

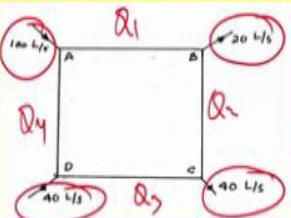
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**Lecture – 47**  
**Pipe Networks (Contd.)**

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**Class Problem (Hardy Cross)**

For the square loop shown, find the discharge in all the pipes. All pipes are 1 km long and 300 mm in diameter, with a friction factor of 0.0163. Assume that minor losses can be neglected.



The diagram shows a square loop of pipes. Vertex A is at the top-left, B is at the top-right, C is at the bottom-right, and D is at the bottom-left. Pipe AB is vertical, pipe BC is horizontal, pipe CD is vertical, and pipe DA is horizontal. Arrows indicate flow direction: Q1 enters at A and leaves at B; Q2 enters at B and leaves at C; Q3 enters at C and leaves at D; Q4 enters at D and leaves at A. Each pipe is labeled with its length (1 km) and diameter (300 mm). The friction factor is given as 0.0163. The text states that minor losses can be neglected.



Welcome back students. This is the last lecture of this module; pipe flow or viscous pipe flow and in the last lecture we have studied the basic concepts of Hardy Cross Method which is a way of solving the pipe networks. It is an iterative procedure but a very well laid out systematic procedure to solve the flow in the pipe. So, currently we have a question at hand in your, as you can see on the screen.

A discharge of 100 litres per second is entering from here and there is an outflow of 20 litres per second here, there is an outflow of 40 litres per second here and there is again an outflow 40 litres per second here and using the Hardy Cross Method, what we have to do; we have to find Q1, Q2, Q3 and Q4.

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Solution 15:

- Assume value of  $Q$  to satisfy continuity equations at all nodes
- The head loss is calculated using  $HL = K Q^2$
- $HL = hf$  then  $\Rightarrow$  neglected minor losses  $h_m = 0$
- Thus  $HL = hf$

$hf$  can be calculated using Darcy-Weisbach Equation

$$hf = \frac{\lambda L V^2}{D 2g} \quad (1 \text{ km long, } 300 \text{ mm dia})$$

$$hf = \frac{\lambda L V^2}{D 2g} \quad (\lambda = 0.0163)$$

$$HL = \frac{\lambda L V^2}{D 2g} = 0.0163 \times \frac{1000}{0.3} \times \frac{V^2}{2 \times 9.81} = \boxed{2.77 V^2}$$



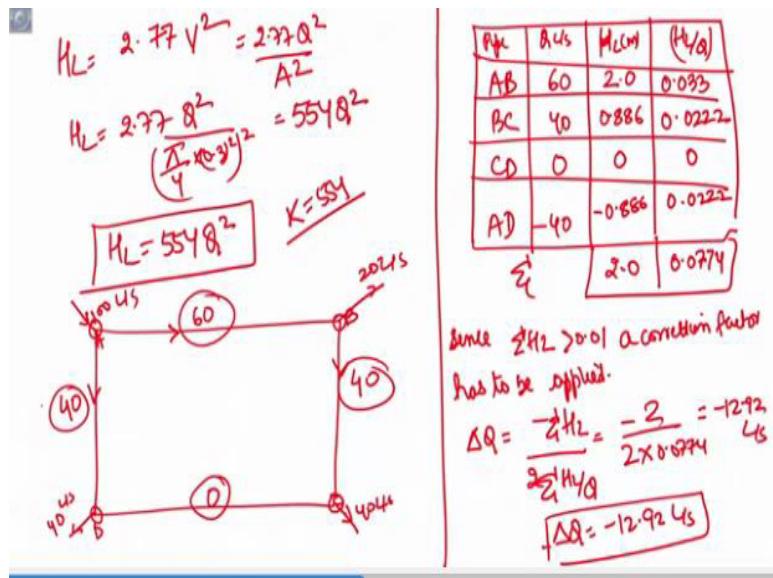
So, let us start by the, so solution 15, so because this is the first problem in Hardy Cross I will write everything. Assume value of  $Q$  to satisfy continuity equations at all nodes. So, there were 4 nodes also the head loss is calculated using  $HL$  is written as  $K1 Q$  square,  $K$  dash  $Q$  square. This  $HL$  actually could be sum of major and minor losses both. In current case, we have neglected minor losses, I am talking in general.

So,  $h L_m$  is 0, thus head loss is only  $hf$ , major losses due to friction. Now, this  $hf$  can be calculated using Darcy Weisbach equation. How? See, using the Darcy Weisbach equation our idea is to arrive at a suitable  $Q$ , in terms of a suitable  $K$ , we want to arrive at something like this, so that for each of the pipe we can do that because they are not changing that much.

So,  $hf$  can be written as  $\lambda L$  by  $D$  into  $V$  square by  $2g$ , in the question we have been given that pipes are 1 kilometre long, 300 millimetre in dia and  $\lambda$  or friction factor is 0.0163,  $\lambda$  or  $F$ , whatever you want to call it. This we have already seen, so the head loss, total  $H$  loss is  $\lambda L$  by  $D$   $V$  square by  $2g$ , so let us put the values,  $\lambda$  is 0.0163 into length is 1000 meters, diameter is 0.3 meters into  $V$  square by 2 into 9.81.

Now, the next step; so, if we do this we are going to get  $2.77 V$  square, the  $HL$ , but we need to have it in form of  $Q$ .

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So, we write  $HL$  is  $2.77 V^2$  or  $2.77 Q^2$  by  $A$  square, so  $2.77 Q^2$ . What is area? Area is nothing but  $\pi$  by 4 into 0.3 whole square to whole square, so this comes to be  $554 Q^2$ . So, you see this is our expression for  $HF$  or  $HL$ , where  $K$  we have seen is 554, we have derived because all the properties of the pipe were given. So, now we make the, so we, what we do is; let us say for the first iteration, this is point A, this is point B, this is point C and this is point D.

100 litres were coming here, this is given first and then we say 20 litres per second and this is 40 litres per second. Let us say when 100 is coming, this pipe AB gets 60, like this and this say is 40 and AD is in this direction 40 and therefore this will be, so 40 comes 40 goes out and this will be 0, this is our first assumption, we are not saying this is the correct answer.

But let us say, this is the first assumption we have used the equation of continuity at each of these nodes, this is what we have done. So, as I said first make a table like this and we write pipe name here, we write  $Q$  into litres per second, we also write head loss in meters and we also find  $HL$  by  $Q$ , these are the 4 things, so 1, 2, 3 and 4. So, let us write AB, BC, CD and AD.

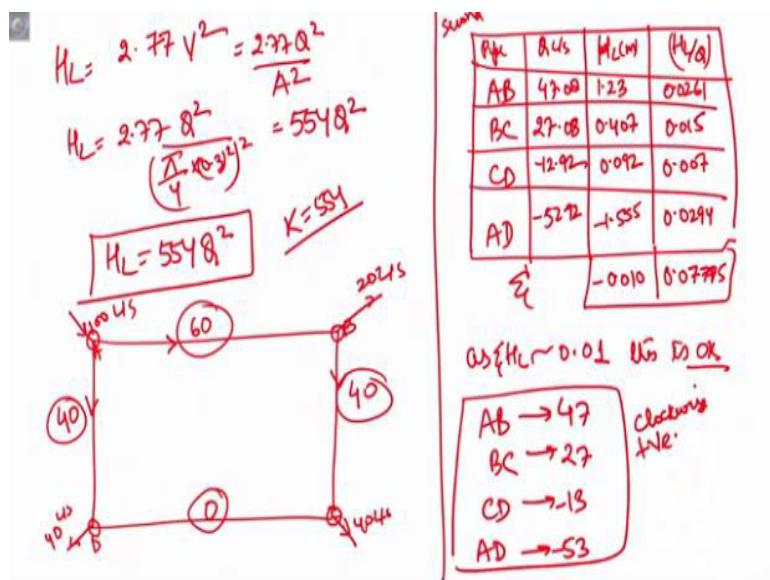
This we have already found, done through using the equation of continuity, so we simply write down 60, 40, 0, -40, you see that the clockwise we have assumed positive that is why AD is minus 40 and using  $554 Q^2$  and converting it into meter cube per second because it is given in litre, the head loss is going to be; you do the calculation  $554$  into  $60$  whole square divided by  $1000$ , it will come out to be  $2.0$ .

And this will come out to be 0.886, since this is 0, this will be 0 and this comes  $-0.886$ . You also calculate 2 by 60 in meters, so I mean 2 by 60 because this is in meter, this is in litre second, so we calculate  $HL$  by  $Q$  0.033, this will come 0.0222, this will come 0 and this will come 0.0222 same, fine. So, we have to do the summation here, we have to find the summation of only these 2 quantities.

If you sum this, you will find, this is  $HL$  is 2.0 and  $HL$  by  $Q$  is coming to be 0.0774, so since the sigma of head loss is greater than 0.01, a correction factor has to be applied. And what is that correction factor? We know the formula, the formula is  $-HL$  divided by 2 times sigma  $HL$  by  $Q$  and this will come out to be  $-2$  divided by 2 into 0.0774 or  $-12.92$  litres per second.

So,  $\Delta Q$  will come to be  $-12.92$  litres per second. Now, this  $\Delta Q$  that we have obtained should be applied to discharges before the next trial here. So, this 60 will become  $60 - 12.92$ , this will become  $40 - 12.92$ , 0 will become  $0 - 12.92$  and 40 will also become  $40 - 12.92$ . So, I will not rub the entire thing but I will rub the right hand side, so now this is the second trial.

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So,  $Q$  here will be  $60 - 12.92$  because  $\Delta Q$  was negative, so it becomes 47.08, first step is to write down the values after the correction,  $-12.92$  and this  $-52.92$ . Similarly  $HL$  if we calculate, we will get 1.23, we get 0.407, we get 0.092 and we get  $-1.555$  because  $HL$  is 554 into  $Q$  square and  $HL$  by  $Q$ , this  $HL$  by  $Q$  1.23 divided by 47.08, if you use the calculator, 0.0261, 0.015, 0.007, 0.0294.

And if you do this sum, this is going to be -0.010 and this is 0.07775, so as sigma HL is approximately equal to 0.01, this is okay. Therefore, these will be the final discharges in the pipe AB, BC, CD and AD. So, AB is going to be 47 approximately, I am writing, 27, this is 13 and this is 53. Of course, this is with minus sign and this is also with minus sign considering clockwise as positive.

So, this concludes our solution to this particular question on Hardy Cross Method, you see how we have calculated K that was 554 and applied this.

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### Class Problem

• For the network given below the discharge at the nodes are known. Verify whether the following suggested distribution of discharges in the pipelines of the network as given below in the table, is satisfactory. If not, adjust the distribution. The head loss in a pipe is given by  $. The values of r for various pipes are indicated in the figure. An accuracy of 0.5 unit of discharge is adequate.$

Line	AB	BC	CA	BE	ED	DC
Suggested discharge [units]	60	19	40	41	16	34

So, we will see yet another question and that will be the last question that we are going to solve in the class. But before finishing this lecture I will give you a problem that you will have to attempt at home and will be based on Hardy Cross Method. So, this is the network that is, so okay, so this is the class problem. So, this is the network given below, the discharge at the nodes are known.

So, instead of K, we say R, you see here we have given the values of R here, R, R, everything is given, so you do not have to worry about calculating the major or the minor head loss, everything has been given here. So, these discharges at the nodes are known, so 100 is coming inside here, this 25 is leaving here, 50 is leaving, this is 25 B that we do not know.

The, if not, so we have to check if these values, so we have been given that AB is here is given 60, BC is 19 is given, CA is 40, BE is given 41, ED is given 16 and DC is given 34, we

simply have to check that if this satisfies. So, there are 2 things that need to be checked. So, the first thing is that the continuity equation is satisfied or not and second thing we have to check of course, this is going to satisfy the continuity equation, 60 is coming, 41 is going, 19 is coming here.

Here, see, 34 is coming, 19 is coming, so 19, 30 + 34 is coming and 25 and 40 are leaving, I think, there is a little problem here, DC 34, ED 16 but anyways, whatever these values are we have to check the Hardy Cross Method, if it is applicable there or not. So, the continuity equation actually this should not be, so 19 it should be 20, 19, if it is like this, because if 100 is coming here, then 60 is going, 40 is coming here.

We have to readjust the values a little bit here, but more important is the procedure that we are going to adopt here. So, let us start with the solution. So, as you see there are 2 loops; one is loop 1 and one is loop 2, it seems by this that these actually, they are not going to satisfy this because the continuity equation itself is not satisfied because first and foremost thing is this, this, this and this should be satisfied.

So, based on the figure here, it does not seem to get satisfied but I will lay out the procedure for solving these types of questions.

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Solution:

- 1) The suggested discharge must satisfy continuity equation at all nodes (each node).
- 2) The flow direction is assumed positive clockwise.
- 3)  $H = rQ^2$ ,  $\Delta Q = \frac{1}{2} \frac{H_L}{r}$

Links	$Q$	$rQ^2$	$\Delta Q = \frac{1}{2} \frac{H_L}{r}$
AB			
BC			
CA			

Links	$Q$	$rQ^2$	$\Delta Q = \frac{1}{2} \frac{H_L}{r}$
BC			
CD			
DE			
EB			

In the diagram  $\rightarrow$  Not Satisfactory

So, the solution for this type of question is, the suggested discharge must satisfy continuity equation at all nodes, each node that is the first thing. Second thing, the flow direction is

assumed positive clockwise. Third is you have to calculate  $HL$  as  $r Q$  square here and we also have to calculate  $\Delta Q$  as minus, you know,  $\sigma HL$  divided by  $2 \sigma HL$  by  $Q$ .

So, we have to make tables, like this for both the loops. I am just laying out the procedure here, so the lines are going to be; lines here will be  $AB$ ,  $BC$ ,  $CA$ , there is going to be  $Q$  that you have assumed and then you have  $r Q$  square, then or  $HF$  is equal to and then you have  $\sigma HF$  by  $\sigma HL$  by 2, sorry, this is  $\sigma HL$  by  $Q$ . Similarly, here, there will be 4 lines, in the second loop there are 4, lines will be  $BC$ ,  $CD$ ,  $DE$  and  $EB$ .

And you have to have written  $Q$ , you have to calculate  $r Q$  square and then  $r Q$  square by  $\sigma Q$  fine. So, this is what you have to calculate. In the current problem, we clearly see that the continuity equation is not satisfied at all. Therefore, this means that in the question, the question is the question it says is if we have to check if it is satisfactory, we simply our answer is not satisfactory.

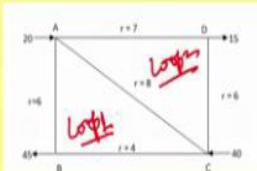
We did not even need to go to the second step, in the first step because the continuity equation was not satisfied therefore we can simply say that the solutions are not acceptable. So, this we have done because we found out the continuity equation was not satisfied at this point. So, we have seen 2 types of problems in Hardy Cross, the problems can be made very complicated.

But since, this was a lot of time, if there will be questions, those questions are going to be quite simple in nature, easy calculations but you should be knowing, what the procedures for Hardy Cross Methods are to be able to solve those problems. If a full problem is given, then the problem will be divided into lot of many parts like at least, it will carry 5 or 10 points out of let us say 50.

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## Home work problem

- For the network shown in the figure the head loss is given by . The values of  $r$  for each pipe, and the discharge into or out of various nodes are shown in the sketch. The discharges are in an arbitrary unit. Obtain the distribution of discharge in the network.



But I will try to give you easy questions where the solutions are not so complex. So, before we end, there is a homework problem. So, the homework problem is there is the network shown in this figure and the head loss is given by, I mean the head loss, you know, is like  $r Q$  square. The values of  $r$  for each pipe is given, here, here and the discharges into or out of the various nodes are shown.

So, 20 is entering here, so the things are given. The discharges are in arbitrary unit. So, obtain the distribution of the discharges in the network. So, you have to solve this question, see there are 2 loops; loop 1 and then loop 2. So, each of these loop must be satisfied, it is a long problem, it will require at least 30 minutes of your time to solve this but I think before you start the lecture on the viscous flow next week, it is good homework problem to solve.

So, I think this is the point where I will end the lecture on viscous pipe flows. Thank you so much for listening and next week, we are going to start viscous pipe flow continued by computational fluid dynamics and in the end, we would wind up the course with inviscid flow or we will derive the basic wave mechanics. Thank you so much for listening and I will see you next week.