Fluid Mechanics
Prof. Subashisa Dutta
Department of Civil Engineering
Indian Institute of Technology-Guwahati

Lec 32: Boundary Layer Approximation II

Good morning. Let us start today on boundary layer approximation, the second part. In the last class, we discussed about what is the utility of the boundary layers concept as well as also demonstrated different type of boundary layers. which we generally feel it not flow past an object. Also the mixing layers, the wake formations, jet formations we commonly encounters boundary layers and those boundary layers how we can solve it a part of a approximation solutions of Navier-Stokes equations. That is what is called boundary layer equations and then we look it for the laminar flow how we can solve it boundary layer equations to get the solutions in terms of boundary layer thickness and the flow Reynolds numbers as well as we try to look at what is the wall shear stress.

Today start with this part. These lectures are designed to look at present context is that we have a lot of computational fluid dynamics tools which we can solve the full Navier-Stokes equations. So boundary layers concept here is given as introductory levels to try to understand what is the boundary layers. But today we need not to go for the lot of approximations or mathematical techniques is necessary to do the boundary layer approximations.

We used to do it almost 70 to 80 years back when there was no the computational facilities what today we have. So looking that I will just going through this Senzel Cimbala books which giving a lot of illustrations about the boundary layers and partly we looking it FM White book but as I said it that we are looking it you should try to understand what is a boundary layer concept not going more details about the mathematical formulations, how we do the approximations for different conditions because that itself is in advanced level of the fluid mechanics. So here as a introductory levels we are talking about this. So today I talk about boundary layer equations which is is a approximations of the basic Navier-Stokes equations okay. That is what today with figures, with the sketches, with the approximations that what we will derive it.

Then I will talk about how boundary layers problems we solve it with a combinations of Euler equations and boundary layers equations and how we can solve the problems and one of the cases for the laminar flow that is what today we will discuss it how we can for a laminar boundary layers we can approximate it and we can get the solutions for the laminar boundary layers. Also we will discuss it what are the assumptions behind these boundary layers equations that is what we should know it. And the next class we may talk about the concept of the displacement thickness, the momentum thickness and the turbulent flat boundary layers. Let us just repeat it what we I discussed in the last class that whenever you take it very simple case is the flat plate okay. Let us see you have a flat plate okay.

Just you have a flat plate and you have a uniform the stream flow is coming uniform the stream flow that is what is coming it okay. It is simplified it as you consider its plate is moving or plate is stationary flow is moving that is the same things okay. So you have the plate flat plate and there is a uniform speed is coming with its velocity v and as you know it you will have a no slip boundary conditions just on this surface that is what the fluid behaviors okay that is what is the fluid behaviors okay. So you have a no slip boundary conditions the velocity particles at these points will be the 0 and you will have the velocity will be increasing still it comes to the the free stream velocity v. The same way if you go along the x, you can see that this is what is called the boundary layer thickness.

This is what is the boundary layer thickness, okay. That is what is representing as delta x. boundary layers thickness what you are getting it okay. So this boundary layers thickness as you see that when you have the flow Reynolds the Reynolds numbers in terms of characteristic length X that is what you have put it subscript here which is rho V x by mu. So this this is what not the d this is the x the distance from the from the point the origins to x distance is that.

So if you look at that the Reynolds numbers what we are getting it from the experiments what we found it that if Reynolds number x is less than 10 to the power 5 that is what 1 lakh. The flow remains in these stretches is laminar natures okay. This is what laminar natures. Then it will have a transitions. So there will be the small vortex presence and all.

Then you can have this fully turbulence behaviors what we will get it. and you can see this velocity profiles at these two points which will be the different that is what we got it from experimentally conducting in wind tunnels which used to conduct a series of experiments in many part of the Europe. So to establish this boundary layer approximations the understanding of what is the drag force, what is the screen frictions are working on a plate if you have a velocity v. That is a basic concept what we used for designing any automobile sectors, the spacecraft all we look at that part. So this is the unstable problem it was that time to try to look it as a boundary layers conduct a series of experiments from that they found it that Reynolds numbers in a characteristics length of x when you less than 10.

5 the flow is laminar and when Reynolds numbers greater than 3 into 10 to the power 6 is 3 millions okay then the flow will be the turbulent. That is what is I am emphasize is that you try to understand it when the flow go through a flat plate the very leading edge part you will have a laminar as soon as the Reynolds numbers 10 to the power 5 the flow becomes changes from laminar to transitionals where there will be laminar characteristics as well as the characteristics of the turbulence as we discuss in a pipe flow. Then you will have the turbulent flow as we can see it, it is what it happens after the Reynolds numbers more than and we can understand it how complexity will be there as you will have the laminar flow. So mostly we are looking at what will be the characteristics of laminar flow, what is the thickness of boundary layer thickness as I discussed that this is the thickness what we are defining which is having the velocity the u component close to the 0.99 of free stream velocities okay.

That is what is 99% of the free stream velocity to simplify the problems okay. To look it the locations where you will have a velocities in is a close to the 0.99% of you okay. But for engineering applications as we always do not know about much about transitional zones as well as the transitional zones are are responsible for acoustic noise or many other effects is coming because of transitional zone. We try to avoid the transitional zone by introducing a tripwire okay many of the structures if you look at that there they put the tripwires okay which reduce the transitional zones and make it a laminar to transition with a short span.

So it is does not allow to grow as you can see it. So that is what is my point it if you look at engineering point of view we try to avoid this transitional zones which causes enhance local mixing that is what is the turbulence behaviors get the difference that quickly lead to a turbulent boundary layers that is what the turbulent boundary layer which start from here. and this is the reasons where laminar boundary layers okay. Now let us go for simplifications as I said it that we are looking at introductory level of boundary layers because today's we have a tools of the CFD with us. So we do not go to many details about that but if you look it as you can see that and any CFD experiment we also can do it with a different velocity fields and the flat plate okay that is what is a flat plate with different Reynolds numbers that is what the experiment we can do in a CFD one of the examples are there in Sinzel Zimbala book you can just look at that as the Reynolds numbers increases okay.

So you can conceptually also understand it that at Reynolds numbers it increases so your boundary layers thickness is reduces that means it will be the more thinners maybe in order of few millimeters or few centimeters. You can do a CFD analysis or you can have a experiment to conduct it in a wind tunnels. If you have a velocity v put a plate and measure the velocity distributions and find out where you have a 0.99 v And that is what you will give is the thickness that is what is we can easily conduct in any wind tunnel lab to get it what is the boundary layer thickness with functions of Reynolds numbers that is what we can get it okay either wind tunnel or CFD. okay that is what is one of the examples is given in St.

Gilles Simbala book you can see it. The basic concept what we are getting it that as we are going for higher Reynolds numbers the thickness decreases or it becomes more the thinner the boundary layers that is the concept we have to understand it as well as that if you look at the boundary layers does not happen only the flow past a flat object or aerofoils. It also occurs it whenever you have a like jet or the wake or you have a even if you are mixing of two velocity zones. So there is a zone of v2 and v1 at the interfaces of that there will be a boundary layer formations that is what is will be there okay. So this is visible boundary layers we can always look it and see it but this is the two fluids are moving with a v1 and v2 velocity.

So at the interfaces we will have this boundary layer formations which is not visible to us okay. Which is not visible to us but we can see that the presence of same way the laminar and turbulent flow boundary layers formations will happen it. So we can use the boundary layer concept to understand it. So that what you can look it the boundary layers happens for a flow fast and a rigid object just down jet wake formations and the mixing layers that is my point to highlight it just try to look it boundary layer can be used for many of the problems okay. Now you come back to the how to derive this boundary layer equations okay.

So the first part is called the boundary layer layer coordinate system coordinate system that is what is just try to show it. This is a great simplifications but that is the strength of this boundary layer approach method is that let you have the aerofoils okay and you have the flow and you have the streamlines going like this, streamlines going like this. Here another streamlines going like this, another streamlines going this. This is what the arrow falls. This is what the arrow falls.

I am sketching the same figures. So if you look at that, this is the arrow falls. I have the streamlines. This is uniform streamlines that is what is coming to here okay. So from this if you can look it that if I am looking it how these boundary layers formations happens which will be the it is not like have a very much high thickness but just have to magnified it okay to tell it the boundary layer formations will happen like this okay.

as well as the boundary layer formations what is going to happen like this followed by you have a flow separations okay. So you will have flow separations okay those are interest for to know the flow separations how it happens it you can always go through this basic books of the fluid mechanics or you try to put a interest how the flow separations occurs it okay. So these are the reasons you have the laminar and the turbulent boundary condition then you have a flow separations and then you have the wake formations which we consider it for other cases. But now if you look it how these boundary layers coordinate systems are there. The basic idea comes it here that you define the coordinate here which will be the along these curve natures okay it is not be the and perpendicular to that is the y this is x okay.

So anywhere I take this x and the y they will have a Leucalli orthogonal. So x and y they will have a Leucalli orthogonal okay perpendicular to each others okay. This is not simply xy coordinate systems as we discussed for the Cartesian coordinate systems. In a boundary layer coordinate systems what we define it at each point this x and y will have a orthogonality each others. So that is the reasons at the localized coordinate systems which will have x point is like this So, that is a way we can transform this curve to a flat having this y is a perpendicular to here and x is moving this direction.

That is what this boundary layer coordinate systems try to understand it that is what basic way to transform any flow field in a curve like a airfoils we can transform it to the locally orthogonal coordinate system which is you call this boundary layer coordinate systems where the x and the y will have a perpendicular to each other okay. That is the local transformations we used to do it. Now if you look it since this flow where it happens it here the gravity force does not dominate it. So that is reasons always we can neglect this gravity force because there is no free force components okay. The inertia force is much much larger than the gravity force component and it acts on a same horizontal plane.

That is the reasons we neglect this gravity force component which quite justified it unless otherwise you have a big structures okay or you look at the proportionates of the inertia force and the gravity force which talks about flow proud numbers. Mostly this is the boundary layers which is the three-dimensional things but for the simplifications we have made it two-dimensional. okay. But if you look at boundary layer formations of wing of an aircraft, you can see that is a three-dimensional structures composite. But if you take it a cross sections, consider the Z is longer accepting the edge part.

So we can say that more or less you can simplified as a two-dimensional component. That is what is a two-dimensional Cartesian coordinate system which is I can say that boundary layers coordinate systems coordinate systems you have. and we are talking about laminar boundary layers. We can get some analytical solutions for that okay. So if you look at this component that these are the assumptions behind that.

Also we have talked about the steady flow. Basically we do not look at the unsteady part because we just have to look at the drag force and components or maximum drag force, maximum screen frictions what is coming it. So looking that we look it simplified our basic equations which is again I start from Navier-Stokes equations quite intentionally that please remember it that we are not doing much difference except approximating these equations okay that is the reasons I always introduced to a Navier-Stokes equations that we always try to understand that we are doing just a approximations okay what we did it in last few classes. The same we are just doing the approximations with certain degree of assumptions okay for a particular problems okay that is what we are doing it. So the basic equations if it is a steady okay so that means we do not have a local accelerations components.

So you will have the component of rho as we bacterial forms is equal to minus great P if you remove the gravity components gravity is neglected okay. if gravity is neglected the viscous force components will be the mu del square p. That is what the components what we have a Navier-Stokes equations just removing the part of local accelerations component and the gravity component. So we will have a equations in this form. what we do it order of the magnitude of analysis okay just order of magnitude analysis which is many of the times we look it like for examples as you have seen it now we have this adaptive part of accelerations component, we have the pressure gradient component, we have the viscous force components.

We always try to look it if I expand these terms okay if I expand these terms okay which as I did it earlier it will have a big expressions. From that which are the terms having what is the order? That means if I normalize okay, we look it to normalize the components okay, if you non-dimensionalized okay. that means how you do this non-dimensionalization okay. More details I am coming back that. So we try to find out the characteristics velocity like v okay.

We normalize as v star by a stream velocities okay. This is the vectors okay. We So this is what we normalize it with a characteristic velocity this is what please remember this is the characteristic velocity okay I can make it a small u. Same way the x star the non-dimensionalized the space can define x by L. So all the terms of the pressures, the velocity and the space coordinates we always can make a non-dimensionalized and once you know these non-dimensional forms v star okay it is a vector form.

So you will get a non-dimensional equations. We try to look it in a when we do this dimensional analysis to look it what is the functions of as function of Reynolds numbers and the Euler s numbers okay that is what we got it. Now if you look at the same concept of non-dimensionalization and look at how we can derive bounded layer equations that is what is our idea. Now taking that non-dimensional things we are again looking it that same concept we will hold it to try to look it at the component wise that what is the order of each terms of the Navier-Stokes equations okay that is what we are talking it. So that means if you look at the magnitudes of order of magnitude analysis is for a boundary layer flow problems okay, u the velocity component in the x directions will have a this is the symbol of order the same order of u not the exactly equal that is what is to remember it. The u is not equal to the free stream velocity it say they will have a same order they will have a u the velocity of x directions velocity component will have the same order of the free stream velocities okay.

Same way if you look at p, p at the infinity will have a same order of rho u square okay. rho u squares both will be the difference of the pressures okay at the infinity at the pressure point that difference will have the same order of rho u squares okay. Same way delta by delta x will have a order of 1 by L and delta by delta y will have a order of 1 by delta which is the boundary layer thickness. So please try to look at these techniques which is really we use for many other studies in social science or look at other problems to understand the order of each equations. To do that like for these boundary layers you can know it that u directions velocity the x component of u scalar component that will be at the same order of the free stream velocity.

The pressure difference that will have a order of rho u squares you can just substitute the dimensions you check it del by del x will be the 1 by L del by del y because y dimensions we have a maximum is 1 by delta that is boundary layer thickness. So del by del y is will have a order of 1 by delta. So this is what is giving us the order not the equals that is what again repeating this is the order the same order that means if this is a something like the velocity l t is in a meters. So your del by del x will be the meters. If a delta as I say that it is in a order of millimeters.

So it will be in order of millimeters. That is the understanding. That is the understanding is that if in terms of not the unit I am talking about in terms of meters 5 meter, 10 meter, 20 meters your l x will be also varies on that order. similar way if you look at the millimeters the boundary layer thickness will be in terms of millimeters 5 mm, 6 mm, 10 mm. So your y will have the thickness of the y that is what it will be in terms of only this boundary layer thickness. So you have to try to understand it how we are matching the orders because that information is necessary to simplify this Navier-Stokes equations.

First as we have the component in boundary layers, so this is x, this is u, this is y direction, this is your thickness and this is the v, the velocity components as we are looking at the thickness like this. Now if you look it because it is a two-dimensional flow, two-dimensional and incompressible. Again, I am writing the basic equation as I put it many times. That is what you have to remember it. As a two-dimensional, you can have a practice to write this part.

it is too easy okay. So it is too easy to have a do not have to remember the intermediate steps you remember the basic equations is the conservations of mass, the continuity equations in incompressible flow is a velocity divergence that is what if you expand it I will get these terms. Now I am looking it that what would be the order of this velocity component am just substituting order of these equations because that should be equal they will be the same. So order of not I am substituting the value please remember it I am just order okay order of this magnitudes what I am such that if you look at that delta v by delta x what will be the order 1 by L u will be capital U that is what is order 1 by L y capital U that is what having a order of we do not know about the order of small v but we know the order of y will be the boundary layer thickness. So that means the order of the velocity in the y directions will have a order of u delta by L just substituting.

This is order equating. So please do not have as equal signatures. So that means order of the second velocity component will have a free stream velocity order into boundary layer thickness divided by the L. divide by the l. As you know it as I try to emphasize that always this is too low okay. This ratio is always much much lesser than the one values okay.

Maybe 0.001, 0.005 like this. Maybe 1 percent, 2 percent, less than 1 percent. So if it is that And if you look at this order that means this value is much much lesser than 1. So v is u order that means the velocity v will be much much lesser than the u value. This is just a order analysis. is just a order analysis to know it the velocity component which is perpendicular okay to these boundary layer formations okay in the y directions that will be much much lesser than the velocity in the x direction.

That is the order analysis that is what order analysis from by just analyzing the incompressible continuity equations. Now if I go for the next levels to know it order of other components okay that is what is the approximations what we look it. That what we got it that in non-dimensional form as I said it is a non-dimensional x by again I am just writing it you to think it. Everything is I am putting a non-dimensional forms why here the boundary layer thickness u starts I am putting the u by free stream velocity the v starts I am putting it as I derived from now continuity equations that what will be the L by just u delta just you looking the non-dimensional form of the pressures will be the p minus p at the infinity pressure difference with a from a reference by rho u square just I repeat it to try to understand it we do a non-dimensional forms.

because each one having same order is a 1. This is having word order 1 because they are the order 1. This is having the all these non-dimensional forms having the order 1. That is the strength we do it a nonlinear Navier-Stokes equations we are trying to look it which are the terms are dominating which are terms are can be neglected that is what the strategy that if Each non-dimensional terms we have the order of 1 but some of the parameters are less than the value. So we can understand it those are the parameters we can drop it. We can so that means the Navier-Stokes equations we can simplify it by dropping some of the parameters which are some of the terms which are not dominating.

are having a very less order values like these are the 1 1 values if you are something is 0.0001 should I consider not if a somebody it is at a 0.0000005 I should not consider that. So that way we do a order analysis to drop the non-linear or linear terms on Navier-Stokes equation for simplifying for this boundary layer flow. Now if you look it, we are using these terms as I rewritten on the board.

If I substitute in again in y component of Navier-Stokes equation, again we are coming y component of an S, Navier-Stokes equations. That is what will you commit as you have neglected. So just I can write it u dv by dx because we are talking about y components. That is what you can have. this is convective acceleration terms okay and rho we have taken to this side.

Then kinematic viscosities as we are putting it rho v rho s square plus rho v rho s square. See this is the velocity in the y directions okay that is what it you are looking at velocity in the y directions. Now I will be substituting all these u term as a non-dimensional form that is what would be u star into the free stream. Same way I can apply this del by del x as del by del star into L.

So each component I can just put it as a non-dimensional forms okay. So this is the non-dimensional variables I will apply it each one del pi by del x 1 by rho, rho is a constants. So I have the velocities I am just applying this component for each one del by del x is 1 by L del by del y is in terms of your boundary layer thickness just you substitute this and in a non-dimensional forms if you rearrange it then it is a very interesting equations comes okay. This is the non-dimensional form of the y momentum equations. So you can see that this is non-dimensional starts indicates the non-dimensional form the u starts indicate the non-dimensional forms.

So non-dimensional forms of the v stars. So you have a components here, you have a components here, you have a components here. Now if you look it this is what we should inspect each terms now. these all are non-dimensional order of 1, 1 is to 1, this is 1, 1 is to 1, this is 1, okay. So all are non-dimensionals but there are three terms are coming it here, okay. With the multiplications of non-dimensional parameters, okay, the order of 1, this is what is the Reynolds numbers, 1 by Reynolds numbers, that is what is we are getting it in terms of is the Reynolds number this is the geometry okay that is the length of boundary layers and thickness of boundary layers that is what is the geometry of these boundary layers.

Now if you look at each terms that as we do the boundary layer thickness we were talking about we are talking about very higher Reynolds numbers okay is close to the 1 is to 10 to the power 5. So u1 by Reynolds numbers you can see that these values are very very significantly less orders. So the laminar critical Reynolds number is 1 is to 10 to the power 5. So if you look at 1 by Reynolds number that is what is representing it.

So that means these the terms they do not dominate it. These the terms they do not dominate it. and these are all these first order terms that means this part has to be close to the 0 because we know it the sigma by L is a very less values. The reverse of that if I am making the squares that means the square of that values multiplied something should close to the one order the same order that is what only will be possible see it only if the order of dp by dy close to the 0 okay. in a non-dimensional forms it should be close to the 0 otherwise it cannot close to the one order because as I said it these are a very very small numbers as one by Reynolds numbers are a multiplications are there these are the small so that we can neglect these two terms. But what we can do it we can inspect these two that since there is a multiplications L by the delta which is boundary layer thickness that what has to be since it is a larger numbers and we do the square that means multiply it something that should also be a very very close to the 0.

That means the normal pressure gradient across the boundary layer should equal to the 0. So del P by del Y should be close to the 0 otherwise the boundary conditions will not this is what is showing it from this when you look at the order analysis of this part that is what interpreting it that is if I draw it that means if I have a point here I have the boundary layers sigma x is a boundary layers okay. boundary layers and this is what the wall or the surface. So if you take a point P x1 and the x2 if you have a pressure P1 so as they does not vary along these y directions everywhere should be P1 value. okay every point should have a P1 values similar way the P2 the pressure should be happen in all the locations that means it is indicating it that the pressure across a boundary layers nearly constants that is what it happens it is a pressure across the boundary layers really constants Due to the thin boundary layer the streamlines within the boundary layers have a very negligible curvatures okay.

The scale of the boundary layers no significant pressure gradient across the boundary layer as the corp streamlines requires pressure gradients we are not pressure at the outer edge of the wall as same as the that is what I saying it if you have a outer edge and this they will have the same pressure gradient. Now if you look it look at the x momentum equations the same way if I look at the x momentum equations okay x momentum component of Navier-Stokes equations along the x directions okay And that is what is component as we have derived it and you put it substitutes each non-dimensional terms okay. Non-dimensional terms you just substitute it and if you rearrange it with a multiplications of L by U squares you will get again the same way as we got it for slightly different forms, we are getting the same way of non-dimensional form of Navier-Stokes equations in the x direction. As explaining the same things, this is all having order 1. Now if you look it, always you can think it, these terms supposed to have a less because it is a 1 by the Reynolds number, so this will be much lesser.

this is also will be much lesser values. But try to understand it if I am going to drop this part this is we have to have a physical understanding of that. If I dropping this part because this the part is showing the viscosity. So viscosity we cannot neglect it in the boundary layers. So we cannot drop this part that means what it indicates is that this value should be equal to 1 order of 1. Otherwise if I drop this part again I am coming back to the Euler equations format which is not justifiable because Euler equation does not satisfy in the boundary layers.

So I need to have a viscous terms when this order will be justified it if these multiplications components will be closer to the order 1 closer to the order 1 that is what he is saying. If you have a that and if you just rearrange these terms you will get it the ratio between the boundary layer thickness of i L is a functions of square root of Reynolds numbers. So that means from dimensional analysis we are getting it that these terms giving a relationship between thickness and the Reynolds numbers. But now since these terms we made it 0 So only we need it now and we already prove it dP by dy is 0. So that means we are looking P is only function of x substituting this that is what is y momentum equations sorry that is x momentum equations which we are getting it u del u by del x v del u by del y is equal to 1 by rho dp by dx because p is only function of x that is what we got it okay plus the kinematic viscosity rho u square by del y.

This is what our boundary layer equations still we have a pressure components. still we have the pressure components. What we try to do it that since these pressure gradients okay that means if I consider a simple plate as transformations we look it and this is what the boundary layers Beyond that the velocity field will be the close to the constants. This is the streamlines. What we can do it? We can consider a relationship with u, the free stream velocity field with the p.

How we can do it? Along the streamlines, we can use Bernoulli s equations. That is what we can do it. Along the streamline, we can apply the Bernoulli s equations. we can apply the Bernoulli s equations because there is no z component here we are assuming there is no gravity force components here. So we have a p by rho half u square along the streamlines if I put it that is what is constant and if I just differentiate it with respect to x it will give me the terms 1 by rho dp by dx is equal to the minus u is a free stream velocity du by dx.

Just substituting that I will be getting it a boundary layer equations. I am getting the boundary layer equations which I can rewrite here that the two equations we are getting it one is continuity equations two dimensional continuity equations. The second is I am getting a boundary layer equations is dou by dx v dy. So with help of Bernoulli s equations applying along the streamlines which is not having affected by these boundary layers. Thus when you apply it you are getting it the velocity u free stream velocity plus del square y.

okay this is not sorry this is the kinematic viscosity terms. So what you are getting it okay so that means we instead of solving the Navier-Stokes equations we are trying to solve boundary layer equations okay boundary layer. Today it is not a very difficult to solve these boundary layers there are a series of numerical techniques are available. you can solve these equations. It is not so big things because today lot of tools are numerical tools are there.

It is not talking about concept of almost 70 to 80 years back or almost 100 years back. So, now today we have a lot of numerical techniques. You can easily solve these equations as compared to Navier-Stokes equation solvers. So, we can easily get the numerical solutions of these, what is the boundary conditions? That we should try to understand it. What are the boundary conditions are applicable with that? Those things you can try to understand it and more details I am not going it because this is the equation in the parabolic nature which is easy to understand.

solve it as numerically as compared to the Navier-Stokes equations which is elliptical natures. Those things are not going details but try to understand it with order of magnitudes analysis. We can make it the equations which is the parabolic natures and we can easily with a new numerical techniques like many numerical techniques today we have. We can solve these equations to find out how the u varies okay how the velocities varies, how the pressure gradients varies all these things we can get it within the boundary layers okay. We remember it is very very thin layers but that what is matters to lot when you talk about designing automobiles any things like aircrafts and all the things.

So now let me I go for how do you solve it. the boundary layer problems and what are the assumptions we have. As I say that it is a combinations of Euler equation solvers and the boundary layer equations. So what we try to do it like for examples I am looking at the solutions around a big towers okay let me I talk about a concept of okay or coming back to this. very simple way that I have a concept of a simple structures like I am looking for a simple structures like I have the aerofoils. so when you have a consider airfoils near to the airfoils you will have a boundary layers beyond that you have the outer layers.

So what you do it first calculate this u field as we are getting it okay what is the u variations with x that what you use the Euler equations assuming it that there is no viscosity. So as it this surface you consider as a aerofoils consider the uniform flow you can solve it to get it the stream flow then you can get it how the ux varying at the outer layers that is what you can do it. Assuming a thin boundary layers that is the big assumptions we have that boundary layer is very thin which is not making a any changes in the boundary layer domains. That means whatever the structures we have considering that is closely approximated with a boundary layers.

So that is the reasons we should have a thin boundary layers. After solving this ux values using the Euler equations that is what you do it Euler equations then we go to solve the boundary layer equations what I just derived it. Once you solve this boundary layer equations you get this component of u, component of v, delta x, the shear stress all these components you try to compute from this. solutions of boundary layer equations. Once you get these quantities like boundary layer thickness, the shear stress, wall shear stress and the pressure variations if you look at that then you check it whether your boundary layers is a thin or not okay that is if it not thin again you have to look it as we will discuss it in next class how we are try to modify it. So the step wise you move it from calculate the ux the outer velocity fields assuming that region outside approximately no viscosity no irrotational.

So we can use a Euler equations then we transform culminated to get the ux value assume this is the thin again you solve boundary layer equations with a boundary conditions of no slip as always boundary condition u equal to the ux and u is remains a starting at all these locations that is what we look at a boundary conditions power. boundary layer equations solving that we proceeds to go for calculating these things that is as you seeing that you calculate boundary layer thickness shear stress along this total skin friction drag that is what is engineering point of view we are more interested to know it what is the friction drags happens it and then you go for verify that boundary layer approximations is it appropriate for us as we consider the boundary layer is thin otherwise apprehension is not justified it. So but look at that what boundary layers problems when you are talking about okay it is the wall if you have the curvatures that what you look it and you have the thickness and us the free stream velocity x directions are moving it always you look at the dimensions okay. For example, if you have a thickness Reynolds number is thousands you will have a delta by L will be 3 percent okay 0.

3 but 10 thousands you will have a 1 percent only. So, you have to look at that how the boundary layers thickness you are getting it and that what assume the zero pressure gradient we will discuss more this okay which is quite accepted for that. So, that is what it happens it if the wall curvature is smaller magnitudes then this boundary layer thickness okay, smaller magnitudes that that is the centripetal expressions affects the streamline curvature cannot be ignore it. The physically the boundary layer is not thin enough for approach masses to be appropriate when boundary layer thickness is not much much lesser than the r values. So that is what we try to do it and second thing as I think in very beginning I discussed that Reynolds number is if it is too high it is not a laminar boundary layers.

So we will have the transitions towards the turbulence we cannot use that part. So we have to we are not going this is out of scope of this we are not going for turbulent boundary layers how to do the approximation only experimentally what is the characteristics of the turbulent boundary layer that what we will discuss it. As also if this flow separations occurs it is which from experiment from the wind tunnel test we can know it. there also we cannot apply this boundary layer approximation is no longer appropriate in separated flow regions. That is what is having a reverse flow.

So we do not have a parabolic nature of the boundary layers equation is lost. So these are the assumptions of boundary layers equations what we consider it. So with this let today I finished this class. Thank you. Bye.