Hydraulic Engineering
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Lecture – 7
Basics of fluid Mechanics – II (contd.,)

So, welcome back last class, we studied several concepts and concluded our lecture on calculating the stream lines the equation for the stream lines.

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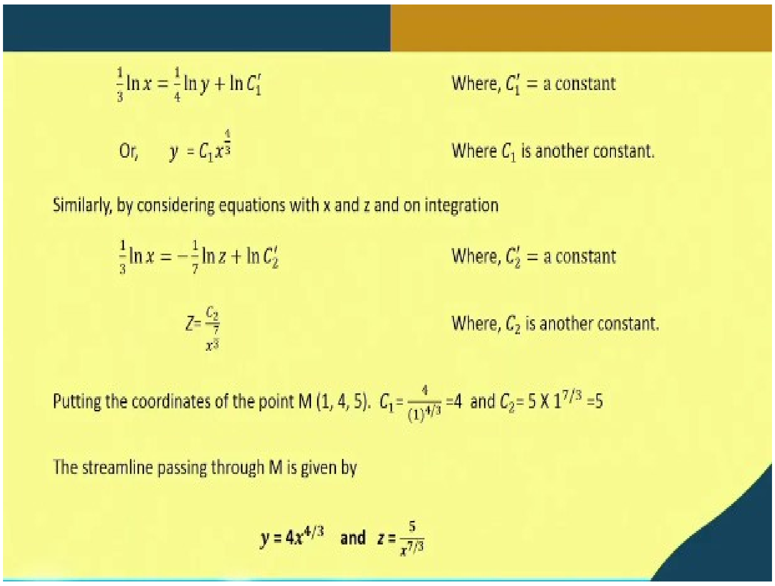

And this is this slide where we finished. So, we will talk about more concepts now.

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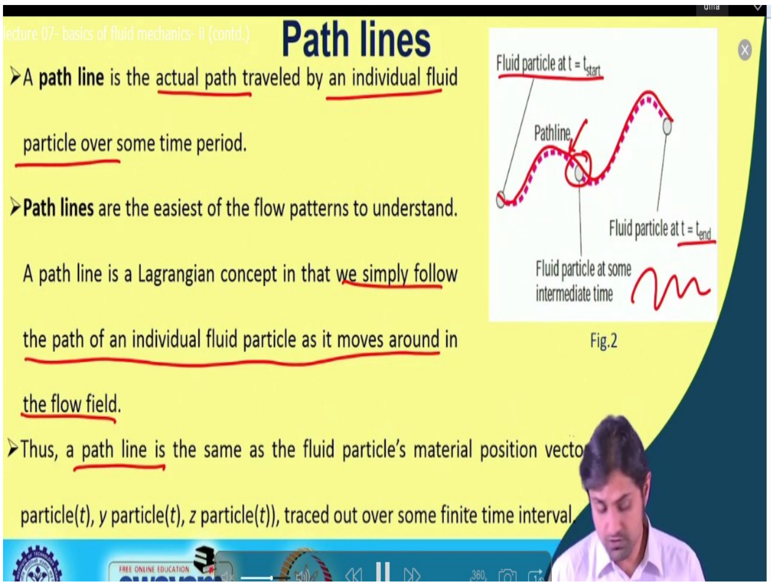

If you remember that we said something about path line in our last lecture that we are going to talk about it in the upcoming slides. So, what actually are path lines? So, path line is the actual path, sorry, I will this should have been chosen the pen, yes. So, a path line is the actual path traveled by an individual fluid particle over some time period. So, this is actually the trajectory of the particle right.

So, the real path on which the particle travels. So, path lines are the easiest of the flow patterns to understand and a path line is a Lagrangian concept in that we simply follow the path of the individual fluid particle as it moves around in the flow field. So, what we are doing, path line is can also be said as a Lagrangian concept because what we do is, we simply follow the path of an individual particle as it moves around in the flow field.

So, for example, we see there is a fluid particle at t, that is, t start or we can also say $t = 0$ for example, and the particle is moving like this. So, this is the path line and if you want to know some intermediate at intermediate point. This is the intermediate point and at the end this is t and this is just to show you how the path line can look, it could be the path line could be any shape, depends upon the fluid velocities and the fluid acceleration of the properties, but it is simply the path traced by the individual fluid particles.

So, what we are doing here, we are simply following that particle as it moves along the fluid. Therefore, it is a Lagrangian concept. So, a path line is the same as fluid particles material position vector, x particle, y particle and z particle t traced out over some finite time interval. So, we integrate and we can obtain that.

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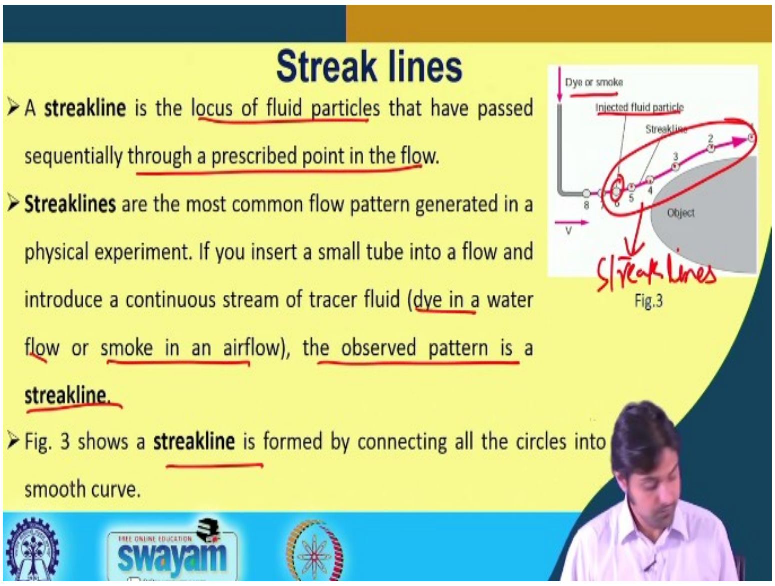

Moving to the next concept is streak lines. So, what are streak lines? So, a streak line is the locus of the fluid particles that have passed sequentially through a prescribed point in flow. So, if we start tracing the fluid particles that have passed sequentially through a prescribed point in the flow. We will explain it a little more. So, streak lines are the most common flow pattern generated in a physical experiment.

For example, if you insert a small tube in the flow and reduce a continuous stream of tracer fluid for example, dye in a water flow or smoke in an air flow we the observed pattern is a streaks line because we are seeing the locus of that particle through a prescribed point in the flow, because that the tracer is one specific point and we are following that movement of that particle.

For example, this if you inject a dye or dye in case of water or smoke in case of an airflow and there is an object here, you know, and this is the particle in which we have injected the fluid particles, this one, so, it moves this one, I mean, and this if we observe this the locus or the path traced by this particle is called the streak lines. You can always go to the YouTube and see one of the streak lines and yeah, it is this figure shows the streak line formed by connecting all circles into a smooth curve.

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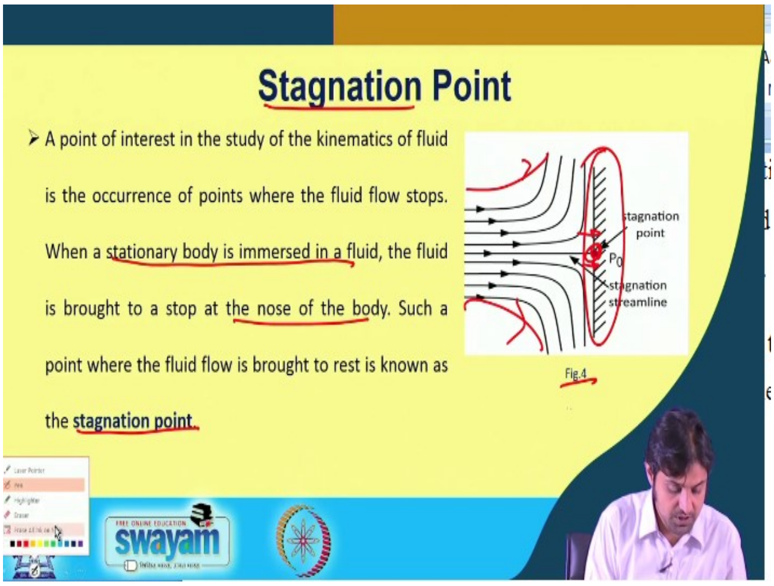

Now, there is something called a stagnation point. So, this is a very interesting point. So, a point of interest in the study of kinematics of fluid is the occurrence of the points where the fluid flow stops. So, as the name suggests stagnation, stagnation means something is stagnant or stopped.

So, those points are of very much importance. So, one of the stagnation point could also be the plate for example, because there also the fluid flow stops right.

So, when a stationary body is immersed in a fluid, the fluid is brought to stop at the nose of the bound nodes of the body, such a point, where the fluid flow is brought to the rest is known as stagnation point. My one question to you is, what creates, what physical phenomenon helps or is related to the occurrence of the stagnation point? Suppose, there is a solid object which is kept fixed in the fluid flow when the fluid touches this body that point is called stagnation point. Why?

Because last lecture, we read about a concept that was called no slip condition because of the no slip condition, the adjacent fluid velocity fluid particle, the velocity will also be zero because of no slip condition. So, this is the stagnation point. In this example, in this figure 4, there is a wall here and then there is air flow or the fluid flow, whatever you want to call it is there when it encounters the wall the velocity here actually is zero, the flow is stopping or diverting. So, in this direction there is no flow and this is called the stagnation point.

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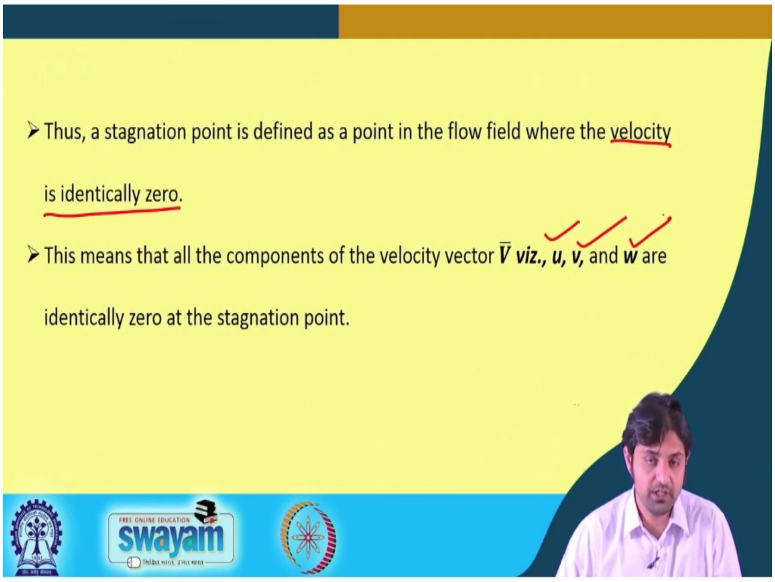

Thus, a stagnation point is defined as a point in the flow field where the velocity is, this is the key point is identically zero. This means, that the all the components of the velocity V that is u, v, and w, are identical is zero at this stagnation point. Some of it we will be going to see in today's lecture or maybe in the weeks coming forward what the stagnation point, where it can be used. Now, after all these points, we move on from the velocity description to the acceleration part.

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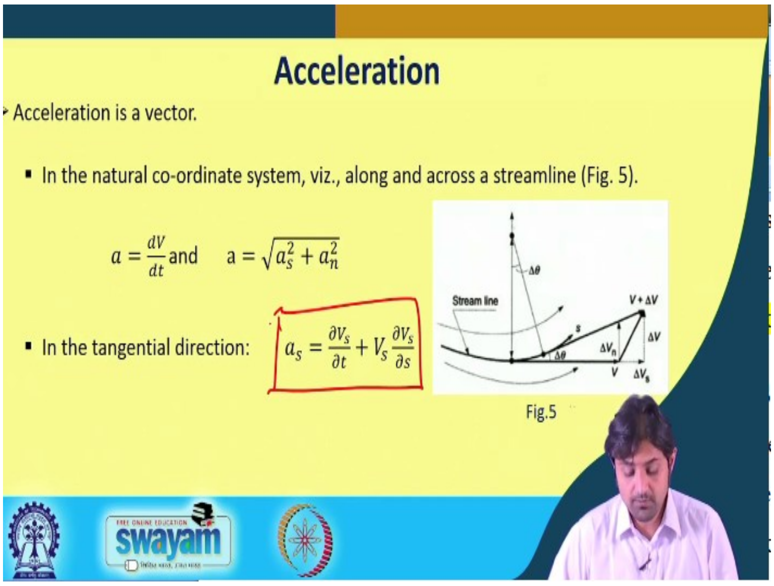

So, acceleration first thing to that you already know is a vector. In natural coordinate system, that is, we describe the acceleration along and across the stream line that we are going to see. Suppose, this is a streamline here, this in figure 5, let me take the laser pointer here. So, this is a streamline and this is a direction s and this is the velocity v direction here and this is the S direction velocity in change in velocity in s direction and this is the velocity change in the direction perpendicular.

So, acceleration when can be given as, so I go back to the pan, total derivative of velocity with respective time, dv dt and not ∂v ∂t. So, the total derivative of the velocity is called the acceleration. In our case here, it will be a vector sum of the acceleration along the stream line and perpendicular to the stream line same as velocity. Suppose, the velocity there are two velocity directions, V_s and V_n with total velocity the speed is going to be $\sqrt{V_s^2 + V_n^2}$.

So, in the tangential direction, this direction, in the tangential direction a s can be written as $\frac{\partial Vs}{\partial t} + Vs\frac{\partial Vs}{\partial s}$ this you already have derived in your fluid mechanics class. So, this is the acceleration in s direction.

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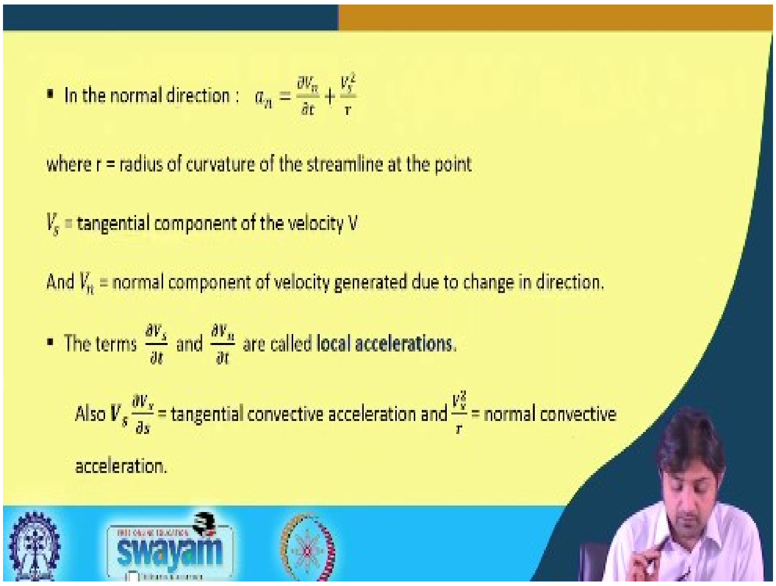

And in the normal direction it is nothing that it is $\frac{\partial Vn}{\partial t} + \frac{V_s^2}{r}$, V_n and V_s are velocities in s and n direction. Here, this r is the radius of curvature of the streamline at that point. This you will we will solve some problems, maybe in the upcoming, you know, lectures. V_s is the tangential component of the velocity V. This we already talked and V_n is the normal component of the velocity generated due to change in direction.

So, normal V_n the velocity, the velocity component that is normal the term $\frac{\partial Vs}{\partial t}$ and $\frac{\partial Vn}{\partial t}$, this one and this one are called local accelerations, and $Vs\frac{\partial Vs}{\partial s}$ is the tangential convective acceleration and $\frac{V_s^2}{r}$ is the normal convective acceleration. This is quite some information to note. So, now what I am going to do highlight the most important one. So, let us go to the previous slide.

So, this is an important equation. So, it is a combination of a s is a combination of local acceleration plus convective acceleration. In the normal direction also the sum of local acceleration plus the normal this is also normal convective acceleration. So, when we need to calculate the total acceleration, we should be calculating the corresponding accelerations, local accelerations and corresponding normal convictive accelerations and the convective acceleration which is tangential and then we will be able to solve the problems.

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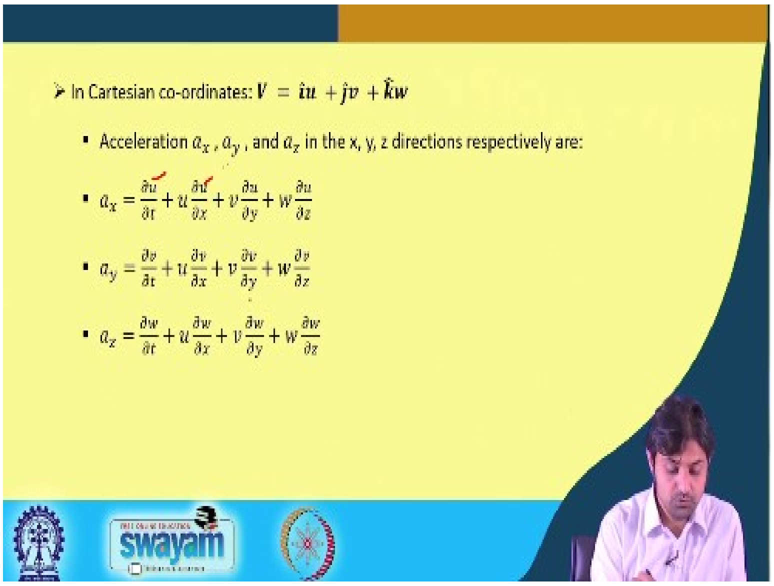

So, again in Cartesian coordinates, if we write the velocity I think, it is best again to write V as $u\hat{i}$, here we have written the velocity according, I mean, $\hat{i}$, $\hat{j}$ before, it is also right, but better to. This is the normal convention. Acceleration a_x, a_y, and a_z in the x, y and z directions respectively are. So, a_x will be written as ∂t plus the this is local acceleration and this part is convective acceleration.

So, let me rub the ink. Similarly, for a_y as well. So, $\frac{\partial v}{\partial t} + u\frac{\partial v}{\partial x} + v\frac{\partial v}{\partial y} + w\frac{\partial v}{\partial z}$ these are in some important equations that you should be remembering, a_z will be ∂w ∂t plus $u\partial w\ \partial z$ So, you see u, u, u, u for a_x, v, v, v, v for a_y and w, w, w, w here for a_z.

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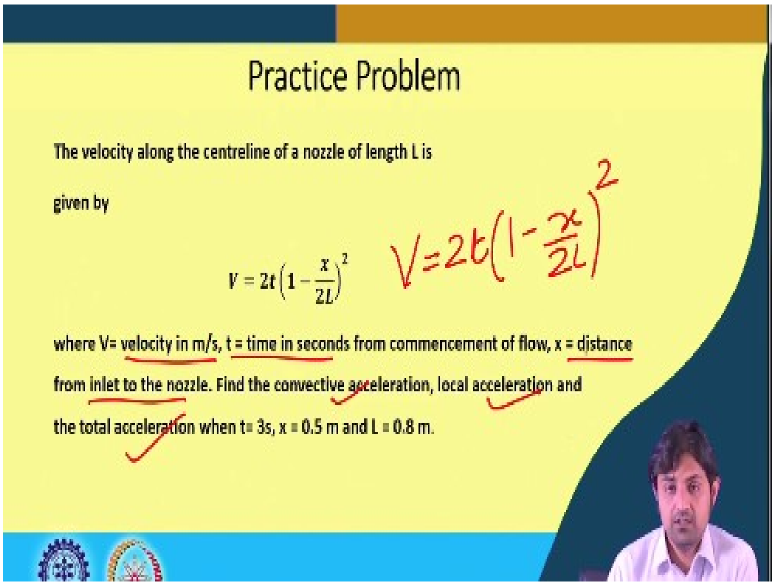

So, now, we will see a practice problem. So, the question is, the velocity along the center line of a nozzle of length L is given by $= 2t\left(1-\frac{x}{2L}\right)^2$ or I will write $2t\left(1-\frac{x}{2L}\right)^2$. Here the V is velocity in meters per second, time t in seconds and x is the, so, t is from t = 0, x is the distance from inlet to the nozzle. So, and so, we have to find the convective acceleration, local acceleration and total acceleration when t = 3 seconds, x = 2.5 and L = 0.8. So, we have seen the equations previously and that we are going to utilize now.

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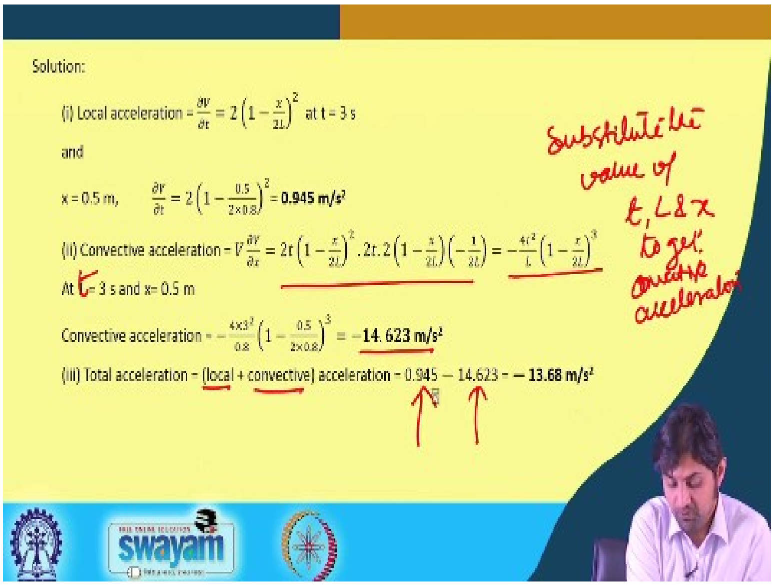

So, the solution procedure goes as like this. Local acceleration is given by $\partial v\ \partial t$. So, so, $\partial v\ \partial t$ if you see this equation here, so, if you multiple if you do $\partial v\ \partial t$, it is going to be 2t, this t will, you know, this will become $2\left(1 - \frac{x}{2L}\right)^2$ and that is what exactly this is and at t = 3 seconds and at x = 0.5 meter, what we have done is we have substituted the value of t = 3 second and x = 0.5 in this equation.

So, it is 2 will remain 2, 1 will remain x is 0.5 and length of length value we have been given 0.8 and this comes out to be as 0.945 very simple multiplication is there. This is the local acceleration. So, now, the convective acceleration here is going to be, you see, there is only one direction x so, there will be only one velocity component, I mean, in convective velocity component and that is going to be $v\frac{\partial v}{\partial x}$, so if V was, so, we knew that, I mean, the original equation says $2t\left(1 - \frac{x}{2L}\right)^2$.

So, when we multiply $v\frac{\partial v}{\partial x}$, so, let us write down V. So, this is V, correct, 2t into $(1 – (x / 2L))^2$ and what is ∂v ∂x. So, if you do if you differentiate ∂v ∂x ,2t will remain 2t. And this 2 will come down here on here and it will be (1 – (x / 2L)) multiplied by this x will become – (1 / 2L) and on simplification I will just erase the. So, this on simplification will give $-\frac{4t^2}{L}\left(1 - \frac{x}{2L}\right)^3$. So, what needs to be done very simply.

Simply, we need to substitute the value of t, L, and x to get our, so, substitute the value of t, L, and x to get convective acceleration. So, at t = 3 second and x = 0.5-meter, convective acceleration will be -14.623 meters per second square. So, the total acceleration is going to be local plus convective acceleration. You can simply do the arithmetic sum I generally prefer to write these accelerations separately. So, if you write this one and this one separately, it is fine. Here, we have simply added them because the total is the sum of both the accelerations.

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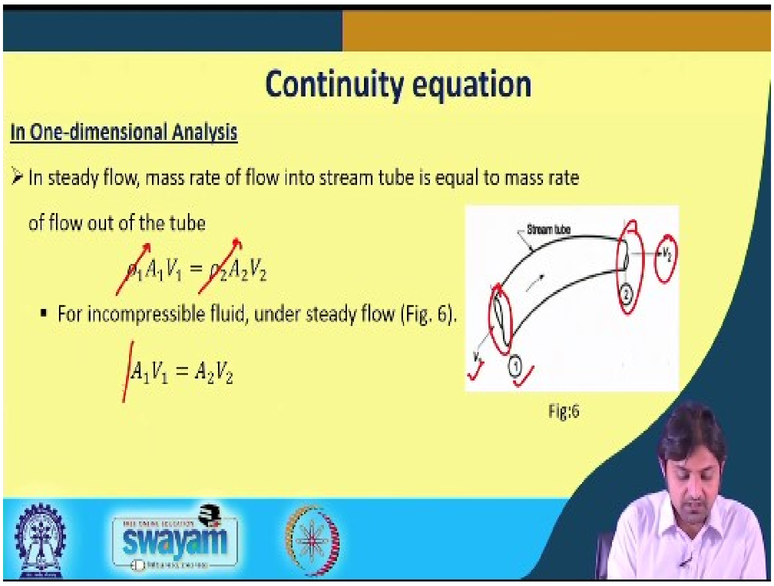

So, going to another concept, that is, continuity equation. So, in one dimensional analysis, what we see is, in a steady flow mass rate of flow into the stream tube is equal to the mass rate of flow out of the tube. So, whatever goes comes in goes out of the tube for example, in a steady flow. So, in one dimensional the continuity equation says $\rho_1 A_1 V_1 = \rho_2 A_2 V_2$, this is the continuity equation.

For incompressible fluid under a steady flow, steady flow we know that not a function of time. But what is incompressible fluid? Density remains constant, so, the density does not changes, so, $\rho_1 = \rho_2$. So, if there is V_1 and this is the velocity V_1 this is the cross sectional area A_1, this is the velocity V_2 and this is the cross sectional area A_2 and for incompressible fluid because ρ is equal on both side we can get it cancelled and we can simply write $A_1 V_1 = A_2 V_2$. So, this is the continuity equation in one dimension.

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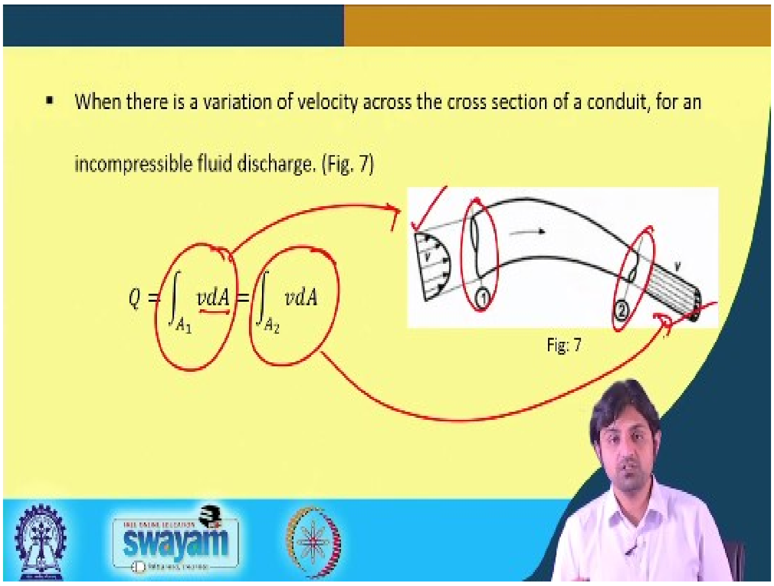

Now, when there is a variation of velocity across the cross section of the conduit for an incompressible fluid discharge like this, you see. So, if these velocity, you see, this is the velocity component, this is V, and this is cross sectional area 1 this is cross sectional area 2 and if this is the profile, we can simply write, still the equation of continuity will hold but what we do instead because the velocity is varying across the cross section, so, instead of writing V_1 A_1 what we do is we integrate the velocity with area for the entire surface and do it on the same the other side as well. So, let us save this is this side A_1 and this is the other side A_2, no change just that we have to use a differential or an integral form here.

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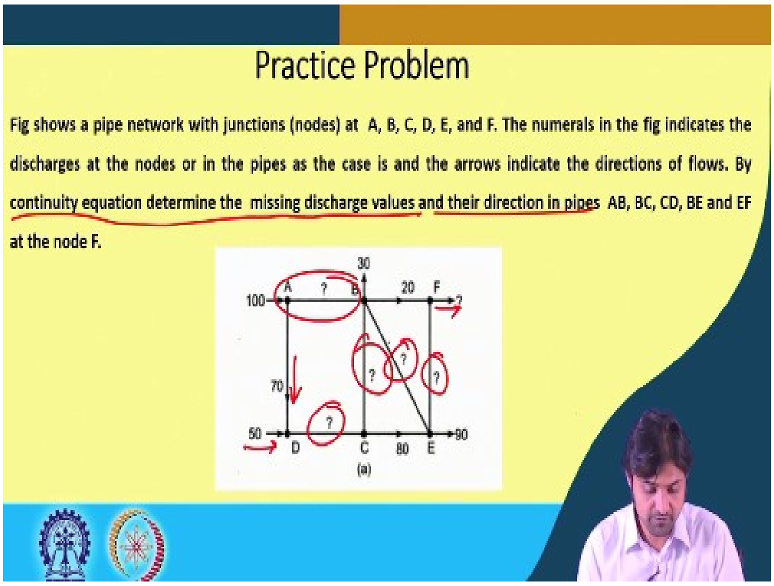

So, we are going to see a practice problem. Now, this figure we are going to show, shows a pipe network with junction nodes at A, this is junction node B, C, D, E and F. The numerals in the figure like this 100, 30, indicates the discharges at the node or in the pipes as the case is and the arrows indicate the direction of the flows. So, this let me just erase this one now. So, you see, this means that the flow is in this direction and this means this flows in this direction but many of like here it is already told that the flow is in this direction.

But other places we have to calculate. So, the question is by continuity equation determine the missing discharge values and their direction in the pipes. So, we are lucky because they have directly given us the discharge value and that is what we have to preserve, A_1V_1 is discharge in the pipes AB which is unknown so question mark BC, CD and EF actually we can calculate BF, BE as well BE yeah, all the 5.

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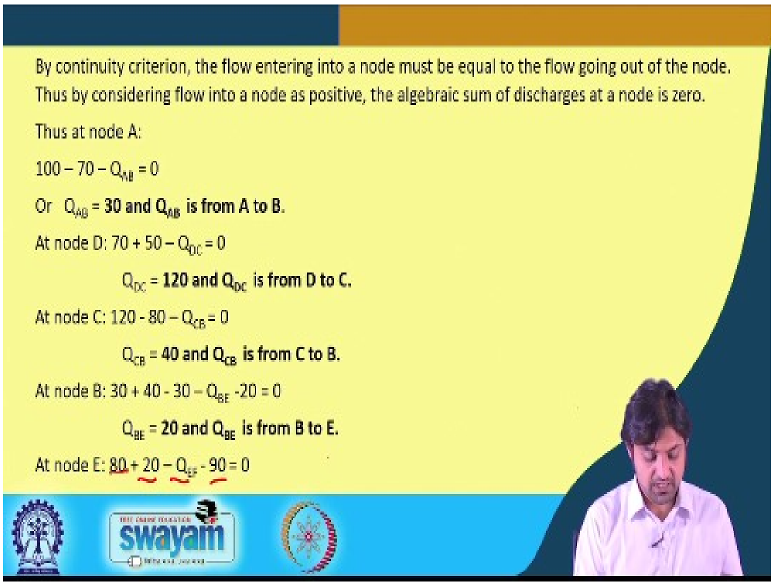

So, we go to the solutions. So, by the continuity criterion, this is quite important, the flow entering into a node must be equal to the flow going out of the node as the continuation of continuity says, thus, by considering the flow. So, what we do is we consider flow in to the node as positive and flow out of node as negative. So, at point, so, there is a point here and whatever goes in for example, x goes in x / 3 and 2x / 3, whatever goes in will go out because of the equation of continuity. Therefore, what this implies is it that the algebraic sum of the discharges at a node is 0, this is quite important. So, this is the fundamental that we calculate thus at node A. So, let us go back and see, if we assume, this is we called Q_{AB}. So, 100 is coming in and 70 is going out and Q_{AB} also going out. If we consider this node here, let me just erase this. This node A 70 was here and we assume Q_{AB} here.

So, this will be 0. So, this is reflected by the equation here, 100 - 70 - Q_{AB} = 0. So, what we get simply using this equation, Q_{AB} is 30 and is from A to B, very simple. So, the first we, now we go to node D, this is node D. So, if we calculate, if we see, so 50 is also coming in this direction, 70 is also coming into this direction and there should be something here that is called Q_{DC}, I think. So, 70 + 50 - Q_{DC} = 0. Therefore, we get Q_{DC} is equal to 120 and it is from D to C.

Similarly, at node C, you go to node C. Let, me erase this. This is node C, this we have already calculated, right, this was in this direction 120, this we already know and this for example, we need to calculate Q_{CB} or Q_{BC} whatever it is there. So, this is known, this is known and this is unknown, right. So, this is 120 going in, 80 was going out and Q_{CV} was also going out therefore, we get Q_{CB} is equal to 40 and it is from C to B.

Now going to the node B, node B, so, now, this is known, this is known, this is known, this is known and we need to calculate this one. So, all most of the things are known for node B because node B will have incoming from here that we have already calculated. This we just calculated. This we know from before and this is already given as 20. So, for node B; 30 + 40 - 30 - Q_{BE} - 20 and this is going to give Q_{BE}, as 20 but direction is from B to E. Let, me just erase all the ink.

And finally, at node E, if we go back again to node E, right, this is we know, this is we know, now, this is also we know from our last calculation. This is the one that we need to calculate now, so only one unknown is there. So, 80 + 20 - Q_F - 90 = 0.

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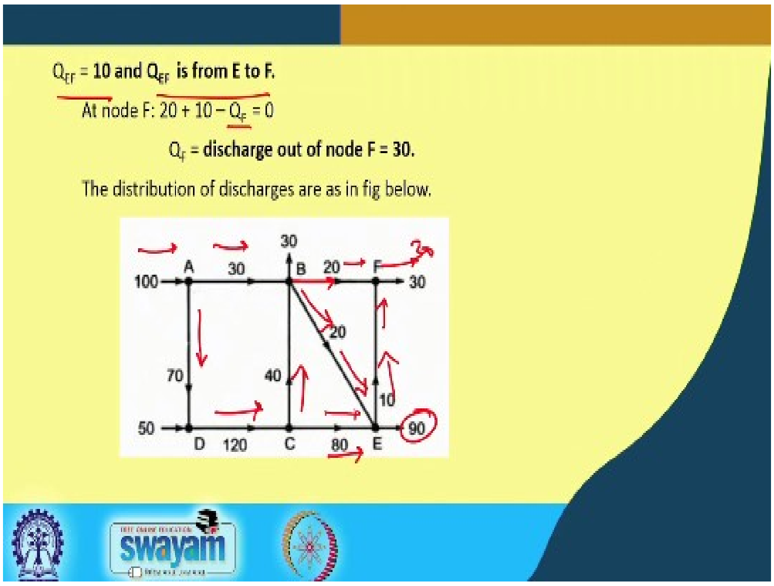

And this gives us Q_{EF} is equal to 10 and Q_{EF} is from E to F. So, at node F we need to calculate our final quantity Q_F then see that is simply discharged out of node F. So, we know this, we know this, 10 was coming here, so, 20 + 10 and that will be simply 30. So, simple application of continuity equation that comes 30. Now, the final distributions of the discharges are in as follows: I will just take you, 100 goes in 30 is distributed, 100 goes in here, 30 distributed here, 70 comes here.

Now, 50 + 70, 120 will go here and at node C, 80 is going in this direction, 40 is going in this direction. At node B, 30 which is coming here will go there, 40 that is going, coming here it is distributed 20 this side, 20 this side. At node E, 80 is coming in this direction and 10 is going in this direction. However, we were also getting 20 from this direction therefore, resulting will be 90. So, at node F, what is happening is, we get 20 from this side, 10 from this side and this total 30 is going from this side.

So, every place the equation of continuity is satisfied. So, this is one of the simplest most problem that you can see about the equation of continuity. Normally, this could also happen in place of the velocities, you know, only the velocity can give in the areas of these pipes could be different, but the procedure will remain the same you have to calculate A_1, V_1, A_2, V_2 and so on and come and equate the discharges as we did here in the simplest example. So, this is for today. I am going to see you again in the next class. Thank you so much.