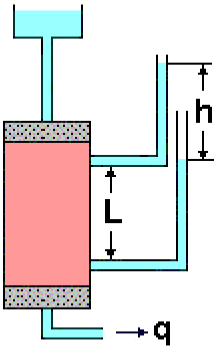
LECTURE 11

Laboratory Measurement of Permeability:

Constant Head Flow

Constant head permeameter is recommended for coarse-grained soils only since for such soils, flow rate is measurable with adequate precision. As water flows through a sample of cross-section area **A**, steady total head drop **h** is measured across length **L**.

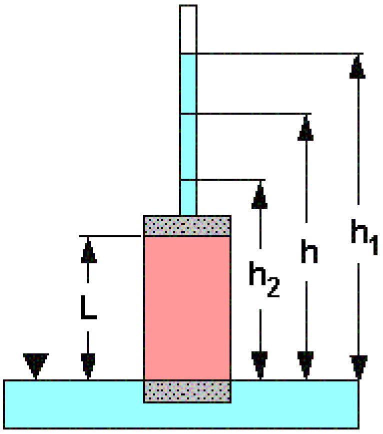


Permeability **k** is obtained from:

$$k = \frac{qL}{Ah}$$

Falling Head Flow:

Falling head permeameter is recommended for fine-grained soils.



Total head **h** in standpipe of area **a** is allowed to fall. Hydraulic gradient varies with time. Heads $\mathbf{h_1}$ and $\mathbf{h_2}$ are measured at times $\mathbf{t_1}$ and $\mathbf{t_2}$. At any time **t**, flow through the soil sample of cross-sectional area **A** is

$$q = k.h.\frac{A}{L} \quad \text{--------------------- (1)}$$

Flow in unit time through the standpipe of cross-sectional area **a** is

$= a \times \left(-\frac{dh}{dt} \right)$ ----------------- (2)

Equating **(1)** and **(2)** ,

$$-a.\frac{dh}{dt} = k.h\frac{A}{L}$$

or $$-\frac{dh}{h} = \left(\frac{kA}{La} \right) dt$$

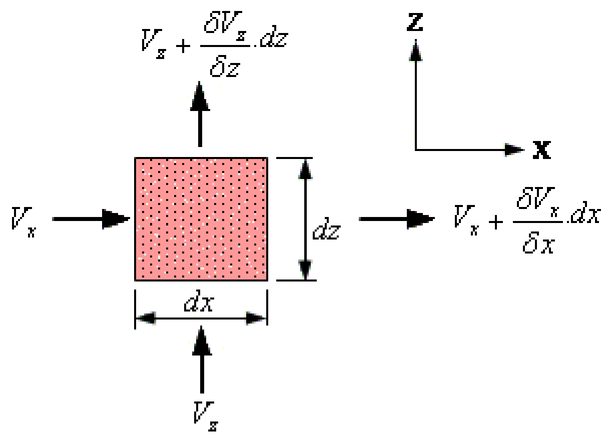
Integrating between the limits,

$$\log_e \left(\frac{h_1}{h_2} \right) = \frac{k.A}{L.a}(t_2 - t_1)$$

$$k = \frac{L.a.\log_e \left(\frac{h_1}{h_2} \right)}{A(t_2 - t_1)}$$

$$= \frac{2.3L.a\log_{10} \left(\frac{h_1}{h_2} \right)}{A(t_2 - t_1)}$$

Seepage in Soils:



A rectangular soil element is shown with dimensions ***dx*** and ***dz*** in the plane, and thickness ***dy*** perpendicuar to this plane. Consider planar flow into the rectangular soil element.

In the **x-direction,** the net amount of the water entering and leaving the element is

$$\frac{\delta V_x}{\delta x}.dx.dy.dz$$

Similarly in the **z-direction,** the difference between the water inflow and outflow is

$$\frac{\delta V_z}{\delta z}.dz.dx.dy$$

For a two-dimensional steady flow of pore water, any imbalance in flows into and out of an element in the z-direction must be compensated by a corresponding opposite imbalance in the x-direction. Combining the above, and dividing by***dx.dy.dz*** , the **continuity equation** is expressed as

$$\frac{\delta V_x}{\delta x}+\frac{\delta V_z}{\delta z}=0$$

From Darcy's law, $V_x = k_x.\frac{\delta h}{\delta x}$, $V_z = k_z.\frac{\delta h}{\delta z}$, where **h** is the head causing flow.

When the continuity equation is combined with Darcy's law, the equation for flow is expressed as:

$$k_x.\frac{\delta^2 h}{\delta x^2}+k_z.\frac{\delta^2 h}{\delta z^2}=0$$

For an isotropic material in which the permeability is the same in all directions (i.e. $k_x = k_z$), the **flow equation** is

$$\frac{\delta^2 h}{\delta x^2}+\frac{\delta^2 h}{\delta z^2}=0$$

This is the **Laplace equation** governing two-dimensional steady state flow. It can be solved ***graphically, analytically, numerically, or analogically.***

For the more general situation involving ***three-dimensional*** steady flow, Laplace equation becomes:

$$\frac{\delta^2 h}{\delta x^2}+\frac{\delta^2 h}{\delta y^2}+\frac{\delta^2 h}{\delta z^2}=0$$