Solution:
Chainage of apex V = 1190 m, Deflection angle D = 36°, Radius R = 300 m, Peg interval = 30 m.

Length of tangent = R tan $\Delta/2$
= 300 tan 36/2 = 97.48 m

Chainage of T_1 = 1190 – 97.48 = 1092.52 m = 36 chains of 30 m + 12.52 m
C_1 = 30 – 12.52 = 17.48 m, and C_2 = 30

Length of curve = $R\Delta (\pi/180)$
= 300 * 36 (π/180) = 188.50 m
$C_3 = C_4 = C_5 = C_6$ = 30 m
$C_n = C_7$ = 188.5–17.48– 30*5 = 21.02 m
Chainage of T2 = 1092.52 + 188.50 = 1281.02 m
Ordinates are $O_1 = C^2_1/2R = (17.48)^2 /2 *300$ = 0.51 m
$O_2 = C_2 (C_2 + C_1)/ 2R$ = 30 (30 + 17)/ 2*300 = 2.37 m
$O_3 = O_4 = O_5 = O_6 = 30^2/ 300$ = 3.0 m
O_7 = 21.02 (21.02 + 30)/ 2*300 = 1.79 m

Example 2.10:
Tabulate the data needed to set out a circular curve of radius 600 m by a theodolite and tape to connect two straights having a deflection angle of 18^0 24. The chainage of the PI is 2140.00 m and a normal chord length of 20 m is to used.

Solution:
Given $\Delta = 18^0 24'$, R = 600 m, normal chord c = 20m, Chainage of PI = 2140.00 m = 21+40.00
Tangent distance, T= R tan (Δ /2) = 600 tan $9^0 12''$= 97.20 m
T=97.20 m = 0 + 97.20
Chainage of PC = 2042.80 m = 20 + 42.80
Next full station on the curve (@20-m intervals) = 20 + 60.00
Therefore, length of initial subchord c = 20 + 60.00 – 20 + 42.80 = 17.20 m
Length of curve = R Δ (π /180) = 600*18.24 (π/180) = 192.68 m
Chainage of PC = 2042.80 m= 20 + 42.80
Length of curve L=192.68 m = 1 + 92.68
Chainage of PT= 2235.48 m = 22 + 35.48
Last full station on the curve (@ 20-m intervals) = 22 + 20.00
Therefore, length of final subchord C_2 = 22 + 35.48 – 22 + 20.00 = 15.48 m
The curve has an initial subchord C_1 of 17.20 m
Eight normal chords of 20 m and a final subchord of 15.48 m
Given the cord length C, the deflection angle from station to station on the is given by Δ =1718.873c / R minutes
Hence, δ_1 =1718.873 C_1 / R = 1718.873*17.20 / 600 = 49.27'
δ= 1718.873 C / R = 1718.873*20 / 600 =57.30'
δ_2 =1718.873 C_2 /R=1718.873*15.48/600=44.35'
Now, the chainage of each station and the cumulative deflection angles from the back tangent to each station on the curve is computed and tabulated below.

Chainage	Chord (m)	Deflection Angle	Total deflection angle	Total def. angle with 20" theodolite
PC=20+42.80	0	0	0	0
20+60.00	17.20	49.27'	$0^0$49.27'	$0^0$49'20"
+80.00	20.00	57.30'	$1^0$46.57'	$1^0$46'40"
21+00.00	20.00	57.30'	$2^0$43.87'	$2^0$44'00"
+20.00	20.00	57.30'	$3^0$41.17'	$3^0$41'20"
+40.00	20.00	57.30'	$4^0$38.47'	$4^0$38'20"
+60.00	20.00	57.30'	$5^0$35.77'	$5^0$35'40"
+80.00	20.00	57.30'	$6^0$33.07'	$6^0$33'00"
22+00.00	20.00	57.30'	$7^0$30.37'	$7^0$30'20"
+20.00	20.00	57.30'	$8^0$27.67'	$8^0$27'40"
+35.48	15.48	44.35'	$9^0$12.00'	$9^0$12'00"= Δ /2 (check)

Example 2.11:
Tabulate the necessary data for setting out a simple circular curve. The angle of intersection is 144^0, chainage of point of intersection is 1390 m, radius of the curve is 50 m. The curve is to be set out by offsets from chords produced with pegs at every 10 m of through chainage.

Solution:
Chainage at the point of intersection= 1390 m, Angle of intersection= 144^0, Radius of curve = 50 m.
Deflection angle (Δ)= 180^0-144^0= 36^0
Tangent Length T= R tan Δ /2 = 50 tan 36^0/2 = 16.25 m
Length of the curve (*l*) = π R Δ/ 180= 50 * 36 π/ 180 = 31.42 m
Chainage at point of curve (T_1) = Chainage at the point of intersection – tangent length
= 1390.00 – 16.25 = 1373.75 m
Chainage at point of tangency (T_2) = Chainage at point of curve (T_1) + Length of curve
= 1373.75 + 31.42 = 1405.17 m
Length of initial sub chord (C_1) = 1380 -1373.75 = 6.25 m
Length of final sub chord (C_n) = 1405.17 - 1400.00 = 5.17 m
Length of normal chord (C) = 10 m
No. of normal chords = (1400-1380) / 10 = 2
Total no of chords = 1 + 2 +1 = 4

Offset for the initial sub-chord O_1= C_1^2 / 2R = 6.25^2 / 2 * 50 = 0.39 m
Offset for the second chord O_2 = C (C + C_1) / 2R = 10 (10 + 6.25) / 100 = 1.625 m
Offset for the third chord O_3 = C_2 / R = 102 / 50 = 2.0 m
Offset for the fourth chord O_4 = C_4 (C + C_4) / 2R = 5.17 (10 + 5.17) / 100 = 0.78 m

Example 2.12:
Two tangents intersect at chainage 1190 m, with 36^0 deflection angle. Calculate all the required data for setting out a simple circular curve of 60 m radius by the deflection angle method. Take peg interval as 10 m.

Solution:
Tangent Length T= R tan Δ /2 = 60 tan 36 / 2 = 19.50 m
Length of the curve (*l*) = π R Δ/ 180 = 60 * 36 π / 180 = 37.70 m
Chainage at point of curve (T_C) = Chainage at the point of intersection – tangent length
= 1190.00 – 19.50 = 1170.50 m
Chainage at point of tangency (T_T) = Chainage at point of curve (T_C) + Length of curve = 1170.50 + 37.70 = 1208.20 m

Length of initial sub chord (C_1) = 1180 -1170.50 = 9.5 m
Length of final sub chord (C_n) = 1208.20 - 1200 = 8.2 m
Length of normal chord (C) = 10 m (peg interval)
No. of normal chords = 1200 – 1180 / 10 = 2
Total no of chords = 1 + 2 + 1 = 4
$\delta_1 = 90\ C_1 / (\pi\ R) = 90 * 9.5 / 60\ \pi = 4^0\ 32'\ 9.3''$
$\delta_2 = \delta_3 = 90 * 10\ /\ 60\pi = 4^0\ 46'\ 28.73''$
$\delta_4 = 90 * C_4\ /\ 60\pi = 3^0\ 54'\ 54.76''$

Point	Chainage (m)	Chord Length (m)	Tangential angle	Deflection angle	Actual Theodolite reading
T_C	1170.50		$0^0\ 00'\ 00''$	$0^0\ 00'\ 00''$	$0^0\ 00'\ 00''$
P_1	1180	9.5	$\delta_1 = 4^0\ 32'\ 9.3''$	$\Delta_1 = \delta_1 = 4^0\ 32'\ 9.3''$	$4^0\ 32'\ 00''$
P_2	1190	10	$\delta_2 = 4^0\ 46'\ 28.73''$	$\Delta_2 = \Delta_1 + \delta_2 = 9^0\ 18'\ 38.03''$	$9^0\ 18'\ 40''$
P_3	1200	10	$\delta_3 = 4^0\ 46'\ 28.73''$	$\Delta_3 = \Delta_2 + \delta_3 = 14^0\ 5'\ 6.76''$	$14^0\ 05'\ 00''$
T_T	1208.20	8.2	$\delta_4 = 3^0\ 54'\ 54.76''$	$\Delta_4 = \Delta_3 + \delta_4 = 18^0\ 00'\ 1.52''$	$18^0\ 00'\ 00''$

Check **:** $\Delta_4 = \Delta/2 = 36/\ 2 = 18^0\ 00'\ 00''$

Example 2.13:
Two straights intersect at a chainage of 1764 m with a deflection angle of 32^0, which are to be joined by a 5^0 curve. Work out the data required to set out a curve by the deflection angle method. Take length of chain as 30 m, peg interval at 30 m, and least count of theodolite as 20".

Solution:
For a 30 m chain length, R = 1719 / Degree of curve of Degree = 343.8 m
Tangent Length T= R tan Δ /2 = 343.8 tan 32 / 2 = 98.58 m
Length of the curve (l) = π R Δ/ 180
= 343.8 * 32 π/ 180 = 192.01 m
Chainage at point of curve (T_C) = Chainage at the point of intersection – tangent length
= 1764 – 98.58 = 1665.42 m
Chainage at point of tangency (T_T) = Chainage at point of curve (T_C) + Length of curve
= 1665.42 + 192.01 = 1857.43 m
Length of initial sub chord (C_1) = 1680 – 1665.42= 14.58 m
Length of final sub chord (C_n) = 1857.43 – 1830.00 = 27.43 m
Length of normal chord (C) = 30 m (peg interval)
5 30 No. of normal chords = (1830 – 1680) / 30 = 5
Total no of chords = 1 + 5 +1 = 7
$\delta_1 = 90\ C_1 / (\pi\ R) = 90 * 14.58 / 343.8\ \pi = 1^0\ 12'\ 53.68''$
$\delta_2 = \delta_3 = \ldots\ldots\ldots\ \delta_6 = 90 * 30\ /\ 343.8\pi = 2^0\ 29'\ 59.34''$
$\delta_7 = 90 * C_7\ /\ 343.8\pi = 2^0\ 17'\ 8.39''$

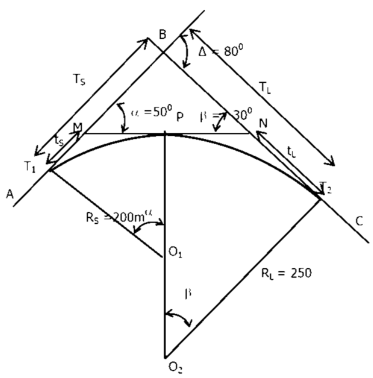
Point	Chainage (m)	Chord Length (m)	Tangential angle	Deflection angle	Actual Theodolite reading
T_C	1165.43		$0^0\ 00'\ 00''$	$0^0\ 00'\ 00''$	$0^0\ 00'\ 00''$
P_1	1680	14.58	$\delta_1 = 1^0\ 12'\ 53.68''$	$\Delta_1 = \delta_1 = 1^0\ 12'\ 53.68''$	$1^0\ 13'\ 00''$
P_2	1710	30	$\delta_2 = 2^0\ 29'\ 59.34''$	$\Delta_2 = \Delta_1 + \delta_2 = 3^0\ 42'\ 53.02''$	$3^0\ 43'\ 00''$
P_3	1740	30	$\delta_3 = 2^0\ 29'\ 59.34''$	$\Delta_3 = \Delta_2 + \delta_3 = 6^0\ 12'\ 52.36''$	$6^0\ 13'\ 00''$
P_4	1770	30	$\delta_4 = 2^0\ 29'\ 59.34''$	$\Delta_4 = \Delta_3 + \delta_4 = 8^0\ 42'\ 51.7''$	$8^0\ 43'\ 00''$
P_5	1800	30	$\delta_5 = 2^0\ 29'\ 59.34''$	$\Delta_5 = \Delta_4 + \delta_5 = 11^0\ 12'\ 51.04''$	$11^0\ 13'\ 00''$

P_6	1830	30	$\delta_6 = 2^0\ 29'\ 59.34''$	$\Delta_6=\Delta_5+\delta_6= 13^0\ 42'\ 50.38''$	$13^0\ 43'\ 00''$
T_T	1857.43	27.43	$\delta_7 = 2^0\ 17'\ 8.39''$	$\Delta_7=\Delta_6+\delta_7=15^0\ 59'\ 58.77''$	$16^0\ 00'\ 00''$

Check : $\Delta_7 = \Delta/2 = 32/2 = 16^0\ 00'\ 00''$

Example 2.14:
A right hand compound curve, with radius of the first arc as 200 m and the radius of the second arc as 250 m, has a total deflection angle is 80^0. If the chainage of the point of intersection is 1504.80 m and the deflection angle of the first arc is 50^0, determine the chainages of the starting point, the point of compound curve and the point of tangency.

Solution:



Given- R_S = 200 m, R_L = 250 m, $\Delta = 80^0$, chainage of the intersection point = 1504.80 m, and ∠BMN = $\alpha = 50^0$.
Let, BA and BC be the back tangent and forward tangent.
$\Delta = \alpha + \beta = 80^0$
Deflection angle of the second arc ∠BNM = $\beta = \Delta - \alpha = 80^0 - 50^0 = 30^0$
$MP = MT_1 = t_S = R_S \tan \alpha/2 = 200 \tan 50/2 = 93.26$ m
$NP = NT_2 = t_L = R_L \tan \beta/2 = 200 \tan 30/2 = 67.00$ m
MN = MP + NPN = 93.26 + 67.00 = 160.26 m
In triangle BMN,
∠BMN + ∠BNM + ∠MBN = 1800
∠MBN = 180^0 – ∠BMN – ∠BNM = $180^0 - 50^0 - 30^0 = 100^0$
Applying sine rule in triangle BMN,
(BN / sin α) = MN / sin MBN
BN = MN sin α / sin MBN
= 160.26 sin 50 / sin 100
BN = 124.66 m
(BM / sin β) = MN / sin MBN
BM = MN sin β / sin MBN
= 160.26 sin 50 / sin 100
BM = 81.23 m
Tangent length $BT_1 = (T_S) = BM + MT_1 = 81.23 + 93.26 = 174.49$ m
Tangent length $BT_2 = (T_L) = BN + NT_2 = 124.66 + 67.00 = 191.66$ m
Length of the first curve = $R_S\ \alpha\ (\pi / 180) = 200 * 50\ (\pi / 180) = 174.53$ m
Length of the Second curve = $R_L\ \beta\ (\pi / 180) = 200 * 30\ (\pi / 180) = 130.90$ m
Chainage at point of curve (T_1) = Chainage at the point of intersection – tangent length (BT_1)
= 1504.80 – 174.53 = 1330.27 m.

Chainage at point of PCC = chainage at the point of curve + length of first curve
= 1330.27 + 174.53 = 1504.8 m
Chainage at point of tangency (T_2) = chainage at the point of PCC + length of second curve
= 1504.8 + 130.90 = 1635.7 m

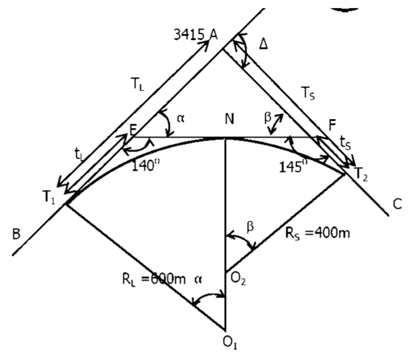
Example 2.15:
Two straights AC and BC are intersected by a line MN. The angles AMN and MNB are 150° and 160° respectively. The radius of the first curve is 650 m and that of the second curve is 450 m of a compound curve. If the chainage of point of intersection C is 4756 m, find the chainage of the tangent points and the point of compound curvature.

Solution:
$\Delta_1 = 180 - 150 = 30°$
$\Delta_2 = 180 - 160 = 20°$
$\Delta = \Delta_1 + \Delta_2 = 30 + 20 = 50°$
$t_1 = T_1M = 650 \tan 30/2 = 174.17$ m
Now, $TL_1 = T1M + MC$
$= R_1 \tan \Delta_1/2 + [R_1 \tan \Delta_1/2 + R_2 \tan\Delta_2/2] (\sin\Delta_2/\sin\Delta)$

$TL_1 = 36 \tan \Delta_1/2 + [36\ 2 \tan \Delta_1/2 + 48 \tan (84.5-\Delta_1)/2\ ] [\sin (84.5-\Delta_1)/ \sin 84.5]$
$38.98 = 36 \tan \Delta_1/2 + 1.0046251 \sin (84.5 - \Delta_1)\ [36 \tan\Delta_1/2 + 48 \tan (84.5-\Delta_1)/2]$
$F(\Delta_1) = 36 \tan \Delta_1\ 2 + 1.0046251 \sin (84.5 - \Delta_1) [36 \tan\Delta_1/2 + 48 \tan (84.5-\Delta_1)/2] - 38.98$
Solving it by trial and error method, when $\Delta_1 = 30°$,
$F(\Delta_1) = -0.0954222$
when $\Delta_1 = 32°$, $F(\Delta_1) = -0.123610$.
If $\Delta_1 = 29°$, $F(\Delta_1) = -0.08106$
If $\Delta_1 = 28°$, $F(\Delta_1) = -0.0665$
If $\Delta_1 = 25°$, $F(\Delta_1) = -0.02419195$
If $\Delta_1 = 23°$, $F(\Delta_1) = 0.008600$
say $\Delta_1 = 23.5°$ for which $F(\Delta_1) = 0.000914 \approx 0$

Thus, the solution is-
$\Delta_1 = 23.5°$, $\Delta_2 = 84.5 - 23.5 = 61°$
Arc length of first curve = 36 * 23.5 (π/180) = 14.765 chains of 30 m length
Chainage of point of junction of the two curves (C) = 30.5 + 14.765 = 45.265 chains of 30 m length
Length of second curve = 48 * 61 (π/180) = 51.103 chains of 30 m length
Chainage of last tangent point (T_2) = 45.265 + 51.103 = 96.363 chains of 30 m length
For first curve: Length of first sub-chord = 31 – 30.5 = 0.5 chains of 30 m length

Example 2.16:
Two straights BA and AC are intersected by a line EF so that angles BEF and EFC are 140^0 and 145^0, respectively. The radius of the first curve is 600 m and the second curve is 400 m. If the chainages of the intersection point A is 3415 m, compute the chainages of the tangent points and the point of compound curvature.



Solution:

In above Figure, R_L = 600 m, R_S = 400m, ∠BEF = 140^0, ∠EFC = 140^0, Chainage of the intersection point A = 3415 m.

∠AEF = α = 180^0 - ∠BEF =180^0 - 140^0 = 40^0

∠AFE = β = 180^0 - ∠EFC =180^0 - 145^0 = 35^0

ET_1 = EN = t_t = R_L tan α /2 = 600 tan 40/2 = 218.40 m

FT_2 = FN = t_s = R_S tan β /2 = 400 tan 35/2 = 126.12 m

EF = EN + FN = 218.40 +126.12 = 344.52 m

Consider, Δ AEF,

∠AEF + ∠AFE + ∠EAF = 180^0

∠EAF = 180^0 – ∠AEF – ∠AFE = 180^0 – 40^0 – 35^0 = 105^0

Applying sine rule to the Δ AEF,

(AE / sin β) = (EF/ sin EAF)

AE = EF (sin β / sin EAF)

AE = 344.52 (sin 35/ sin 105) = 204.60 m

Similarly,

(AF / sin α) = (EF/ sin EAF)

AF = EF (sin α/ sin EAF) = 344.52 (sin 40/ sin 105) = 229.27 m

Tangent length AT_1 = T_L = AE + t_L = 204.60 + 218.40 = 423.40 m

Tangent length AT_2 = T_S = AF + t_s = 229.27 + 126.12 = 355.40 m

Length of the first curve = π R_L α / 180 = 600 * 40 π/ 180 = 418.90 m

Length of the Second curve = π R_s β / 180 = 400 * 35 π/ 180 = 244.35 m

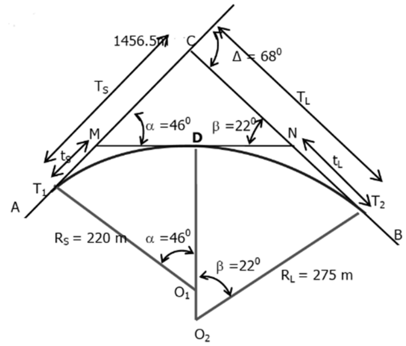
Chainage at point of curve (T_1) = Chainage at the point of intersection – tangent length (T_L) = 3415 – 423 = 2992 m.

Chainage at point PCC (N) = Chainage at the point of curve + Length of first curve = 2992 + 418.90 = 3410.9 m

Chainage at point of tangency (T_2) = Chainage at the point of PCC + Length of second curve = 3410.9 + 244.35 = 3655.25 m

Example 2.17:

Two straights AC and CB have bearing of 59^0 30' and 127^0 30' intersect at C at a chainage of 1456.5 m. Two points M and N are located on AC and BC so that the bearing of MN is 105^0 30'. The straights AC and CB are to be connected by a compound curve consisting of arcs of radii 220 m and 275 m, respectively. If the line MN is the common tangent to the two arcs, compute the chainages of the starting point, point of compound curvature (PCC) and the end point of the curve.



Solution:

Bearing of AC is 59^0 30', of CB is 127^0 30', and of MN is 105^0 30'. Chainage of the intersection point is 1456.5 m. Radius of the first arc (R_S) is 220 m, and that of second arc (R_L) is 275 m. Let D be the point on the line MN.

Δ = Bearing of CB – Bearing of AC
$\Delta = 127^0\ 30' - 59^0\ 30' = 68^0$
CMN = α = Bearing of MN – Bearing of AC
$= 105^0\ 30' - 59^0\ 30' = 46^0$
$\Delta = \alpha + \beta = 68^0$
$\beta = \angle CNM = \Delta - \alpha = 68^0 - 46^0 = 22^0$.
$MD = T_1M = t_S = R_S \tan \alpha/2$
= 220 tan 46/2
= 93.38 m
$ND = T_2N = t_L = R_L \tan \beta/2$
= 275 tan 22/2
ND = 53.45 m
MN = MD + ND = 93.38 + 53.45 =146.83 m
In triangle CMN,
$\angle CMN + \angle CNM + \angle MCN = 180^0$
$\angle MCN = 180^0 - \angle CMN - \angle CNM = 180^0 - 46^0 - 22^0 = 112^0$
Applying, sine rule to the triangle CMN,
(CM / sin β) = MN /sin MCN
CM = 146.83 sin 22 / sin 112
CM = 59.32 m
Similarly, (CN / sin α) = MN /sin MCN
CN = 146.83 sin 46/ sin 112
CN = 113.92 m

Tangent length $CT_1 = t_S = CM + MT_1$ = 59.32 + 93.38 = 152.7 m
Tangent length $CT_2 = t_L = CN + NT_2$ = 113.92 + 53.45 = 167.37 m

Length of the first curve = $R_S\ \alpha\ (\pi / 180)$ = 220 * 46 (π / 180) = 176.63 m
Length of the Second curve= $R_L\ \beta\ (\pi / 180)$ = 275 * 22 (π / 180) =87.16 m
Chainage at point of curve (T_1) = Chainage at the point of intersection – tangent length (CT_1)
= 1456.5 – 157.7 = 1298.8 m
Chainage at point of PCC = Chainage at the point of curve + Length of first curve
= 1298.8 + 176.63 =1475.43 m.
Chainage at point of tangency (T_2) = Chainage at the point of PCC + Length of second curve

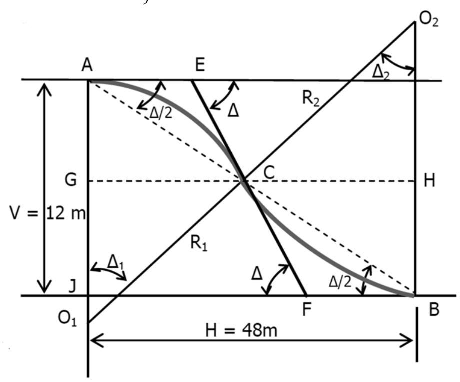
= 1475.43 + 87.16 = 1562.59 m

Example 2.18:

Two parallel railway lines 12 m apart are to be connected by a reverse curve of same radius throughout. If the maximum distance between tangent points measured parallel to the straight is 48 m. Find the maximum allowable radius. Also calculate the radius of 2^{nd} branch, if both the radii are different, and that of the 1^{st} branch is 60 m. Also calculate the length of both the branches.

Solution:

Let A and B be the points of tangencies and C point of reverse curvature (PRC) in the figure. The distance between the lines V=12 m, and h=48 m.



Tan Δ / 2 = V / h = 12 / 48 = 0.25

So Δ = 28^0 04' 20.95"

(i) when the radius is same

h = R_1 sin Δ + R_2 sin Δ

Here R_1 = R_2 = R, so h = 2 R sin Δ

R = h / (2 sin Δ)

R = 48 / 2 sin 28^0 04' 20.95"

R = 51 m Ans.

(ii) when the radius is different

Using the above relationship for h

48 = 60 sin 28^0 4' 20.95" + R_2 sin 28^0 4' 20.95"

R_2 = (48 - 60 sin 28^0 4' 20.95") / sin 28^0 4' 20.95"

R_2 = 42 m Ans.

(iii) Length of curves

Length of first curve = R_1 Δ (π/180)

= 51 * 28^0 4' 20.95" (π/180) = 29.40 m Ans.

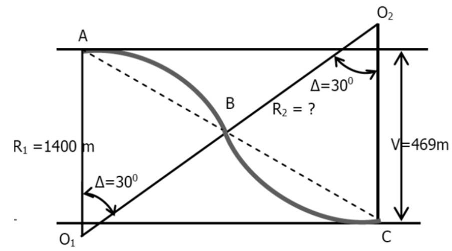
Length of second curve = R_2 Δ (π/180)

= 42 * 28^0 4' 20.95" (π/180) = 20.58 m Ans.

Example 2.19:

Two parallel lines 469 m apart are joined by a reverse curve ABC with two different arcs which deflects to the right by 30^0 angle from the first straight. If the radius of the first arc is 1400 m, calculate the radius of the second arc. Also calculate the chainage of points B and C, if the chainage of A is 2500 m

Solution:



In given Figure, A and C are the points of tangencies and B is the point of reverse curvature (PRC). The distance between the lines V = 469 m

$V = (R_1 + R_2)$ versin Δ

$469 = (R_1 + R_2)(1 - \cos\Delta)$

$469 = (1400 + R_2)(1 - \cos 30^0)$

$R_2 = 2100.66$ m Ans.

Length of first curve AB = $R_1 \Delta (\pi/180)$

$= 1400 * 30^0 (\pi/180)$

= 733.04 m

Length of second curve BC = $R_2 \Delta (\pi/180)$

$= 2100.66 * 30^0 (\pi/180)$

= 1099.9 m

Chainage at B = chainage at A + length of first curve

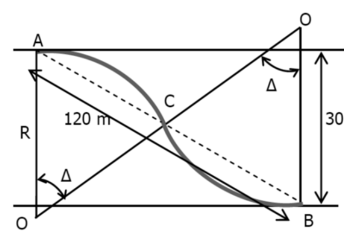
Chainage at B = 2500 + 733.04 = 3233.04 m Ans.

Chainage at C = chainage at B + length of second curve

Chainage at C = 3233.04 + 1099.9 = 4332.94 m Ans.

Example 2.20:

A reverse curve ACB is to be set out between two parallel tangents 30 m apart. The distance between the tangent points A and B is 120 m. Find the radius R, if R_1 is equal to R_2, as well as the radius R_2, if R_1 is 100 m. Also calculate the deflection angle.



Solution:

In the Figure, the length between tangent points A and B, L = 120 m

V = 30 m.

(i) when the radius is same

$L^2 = 2V\ (R_1 + R_2)$

If $R_1 = R_2 = R$, then $L = \sqrt{(4RV)}$

$120 = \sqrt{(4 * 30 * R)}$

R = 120 m Ans.

(ii) when the radius is different

$L^2 = 2V\ (R_1 + R_2)$

$120^2 = 2 * 30\ (100 + R_2)$

$R_2 = 140$ m Ans.

(iii) The deflection angle

$V = (R_1 + R_2)$ versin Δ
$30 = (100 + 140)(1 - \cos\Delta)$
$\Delta = \cos^{-1}(0.875)$
$\Delta = 28^0\ 57'\ 18''$ Ans.

Example 2.21:
A reverse curve is to be set out between two parallel tangents 10 m apart. The distance between the tangent points measured parallel to the tangents is 80 m. If the radius of the first curve is 150 m, calculate the radius of the second curve as well as the lengths of the two branches.

Solution:
v = 10 m, h = 80 m, R1 = 150 m
$\tan \alpha/2 = v / h = 10 / 80$
$\alpha = 14.25°$
$h = R_1 \sin \alpha + R_2 \sin \alpha$
$80 = 150 \sin 14.25 + R_2 \sin 14.25$
$R_2 \sin 14.25 = 43.077$
$R_2 = 175$ m Ans.

Length of first curve = $R_1\ \alpha\ (\pi/180)$
$= 150 * 14.25\ (\pi/180) = 37.31$ m Ans.
Length of second curve = $R_2\ \beta\ (\pi/180)$
$= 175 * 14.25\ (\pi/180) = 43.52$ m Ans

Example 2.22:
A transition curve of the cubic parabola type is to be set out from a straight centreline such that it passes through a point which is 6 m away from the straight, measured at right angles from a point on the straight produced 60 m from the start of the curve. Compute the data for setting out a 120 m length of transition curve at 15-m intervals. Calculate the rate of change of radial acceleration for a speed of 50 km/h.

Solution:
L = 120 m
From expression for a cubic parabola: $y = x^3 / 6RL = cx^3$
$y = 6$ m, $x = 60$ m
$c = y / x^3 = 6 / 60^3$
$= 1 / 36000$
But $c = 1 / 6RL = 1/36000$
$1/RL = 1/6000$
The offsets are now calculated using this constant:
$y_1{}^3 = 15^3 / 36000 = 0.094$ m
$y_2{}^3 = 30^3 / 36\,000 = 0.750$ m
$y_3{}^3 = 45^3 / 36\,000 = 2.531$ m and so on
Rate of change of radial acceleration $q = V^3 / (3.6^3 RL)$
$q = 50^3 / (3.6^3 * 6000)$
$q = 0.45$ m/s^3

Example 2.23:
A compound curve AB and BC is to be replaced by a transition curve of 100 m long at each end. The chord lengths *AB* and *BC* are respectively 661.54 m and 725.76 m and radii 1200 m

and 1500 m. Calculate the single arc radius: (a) If *A* is used as the first tangent point, and (b) If *C* is used as the last tangent point.

Solution:

T_1 = AB, T_2 = BC, R_1 = 1200 m and R_2 = 1500 m

Let I be the intersection point

Chord AB = $2R_1 \sin \Delta_1/2$

$\sin \Delta_1/2 = AB * R_1 /2 = 661.54 / 2*1200$

$\Delta_1 = 32°$

Similarly, $\sin \Delta_2/2 = 725.76 / 2*3000$

$\Delta_2 = 28°$

Distance $At_1 = t_1B = R_1 \tan \Delta_1/2$

t1B = 1200 tan 16° = 344 m

Similarly, $Bt_2 = t_2C = R_2 \tan \Delta_2/2$

t_2C = 1500 tan 14° = 374 m

t_1t_2 = 344 + 374 = 718 m

By sine rule in triangle t_1It_2

t_1I = 718 sin 28 / sin 120

t_1I= 389 m

and t_2I = 718 sin 32 / sin 120

t_2I = 439 m

AI = $At_1 + t_1I$ = 733 m

CI = $Ct_2 + t_2I$ = 813 m

To find single arc radius:

(a) From tangent point A

AI = (R + S) tan Δ /2 + L/2

where $S = L^2 / 24R$ and $\Delta = \Delta_1 + \Delta_2 = 60°$, and L = 100 m

So 733 = $[R + (L^2/24R)] \tan 60/2 + 100/2$

R = 1182 m

(b) From tangent point C

CI = (R + S) tan Δ /2 + L/2

813 = $[R + (L^2/24R)] \tan 30 + 50$

R = 1321 m

Example 2.24:

Find the length of the vertical curve connecting two grades +0.5% and –0.4% where the rate of change of grade is 0.1%.

Solution:

Length of vertical curve = (0.5- (-0.4) * 30) / 0.1

= ((0.5 + 0.4) * 30 *10) / 1

= 0.9 * 30 * 10

= 270 m

Example 2.25:

Find the length of vertical curve connecting two uniform grades from the following data: (i) + 0.8% and - 0.6%, rate of change of grade is 0.1 per 30 m

(ii) – 0.5% and +1%, rate of change of grade is 0.05 per 30 m

Solution:

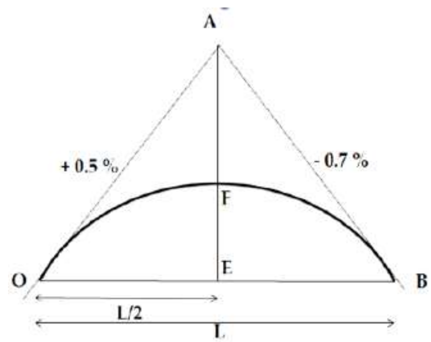
(i) $L = (g_1 - g_2)\ r = (0.8 - (-0.6))\ 0.1 * 30$
$= 420$ m

(ii) $L = (-0.5 - (+1))\ 0.05 * 30$
$= 900$ m

Example 2.26:

Calculate t6e RL of various stations on a vertical curve connecting two uniform grades of + 0.5% and -0.7%. The chainage and RL of point of intersection is 500 m and 350.750 m, respectively. The rate of change of grade is 0.1% per 30 m.

Solution:



Length of vertical curve $L = (g_1 - g_2)\ r * 30$
$= (0.5 - (-0.7))\ 0.1 * 30 = 360$ m
$L/2 = 180$ m
Chainage of O = chainage of A – L/2 = 500 – 180 = 320 m
Chainage of B = chainage of A + L/ 2 = 500 + 180 = 680 m
RL of point of intersection, A = 330.75 m
RL of O = $330.75 - g_1 L / 200$
= 330.75 - 0.5 (360)/ 200
= 329.85 m
RL of B = $330.75 - g_2 L/ 200$
= 330.75 - 0.7 (360)/ 200
= 329.49 m
RL of mid-point E of chord OB = ½ (RL of O + RL of B)
= ½ (329.85 +329.49) = 329.67 m
RL of F (vertex of curve) = ½ (RL of B + RL of A)
= ½ (330.75 + 329.67) = 330.21 m
The difference between A and F = 330.75 – 330.21 = 0.54 m

Check: $AF = (g_1 - g_2)\ L/ 800$
$= [0.5 - (-0.7)] * 360/ 800 = 0.54$ m Checked

1st point on the curve chainage = 350 m , at x= 30 m
RL of 1st point on tangent = $329.85 + (g_1 x /100) = 330$ m
Tangent correction $y = (g_1 - g_2) * x^2/ 200 L$
= 0.015 m
RL of 1st point on the curve = 330 m – 0.015 = 329.985 m
Data is shown in table below:

Station	Chainage (m)	Grade Elevation (m)	Tangent Correction (m)	Curve Elevation (m)	
0	320	329.85	0	329.85	Start of V.C
1	350	330.00	− 0.015	329.985	
2	380	330.15	− 0.06	330.090	
3	410	330.30	− 0.135	330.165	
4	440	330.45	− 0.240	330.210	
5	470	330.60	− 0.375	330.225	
6 (F)	500	330.75	− 0.54	330.210	Vertex of V.C
7	530	330.54	− 0.375	330.165	
8	560	330.33	− 0.240	330.090	
9	590	330.12	− 0.135	329.985	
10	620	330.91	− 0.06	329.850	
11	650	330.70	− 0.015	329.685	
12 (B)	680	330.49	0	329.490	End of V.C

Example 2.27:
A 1% grade meets a +2.0% grade at station 470 of elevation 328.605 m. A vertical curve of length 120 m is to be used. The pegs are to be fixed at 10 m interval. Calculate the elevations of the points on the curve by (a) tangent corrections and (b) by chord gradients.

Solution:
Station PVC:
Xo = Sta PVI – L/2 = 470 – (120/2) = 410.0 m

Elevation PVC:
Yo = Elevation at PVI – (g1*L/2) = 328.605 – (-0.01*120/2) = 329.205 m

Equation of Curve
Rate of change of grade, k = (g2-g1)/L = 0.00025
Let x be the distance of any point on the curve, from PVC (BC), and let Y be its elevation.

Equation of an equal tangent vertical parabolic curve is given as,
$Y = Yo + g1*x + k*x^2/2 = 329.205 + (-0.01)*x + (0.00025)*x^2/2$

1st Full Station is at 420.0 m, at x = 420.0 – 410.0 = 10.0 m from PVC

Put x = 10.0 m in above parabolic curve equation-
Elevation @(420.0) = $329.205 + (-0.01)*(10.0) + (0.00025)*(10.0)^2/2 = 329.118$ m

Low Point:
Low point is given at, x = -g1*L/A = 40.0 m from BVC.
hence station (Low point) = Sta_PVC + x = 410.0 + 40.0 = 450.0 m
Elevation at Low point = $329.205 – 0.01*40.0 + (0.00025)*40.0^2/2 = 329.005$ m

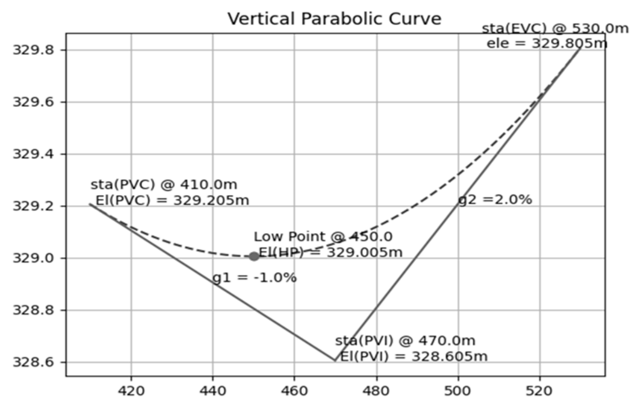
End Station:
End Station, EVC = PVC + L = 530.0 m at x = L = 120.0 m from PVC

Put ‘x’ in parabolic curve equation, Elev EVC = 329.805 m

Similarly, put x values of subsequent stations in above parabolic curve equation to get Elevations.

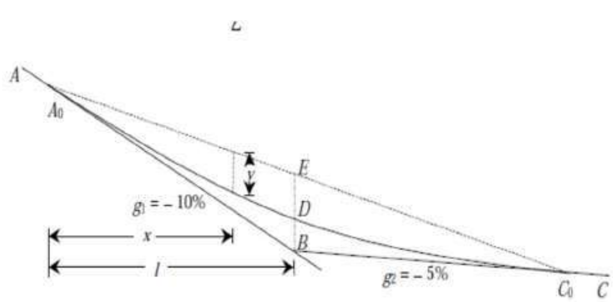
Details are tabulated below:

Station	distance x	g1*x	k*x2/2	Elevation(Y)
410.0	0.0	0.0	0.0	329.21
420.0	10.0	-0.1	0.01	329.12
430.0	20.0	-0.2	0.05	329.06
440.0	30.0	-0.3	0.11	329.02
450.0	40.0	-0.4	0.2	329.01
460.0	50.0	-0.5	0.31	329.02
470.0	60.0	-0.6	0.45	329.06
480.0	70.0	-0.7	0.61	329.12
490.0	80.0	-0.8	0.8	329.21
500.0	90.0	-0.9	1.01	329.32
510.0	100.0	-1.0	1.25	329.46
520.0	110.0	-1.1	1.51	329.62
530.0	120.0	-1.2	1.8	329.8



Example 2.28:
Two straights AB and BC falling to the right at gradients 10% and 5%, respectively, are to be connected by a parabolic curve 200 m long. Compute the parameters for a vertical curve for chainage and reduce level of B as 2527.00 m and 56.46 m, respectively. The peg interval is 20 m. Also calculate the sight distance for a car having headlights 0.60 m above the road level, and the headlight beams inclined upwards at an angle of 1.2°.

Solution:



The total number of stations at 20 m interval (2n) = L / 20

2n = 200 / 20 or n = 5

Fall per chord length e1 = g1 * 20 / 100

e1 = -10 * 20 / 100 = -2 m

e2 = g2 * 20 /100 = 20 * 5 / 100 × × = -1 m

Elevation of the beginning of the curve at A0 = Elevation of B – ne1

= 56.46 – 5 × (– 2) = 66.46 m

Elevation of the end of the curve at C0 = Elevation of B + ne^2

= 56.46 – 5 × (–1) = 51.46 m

Tangent correction with respect to the first tangent h = kn^2 where k = e1 – e2 / 4n

k = -2- (-1) / 4 * 5 = – 0.05

RL of the points on the curve = Tangential elevation – tangent correction = H – h **(**where H is the tangential elevation of a point)

RL on the n'^{th} point on the curve = RL of A_0 + n′ e_1 – kn'^2

RL of point 1 (A_0) = 66.46 + 1 × (–2) – (– 0.05) × 1^2 = 64.51 m

RL of point 2 = 6.46 + 2 × (– 2) – (– 0.05) × 2^2 = 62.66 m

RL of point 3 = 6.46 + 3 × (– 2) – (– 0.05) × 3^2 = 60.91 m

RL of point 4 = 66.46 + 4 × (– 2) – (– 0.05) × 4^2 = 59.26 m

RL of point 5 = 66.46 + 5 × (– 2) – (– 0.05) × 5^2 = 57.71 m

RL of point 6 = 66.46 + 6 × (– 2) – (– 0.05) × 6^2 = 56.26 m

RL of point 7 = 66.46 + 7 × (– 2) – (– 0.05) × 7^2 = 54.91 m

RL of point 8 = 66.46 + 8 × (– 2) – (– 0.05) × 8^2 = 53.66 m

RL of point 9 = 66.46 + 9 × (– 2) – (– 0.05) × 9^2 = 52.51 m

RL of point 10 (C_0) = 66.46 + 10 × (– 2) – (– 0.05) × 10^2 = 51.46 m (Okay)

Chainage of the intersection point B = 2527.00 m

Chainage of A_0 = Chainage of B – 20n = 2527.00 – 20 × 5 = 2427.00 m

Chainage of C_0 = Chainage of B + 20n = 2527.00 + 20 × 5 = 2627.00 m

hainage of the points and the reduced levels of the corresponding points on the curve are tabulated in Table 7.11.

Point	Chainage (m)	R.L. of points on curve (m)	Remarks
0	2427.00	66.46	A_0 (PC)
1	2447.00	64.51	
2	2467.00	62.66	
3	2487.00	60.91	
4	2507.00	59.26	
5	2527.00	57.71	Apex
6	2547.00	56.26	
7	2567.00	54.91	
8	2587.00	53.66	
9	2607.00	52.51	
10	2627.00	51.46	C_0 (PT)

With the car is at tangent point A0, the headlight beams will strike the curved road surface at a point where the offset *y* from the tangent at A_0 = (0.60 + *x* tan 1.2°), where *x* is the distance from A0.

The offset *y* at a distance *x* from A0 is given by y = (g_1 - g_2) x^2/ 400*l*

= (-2 + 1) x^2 / (100* 400) = x^2 / 40000 (ignoring the sign)

0.60 + *x* tan 1.2° = x^2 / 40000

x^2 – 837.88*x* – 24000 = 0

Soling for *x*, we get Sight distance *x* = 865.61 m