is, determine the average height of roughness for a rough pipe of diameter 10 centimeter when the velocity at point 4 centimeter away from the wall is 40 percent more than the velocity at a point 1 centimeter from the wall. So, diameter it says is 10 centimeter, when the velocity at a point 4 centimeter away is 40 percent more than the velocity at a point 1 centimeter away from the wall.

So, what we are going to do? We are going to assume our white screen back again and start solving by writing down what are given.

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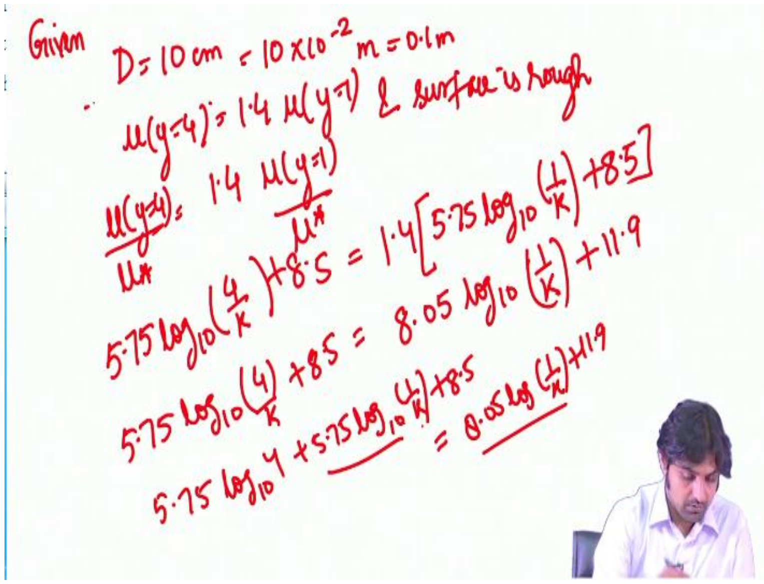

Given is, D is equal to 10 centimeters or 10 into 10 to the power minus 2 meters or 0.1 meter. It is given, u at y is equal to 4 centimeters or 4 is equal to 1.4 times u at y is equal to 1 and the surface is rough. This is what we have already been told. We can therefore, from our equations we can write, u at y is equal to 4 by u star can be written as, 1.4 u at y is equal to 1 divided by u star. So, what we do? We write this, so, we write on the left hand side, 5.75 log base 10, 4 by k, because it is rough, plus 8.5 is equal to 1.4 times 5.75 log base 10 kappa plus 8.5.

This is what is given, I mean, this is what we have know. So, we can write, this will remain as it is, 5.75 log base 10. So, log 4 by k plus 8.5 is equal to this, if you multiply it becomes 8.05 log base 10 one by kappa plus 11.9. So, we will split this log10, log4, I mean, log. So, we can write it, 5.75 log 4 plus 5.75log base 1 by k + 8.5 is equal to we can write 8.05 log 1 by k plus 11.9. And this you see, there is 1 by log 1 by k, here, log 1 by k here, and if you take it to the other side, I am going to use.

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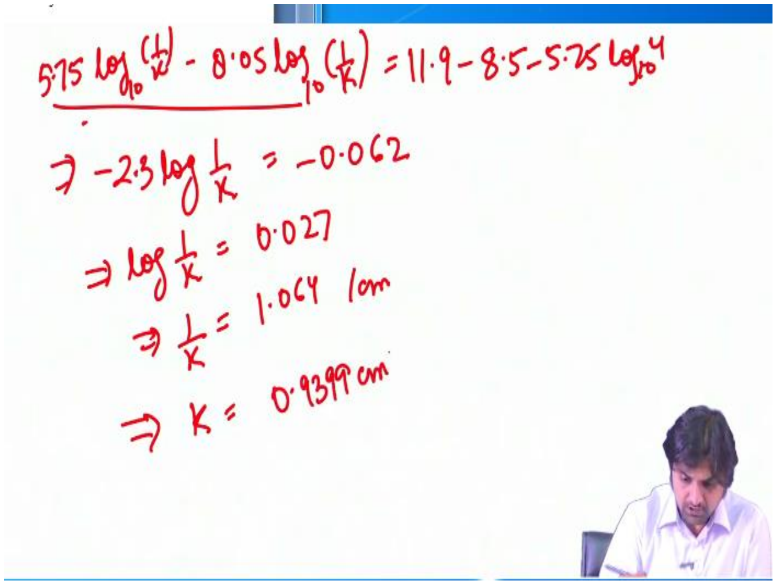

So, we are going to bring it on the other side, 5.75 log 1 by k base 10 minus 8.05 log base 10 1/k is equal to 11.9 minus 8.5 minus 5.75 log base 10 4. So, this will become, the left hand side will become, minus 2.3 log 1 by k is equal to minus 0.062. And therefore, log 1 by k is equal to 0.027, which implies, 1 by k is equal to 1.064 per centimeter, which implies, k is equal to 0.9399 centimeter.

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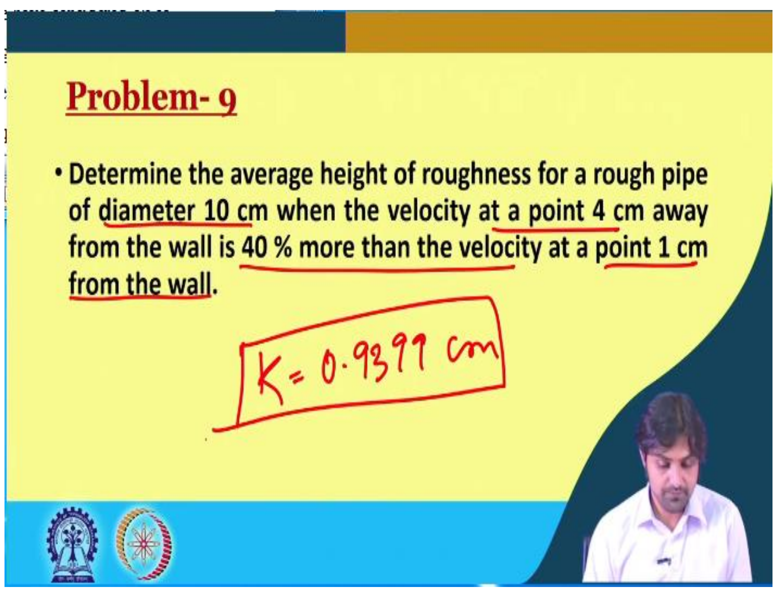

So, this was what was asked. So, k is coming out to be 0.9399 centimeter. This is the answer and how to solve? We have already done that in the sheet.

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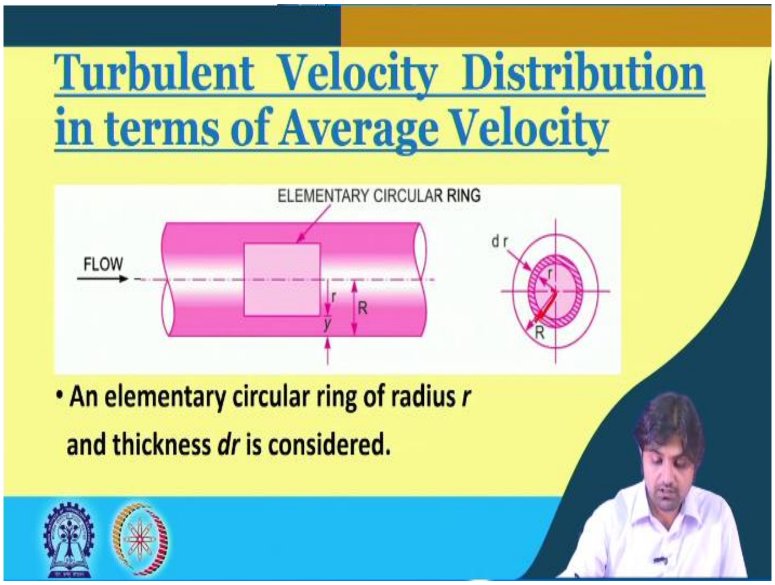

Now, we are going to talk about turbulent velocity distribution in terms of average velocity. So, there is a flow and there is an elementary circular ring here, as you can see, this is of radius R. So, we have an elementary circular ring of radius r and thickness dr which we have considered. So, radius small r and thickness dr, that is, what we have considered.

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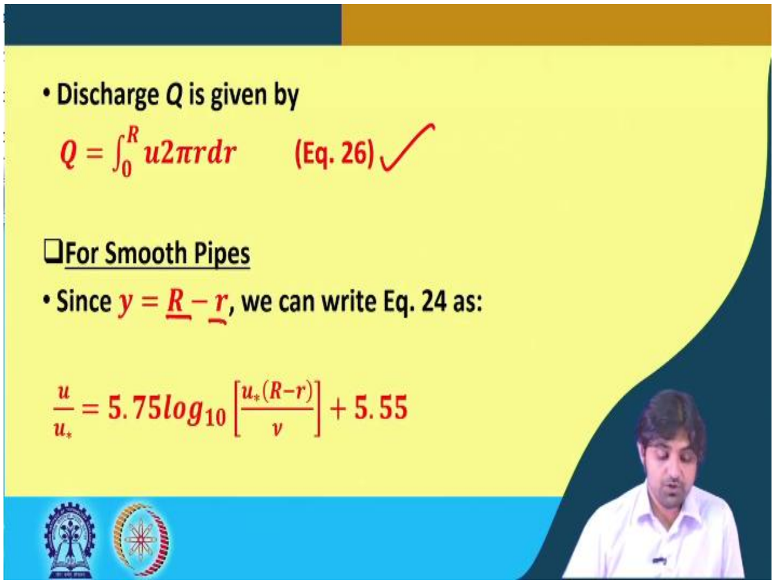

Discharge Q is given by

$$Q = \int_0^R u2\pi r dr$$

. Now, can we calculate it for smooth pipes? Yes, since, y is equal to R minus, capital R minus small r, we can write, equation 24, you know, if you do not remember the equation 24, I will take you to equation 24. So, this was equation 24. Equation 24, u by u star can be written as 5.75 log 10 u star R minus r by nu plus 5.55 and that we have to substitute in Q.

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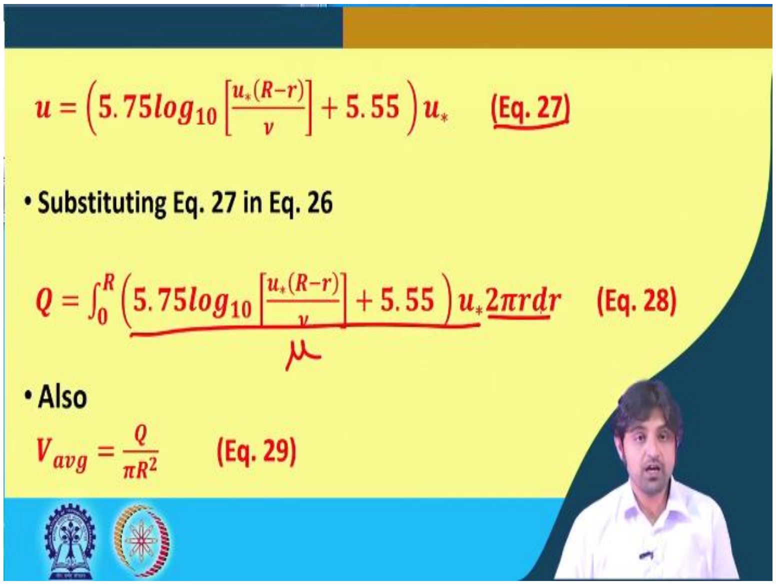

Or we can also write, u is equal to 5.75 log base 10 u star R minus r by nu plus 5.55 y into u star. So, what we did? We took u star that side. Now, if you substitute this equation 27 in equation number 26, so, it will become integral to 0 to R so u this is u into 2 pi r dr, very simple. Also we can write, V average is Q by pi R square. So, we can directly go for the average velocity.

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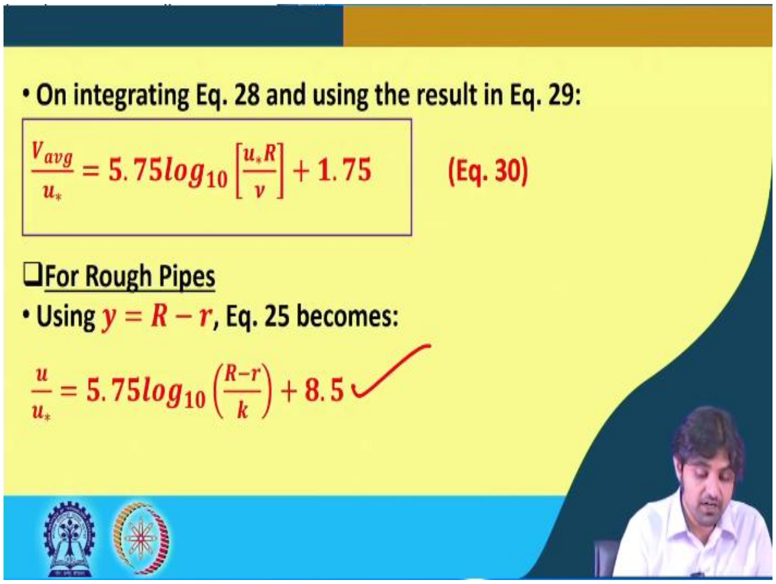

If we integrate this equation number 28 and using the results in equation 29, that is, dividing by pi R square for obtaining V average, we can get,

$$\frac{V_{avg}}{u_*} = 5.75 log_{10} \left[\frac{u_* R}{\nu}\right] + 1.75$$

.

My request to you is that you please try to integrate this equation, it is very simple, it is class 12th 11th and 12th Mathematics. But you please integrate and try to obtain the equation number 30.

Now, for rough pipes similarly, we will use the equation that we have got for the rough pipes and we use the still the same y is capital R because we are calculating the distance from the wall capital R minus small r. So, we are going to get u, we have this equation that we have derived equation number 25, that was, R minus small r, so, instead of, y by R, y by k we have, R minus small r by k.

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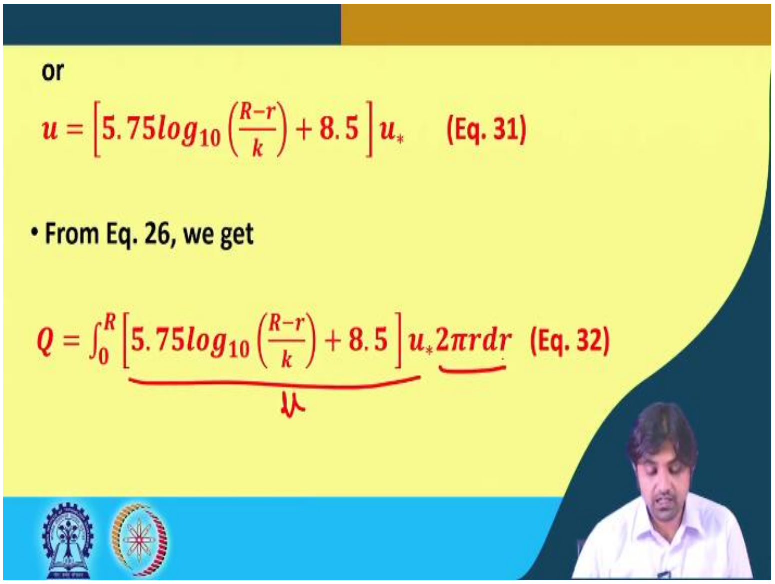

Or we took u star that side and substituted this u in the equation for Q. So, we integrate it from 0 to R, this is u into 2 pi r dr and using the equation 32, we will get a similar equation

$$\frac{V_{avg}}{u_*} = 5.75 log_{10} \frac{R}{k} + 4.75$$

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