Chapter 30: Spectral Acceleration

Introduction

In earthquake engineering, understanding the dynamic response of structures to seismic forces is crucial. One of the most important parameters in this context is **Spectral Acceleration (Sa)**. Spectral acceleration represents the maximum acceleration response of a damped single degree of freedom (SDOF) system to a specific ground motion, as a function of its natural frequency (or period) and damping ratio. This chapter explores spectral acceleration in depth, its derivation, applications, and significance in seismic design and analysis of structures.

30.1 Ground Motion and Structural Response

30.1.1 Ground Motion Representation

- Earthquake ground motion is recorded as a time-history of acceleration at the ground surface.
- These time-histories are characterized by peak ground acceleration (PGA), duration, frequency content, and energy.

30.1.2 Single Degree of Freedom (SDOF) Systems

- A simplified model used to study the dynamic response of structures.
- SDOF systems are idealized with a mass-spring-damper model:

$$m\ddot{x}(t) + c\dot{x}(t) + kx(t) = -m\ddot{u}_g(t)$$

- Where:
  - $\ddot{x}(t)$: Relative acceleration
  - $\dot{x}(t)$: Relative velocity
  - $x(t)$: Relative displacement
  - $\ddot{u}_g(t)$: Ground acceleration

30.2 Definition of Spectral Acceleration (Sa)

- **Spectral Acceleration (Sa)** is the maximum acceleration experienced by a damped SDOF system under seismic excitation.
- It is computed by solving the equation of motion for various periods and damping ratios.

- Units: m/s^2 or **g** (acceleration due to gravity).

$$Sa(T, \zeta) = \max |\ddot{x}(t)|$$

Where:

- T: Natural period of the structure
- ζ: Damping ratio (usually 5% for buildings)
- $\ddot{x}(t)$: Acceleration response

30.3 Response Spectra

30.3.1 Elastic Response Spectrum

- A plot of maximum response (displacement, velocity, or acceleration) of SDOF systems versus period or frequency for a given ground motion.
- For **acceleration spectra**, this yields the Spectral Acceleration curve.

30.3.2 Damping Effect

- The damping ratio significantly affects the shape and amplitude of the response spectrum.
- Common practice: use 5% damping for building design, though variations may apply depending on the structure type.

30.3.3 Design Response Spectrum

- Codified spectrum used for seismic design (e.g., IS 1893 Part 1).
- Represents average spectral acceleration based on probabilistic or deterministic seismic hazard.

30.4 Factors Affecting Spectral Acceleration

30.4.1 Soil Type and Site Conditions

- Soil amplifies or attenuates seismic waves.
- Site classification (Type I to V in IS 1893) significantly affects spectral shape and amplitude.

30.4.2 Seismic Zone

- Spectral acceleration varies by **seismic zone factor (Z)** in design codes.
- Higher Z values increase Sa in response spectrum.

30.4.3 Importance Factor (I) and Response Reduction Factor (R)

- Applied in design base shear calculations, indirectly influencing spectral values.
- These modify the elastic spectrum to derive design spectrum.

30.5 Spectral Acceleration in Seismic Design

30.5.1 Design Base Shear

As per IS 1893:

$$V_b = \frac{ZIS_a}{2Rg} \cdot W$$

Where:

- V_b: Design base shear
- Z: Seismic zone factor
- I: Importance factor
- S_a: Spectral acceleration
- R: Response reduction factor
- W: Seismic weight of the building

30.5.2 Use in Structural Analysis

- Sa provides input for **response spectrum method** of seismic analysis.
- Determines lateral forces at various heights of multistorey buildings.

30.6 Calculation of Spectral Acceleration

30.6.1 From Time History Analysis

- For a given ground motion and SDOF system:
  - Solve numerically (e.g., Newmark-beta method) to find $\ddot{x}(t)$.
  - Obtain peak acceleration → **Spectral Acceleration**.

30.6.2 From Code-Specified Response Spectra

- IS 1893 gives Sa values as functions of:
  - Soil type (e.g., hard, medium, soft)
  - Damping ratio (standardized curves for 5%)
  - Natural period (T)
- Spectra are scaled by PGA for design.

30.7 Spectral Acceleration vs. Other Parameters

Parameter	Definition	Use
PGA (Peak Ground Acceleration)	Max ground acceleration	Basic indicator of shaking
Sa (Spectral Acceleration)	Max acceleration of damped SDOF system	Used in design response spectra
Sv (Spectral Velocity)	Max velocity response	Less commonly used in design
Sd (Spectral Displacement)	Max displacement response	Used for flexible or tall structures

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30.8 Software and Practical Applications

- **ETABS, SAP2000, STAAD Pro**, and other structural analysis tools use Sa from response spectra for seismic design.
- Modal combination methods (e.g., SRSS, CQC) depend on spectral acceleration values for each mode.

30.9 Limitations of Spectral Acceleration

- **Assumes linear elastic behavior**: not valid beyond yield point without modifications.
- **Only for SDOF systems**: approximation for real MDOF structures.
- **Site-specific effects** not fully captured in general code spectra – site-specific analysis may be required for critical structures.

30.10 Spectral Acceleration in Probabilistic Seismic Hazard Analysis (PSHA)

- Sa used in **hazard curves** that relate Sa to probability of exceedance.
- Basis for uniform hazard spectra (UHS) used in performance-based design.

30.11 Site-Specific Response Spectra

While design codes provide general response spectra, certain critical or important structures (like hospitals, dams, or nuclear facilities) require **site-specific response spectra**, which are developed based on:

30.11.1 Geotechnical Investigation

- Detailed soil profile and dynamic soil properties (e.g., shear wave velocity, damping).
- Soil amplification studies.

30.11.2 Ground Motion Selection and Scaling

- Selection of appropriate earthquake records that match the seismic environment of the site.
- Scaling of ground motion to match target spectrum (e.g., uniform hazard spectrum).

30.11.3 Ground Response Analysis

- **Equivalent-linear** or **nonlinear site response analysis** to compute surface motion.
- Final output: Site-specific **Sa vs. T** curve used in dynamic structural analysis.

30.12 Influence of Damping Ratio on Sa

30.12.1 Effect of Damping

- Higher damping → lower spectral acceleration.
- Structures like base-isolated buildings may have damping $> 5\%$ (typically 10–30%).

30.12.2 Code-Based Modification

Most codes provide **damping correction factors**:

$$S_{a,\zeta} = S_{a,5\%} \cdot R_d(\zeta)$$

Where:

- $S_{a,\zeta}$: Spectral acceleration at damping ratio ζ
- $R_d(\zeta)$: Damping reduction factor

30.12.3 Table of Correction Factors (as per IS/Eurocode):

Damping (%)	Reduction Factor $R_d(\zeta)$
0	1.00
5	1.00
10	0.80
20	0.55
30	0.40

30.13 Vertical Spectral Acceleration

While most structural design focuses on **horizontal components**, vertical spectral acceleration becomes important in:

- Long-span bridges
- Piers, cantilever walls
- Structures with sensitive vertical members

30.13.1 Vertical-to-Horizontal (V/H) Ratio

- Typically ranges from **0.3 to 0.7**.
- Design vertical spectrum sometimes specified as:

$$S_{a,V}(T) = \alpha_v \cdot S_{a,H}(T)$$

Where α_v is the V/H ratio.

30.14 Spectral Acceleration Maps

Modern seismic codes use **spectral acceleration maps** rather than just PGA maps.

30.14.1 Example: USGS Maps (for international context)

- Maps show Sa values at different periods (0.2s, 1.0s, etc.) for various probabilities of exceedance (e.g., 2% in 50 years).

30.14.2 Indian Code (IS 1893)

- While IS 1893 primarily uses zone factors and standard spectra, research is ongoing to incorporate spectral maps for enhanced accuracy.

30.15 Use of Spectral Acceleration in Performance-Based Design (PBD)

30.15.1 Beyond Linear Elastic Design

- Sa is used in **pushover analysis** and **nonlinear time-history analysis** to determine demand at different performance levels (IO, LS, CP).

30.15.2 Demand and Capacity Spectrum

- Spectral acceleration is plotted against spectral displacement to form:
  - **Demand spectrum** (from hazard)
  - **Capacity curve** (from structure)
- Their intersection gives performance point.

30.16 Spectral Acceleration in Seismic Isolation and Dampers

30.16.1 Role in Base Isolation Design

- Isolated structures have longer periods → move into region of lower Sa.
- Hence, base-isolation shifts structure out of high Sa zone of response spectrum.

30.16.2 Effect of Energy Dissipation Devices

- Dampers modify effective damping → changes the Sa value used in analysis.
- Sa is recalculated for modified damping using damping correction factors.

30.17 Advanced Spectral Shapes for Special Structures

30.17.1 Tall Buildings and Irregular Structures

- These may require **multi-mode** and **multi-directional** analysis using Sa in all principal directions.

30.17.2 Bridges and Towers

- May require different Sa for different spans/support conditions.

30.17.3 Spectral Matching

- Artificial ground motions are developed to **match a target Sa curve** exactly for use in nonlinear time-history analysis.

30.18 Recent Developments in Spectral Acceleration Research

- **Machine learning** for site-specific spectral estimation.
- **Region-specific spectral models** (e.g., Himalayan region, North-East India).
- Use of **conditional mean spectra (CMS)** in advanced performance-based design.
