Fluid Mechanics
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Lec 26:Stream Function

Good morning all of you for today class on stream functions as we have been discussing on differential analysis of fluid flow. Today we are going to discuss about the stream flow functions okay and which is a as we discuss more details about virtual fluid balls. The same concept we will talk about now extending the same concept we are talking about the stream functions. We will discuss more details about the stream functions. The reference materials what we have with the Sinzel-Simbala MIT courseware and we also have a FM-White book on fluid mechanics. These are the reference materials and most of the contents we are parallelly looking from the fluid mechanics fundamentals and applications part.

Now if you look at the problems what we have solved as a undergraduate fluid mechanics lab conducted in the last years. We try to solve this fluid mechanics problems of the simulations of F-16 fighter plane using ANSYS effluent CFD software. So if you look at the visualizations here this is what the streamlines if you can see that there are the streamlines. and over that we have the velocity magnitudes.

We have the velocity magnitudes which vary from 0 to 550 meter per seconds. That is what is the velocities. So each color representing here the velocity components. So if you can look it very interestingly these figures that how the streamlines patterns are there on a fighter F-16 fighter jets. We are not going details what the turbulence models solved by the ANSYS fluent but I am just visualizing you that you try to look it so complex F-16 flights we can simulate it at the flow at the range of 2.

45 and the range of the velocities if you look at 410 meter per seconds to the 840 meter per second. You can conduct the CFD simulations of F 60 fighter planes and that what we will be showing streamlined patterns and like at this point we have a low velocity zones. So along the streamlines we can see the velocities. Along the streamlines we can find out the velocity. Similar way we can get a pressure gradients act on the streamlines which is looks is very complex figures here, very complex figures.

So but we can get it from this. So nowadays the tools like ANSYS fluent or any CFD softwares available to visual the fluid flow, how it is happening it. For example, like complex flow around a F-16 fighter jet. That is what you conducted by a BTEC students groups. Now if you look at the next applications problems what we have solved it which is quite interesting.

There is a two rotating cylinders okay. So you can understand it. There is a two rotating cylinders are there and the fluid flow is happening from this side to going this. To visualize the stream flows the CFD softwares also have a provisions to incorporated the air bubbles okay. You can inject the air bubbles.

These are the air bubble injections okay. You can look at that how the air bubbles, a series of air bubbles are coming it okay and they are rotating it. There are rotating it and they are dispersing it. So these type of flow visualizations you can do it using the artificially just include the air bubbles which represents a unsteady streamlines. So that is the reasons if you look at that the our understanding of the streamlines should have a understand very complex problems lays that the concept of the virtual fluid walls virtual fluid walls the same way as we try to explain you with the fluid mechanics problems like complex the how the flow process are happening it.

and how we can really simulate it in computational fluid dynamics software and you can use the instead of air bubble that the same concept what we discuss it you can understand the virtual fluid balls and how they trying to move it okay. As the two cylinders are rotating it you can see there the unsteady stimuli and patterns okay generating from this to this how it is happening it. Let us go very basic of the streamline and stream functions which is essential for a undergraduate students that part what I will cover it. So today I will be very basic way we will talk about what is the stream functions okay and we will talk about that in a two-dimensional coordinate systems we can have the stream functions. We also talk about compressible stream functions not only incompressible functions and we will also talk about cylindrical coordinate systems and then we will also solve some examples.

That is what is today plans after demonstrating you the CFD software, how it is showing the streamlines for a rotating cylinders, two rotating cylinders and F-16 flight airplane. Now if you look at the streamlines, as you know it, let I take a very simple examples. There is a hump or you can say it is a buildings okay, there is a big buildings are there and you have a uniform stream flow, uniform stream flow. So if want to draw a streamlines will go like this, go like this, go like this. So these are the streamlines we can artistically draw.

with knowledge of the stream lines how the flow behaviors will happen or how the virtual fluid balls just assuming it that a series of balls are there which is coming in uniform speed okay. Let be the v speed how the balls will move it with time that is what I what I given the examples how the balls are moving it that is what you can visualize it okay. No doubt the flow whatever comes here it can go like this and here you may have a a local vortex presence will be there. So now we are emphasizing the regions where there is less frictional force. For example, the streamlines if I am not taking closer to these structures, closer to this buildings no doubts I will not have this frictional force it will be dominated.

So I can have a basic mass conservation equations and Bernoulli's equations to solve the problems. That is why the Bernoulli s equations is very famous because the regions where you have a very very less frictional force components. very less frictional force components you can use two equations that mass conservation equations and the Bernoulli's equations to solve these problems. That means except these regions okay which is close to these structures we can apply the Bernoulli's equations and the mass conservation equations to solve the problem. when we consider these problems is in a two-dimensional lectures, two-dimensional as I discuss lot of times we can simplify many of the three-dimensional flow components into two-dimensionals, two-dimensional even if the steady flow.

So that means I have the mass conservation equations for a two-dimensional compressible or incompressible flow, let me I put incompressible flow of these regions which is beyond these frictional dominated regions if I consider the flow is incompressible natures the basic equations what I have for conservation of equations is divergence vector is 0 or I can write it this component for two-dimensional flow. That is the reasons even if we have we did not invent high-end computings today what we have that is the reasons I we can able to simulate more than million nodes computational nodes to solve the CFD problems today because if you have a high-end computing systems with us but that was not there almost 200 years back. okay. So that times the stream functions are the tools are used to solve this two-dimensional flow. Stream functions are used to solve these equations.

How we can solve these equations? Because there is a two independent variable x and y, two dependent variables u and v okay and we can apply this Bernoulli's equations that is not a big issue. So, the basically how to draw the streamline how we can solve this two-dimensional equations where you have a two dependent variables in u and v components. Unless otherwise we make it a single functions which is called the stream functions. If I convert these two-dimensional equations to of u and v to dependent variables of scalar velocity component instead of two velocity components if I use a only a single dependent variables then my problems will be sorted out. That is what is a strategy was followed still has been followed to solve many of the fluid flow problems.

Also we can really understand the stream flows for a CFD, very complex CFD problems. If you can understand the stream flows, patterns are happening it, how the pressure gradients are happening it, how those understanding is necessary even if you have a today's era we have the computational fluid dynamics tools with us. but understanding of the stream functions is necessary. But the stream functions was introduced to reduce the dependent variables which was earlier two variables u and v instead of only one functions we can define it to solve this continuity equations. Let us do that derivations part.

If you look at that as I said it if the functions is a very simplified it is a 2 dimensional flow field as given this examples of this case you can have a the mass conservation equations in terms of u and v. That is what we can get it and here very strategically we can define the stream functions. We can define the stream functions as these functions. I will come into further. So we are defining a stream functions which is a function of x and y.

okay functions of x, y and h. The stream functions like for example as I said it in case of flow around a building structures. So these will be my stream functions and these functions will have a different c1, c2, c3, c3. That means I will get a family of curves. I will get family of the curves which each functions are equating with a difference c1, c2, c3.

I am getting a curves. I am getting the stream functions like this. I am getting the stream functions like this. So if I have the stream functions like this and I define it that is what also we can indirectly derive it that the same things is given it here these are stream functions having with different c1, c2 values. So if I define the stream functions just try to understand is u the velocity in the x directions the scalar velocity component is equal to the first gradient or the partial derivative of phi with respect to the y where is the velocity in the y directions the scalar velocity of this will be the minus of in the x direction. Please remember it the size stream functions gradient in y directions represent the velocity scalar field in the x directions.

This is y directions stream functions gradients in represent the velocity in x directions where is in case of the stream functions gradient in x directions negative of that means reverse of that indicates the velocity in the y directions. That is what you can have a reverse the things x that means you have a functions of x directions and y directions. So the velocity the gradient of changing on this y directions okay the y directions will represent is as the u components and the x directions gradients reverse of that reference of the v components that is you can have a physical interpretations. If I have the stream functions are going like this and if I am getting a gradient in these directions that means the y directions I will get this the velocity component in these directions just reverse of that for with a negative sign we will get it what will be the velocity in the v directions that is the basic idea of that if I try to go for a next levels that That is what I said it say way back it was introduced by a mathematicians which is Josephs Louis Lagrange as I said it almost 200 years back okay to introduce the stream functions to solve many fluid mechanics problems. you not that time there was no computers okay there is no CFD softwares which I just demonstrated you to today we have a CFD software with high computing facility with us.

So that time they try to solve these problems these complex problems using a single dependent variable that is why the string functions which is represents a family of the curves which satisfy the continuity equations. That is the basic idea. It is nothing else as we know it the stream functions is showing the functions where representing the streamlines, representing the equations of the streamlines. So, if you look at the stream functions as we know the basic continuity equations again I am writing it because you need to remember only this part of equations okay which is too simple okay. You need not to remember different form of the continuity equations.

I think that is we should have a strategy always to write very simple form of the continuity equations is the velocity divergence field okay. For two-dimensional cases we can write it that is what will be this very simple thing okay. It is very simple things what we have these components. Now as I given a definitions that the U is represent the stream function gradient in the y directions. v is represent minus of stream function gradient in the x directions it is negative signs okay.

So if I substitute these two functions here just substitute these two functions here what I will get it is very interestingly in terms of stream functions I will get this path. That means since we have considered the stream functions and velocity relationship which satisfy the mass conservation equations for compressible flow or incompressible flow we can have a different functions. But for the incompressible flow the mass conservations can satisfy it if we have defined these functions like this. The stream functions if I define like this why can mass conservations equations it just satisfied it that means instead of the solving these problems if I know the stream functions that is what the solve these my problems. Now look at very interesting flow which is having a vortex as you open up a bathtub or water tap you can see the vortex and or you can see the larger vortex which is there in a cyclones.

Many of complex flow we define as a vortex flow. If your stream functions that is a very interesting examples we are giving from MIT lectures that if my stream lines will be functions like this for a vortex flow that means these are the streamlines. These are the streamlines that is what the functions if you can draw in MATLAB you can have a family of functions will like a vertex will happen it and the velocity component if you following this basic relationship we can get this velocity component like this and that is what will be so this type of vortex natures. So if you look at this way there is very complex flow with half of the stream functions we can represent it. Instead of representing the u and v we can represent this stream functions okay.

We can represent the stream functions okay. It can have a going like this. You can see the vertex okay. If you just open a bathtub you can have a So you can see the what and that is what is represent by a simple stream functions is just I am rewriting it to understand it these functions which is looking very simple okay that is what will come it. That is what will be come with these functions and we can just find out these in terms of velocities and if you draw the velocity magnitudes, you will get it this type of vortex flow.

That is very interesting way, very simple problems. We can always look at combinations of that. Now let it, I look at in terms of the definitions of the stream flow. streamlines. So as you know it any streamlines if I take a point this is the streamlines.

If I take a small element on the streamlines so for that I know it if I zoom this part just make it bigger. okay so if I look at the bigger ones I know it at this point whatever I will draw the tangent that is what will you represent the velocity the directions of the velocity field that is so that means at this point if you look at geometrically I will look at these triangles dx by dy or the velocity triangles having this y component and the v component as their isolates triangles, I can always write it dy by dx is equal to v by u. It is a simple to isolate triangles, geometric properties if you look in terms of velocity vectors and the coordinates dx and dy and dr, the r is along this the streamlines that is what if I rearrange it I will get this component Vdx plus Udy. So if I substitute the relationship between velocity and the stream functions again I will get this part is equal to 0. For this that is what is equation the d phi the along the streamline the change of the stream functions is a constant.

So if you look at the first derivative along the streamlines will be the 0. that is what is coming it here that is what is justifying that if I draw the streamlines it shows that the streamlines of a constant value represent me the streamlines functions that is the same derivations we are doing it just to prove it. in terms of getting a streamline function definitions. From the streamline definitions, we can derive it.

That is not a big issue. We can derive the same things. Now if you look at next component, many of the times we consider the incompressible continuity equations. That is what is at the mass flux levels. We look at this is what the equations for compressible fluid flow problem. So many of the times we can also look at the compressible fluid flow problems.

In that case instead of divergence of the velocity we have a divergence of the mass flux rho times of the V and in case of the two dimensional again we are simplifying it we will get these equations which again I am just writing for you it is a very easy to remember it. Now the same way I can define the stream functions. If you remember it earlier I defined the stream fractions of U for incompressible flow. I define the rho is equal to the gradient of that is what why that is what I if you write one relationship other relations you can write it. The stream functions gradient is equal to the scurral velocity u for incompressible but compressible flow you can see it that is the same way I can write it rho times of u that is what is represent of stream functions in the y direction.

If I write these ones definitely I can write for the scalar velocity of v and the rho v that is what is coming to here. So we can also define the stream functions that is what is indicated that functions as a in terms of rho into u or rho into v in case of compressible flow. So that is very interesting part is that we can use the same stream functions concept for compressible flow also incompressible. Only the functions will be the different. For incompressible flow I will have this gradient of stream functions y directions is a scalar velocity function in y directions.

Instead of the velocity, I have the mass flux rho times of u. Rho times of u is a mass flux is representing that. Same way I can write for the scalar component in the y directions is a V component as well as the mass flux in the y directions that we can write it. It is a very easy and I always tell it that please remember the mass conservation equations at this level which is very simple is that the divergence of the mass flux is equal to 0 always you can simplified into next to next level to solve the problems not remembering intermediate steps that is what makes the fluid mechanics very difficult. But fluid mechanics is easy subject if you remember the generic equations okay not the derived equations.

This is the generic equation form which is giving the divergence of the mass flux is equal to 0. Divergence of the mass flux for is equal to 0 for a steady flow and that what we can get it the derivations as just expanding these terms you can get it as well as I am defining that if you can remember this part okay just a simple that that gradient of the stream functions in y directions is equal to the scalar velocity component in the histogram that is what. Other things you can always looking these equations you can write it that is what the same thing but with a negative signs with a negative sign that part is there. So in order to remember too much okay that is what it if you try to generic equations if you try to remember it the fluid mechanics is really interesting subjects and it is in order to mug up lot of equations that is what try to have. Now another interesting problems which again the stream functions which is we are following in the Sinzel Simbala books.

which is quite advanced levels. So you just have a very interesting part that streamline functions what is coming into that. If you have the problems, okay, two-dimensional planar flow, okay, two-dimensional planar flow you have, okay. So you have this r theta plane, okay. So just trying to draw it, okay. So you have x you have y but you have a theta this is what you have r and you have the streamlines okay.

Let me like the examples what I have given it is the rotating cylinder concepts okay. So this is my streamlines. If I that again I will have a take a small delta theta okay representing a smaller segment infinitely smaller sigma segment where we can use all the calculus to define the problems. So that is what will be equal to r delta theta okay that is what is the dimensions in that directions and you have a r dimensions is there. So if you write the conservations of equations mass conservations you will get this form.

okay and I can define this the velocity components okay this is will be velocity component in the r directions and the theta directions. vr and v theta directions that what I can define it as a stream functions of this okay. Only if you look at that we are using it r delta theta okay not only delta theta and we are solving this problem. But if you have axisymmetric problems okay similar way it is a very complex flow it is that the flow is rotational symmetry okay it says moving like this you just try to understand it okay it is moving like this okay. So because of that is a axisymmetric bodies okay this along this body axis it is behaving is a symmetric condition okay.

That case we will have a no component radial theta directions only we will have a radial directions and the z direction. The problems can simplified it you can try to understand it we can make a axisymmetry bodies okay and you can see the flow patterns and that the cases we can have the continuity equations which in form of these where we are writing in terms of r and zeta, z directions okay this is if you look at the z directions this and say rotating it there is no theta component as this axisymmetry problems and we can always write a stream functions for that. Always we can write the stream functions here. This given is here.

So, we can write a stream functions for that cases. So, let me I put it this way. We can draw the stream functions for incompressible flow, two-dimensional incorruptible flow. We can do it similar way we can do for compressible flow, we can do for axis symmetry flow, also we can do for planar flow. So, many of the times we simplify the fluid flow problems into different category and we use a different stream functions to define the flow fields. So, that you can try to understand it, the fluid flow problems that the steam functions we can not only use for two-dimensional incompressible flow, we can do for compressible flow, the planners, we can do for axis symmetry problems.

So, only the functions will be the different, the functions will be the different. incompressible flow, compressible flow, planar flow and the axisymmetric cases. The functions will be the different but we can use the stream functions to make it a single dependent functions which is we can solve it, single dependence functions. So instead of u, v or rho u, rho v, we just load it on a single functions and you can solve the problems.

That is the strength of the stream functions analysis. So, that what we will have with us. Let me look at the stream functions, how we can physically interpretations of the streamlines. If I go for interpretations of the streamlines, okay. Like for examples, I have a corner. So as you know it, the streamlines will come like this, will come like this, come like this.

I am just sketching it, okay. So not exactly as I have a boundary, the L bend is there. the flow is are coming streamlines are like this okay. It is just a sketching of the streamlines not the exact pattern okay. So if you look at the streamlines it is if I just take it regions I can have the streamlines like this.

So this is the streamlines S1, this is S2 streamlines. Just I am taking it maybe small sections okay where I can write the streamlines like this. Now let I interpret it okay. I can follow a control volume concepts okay. As we did it for Bernoulli s equations same concept we can follow it or you can consider as a steam tubes okay. But since a two-dimensional flow let we consider there is a streamlines are there.

I have the S1, S2 and if I consider the control volume that this part as well as these ones it has a two section A and the B the same figure you can look at. So there will be A and B if it is that A and B if you look at these figures that means the flow is coming from here flowing is going out from this. I know very well that as this stream functions stream lines definition says to us there will be no flow across the stream lines. So that means there will be no flow here there will be no flow here.

So no mass or momentum exchange. So that means there is nothing going crossing through these ones, no flow. So whatever the flow is coming from this side that is what is going out. This is not a solid boundary. Do not be confused with a solid boundary. These are streamlines, two streamlines and there is a flow is coming here and going out.

If it is that, if this volumetric flux or the q v dot okay that is a volumetric flux that means meter cube per second this is volumetric flux. How much of volume of flow is coming in volume per unit time that should be go out from this because there is a no change of this control volume mass. So you will have a V dot both or the you can write it Qa and Qb but it is okay as given in most of the international book they do not write the distance they look it right in terms of volumetric flux, the volume per unit times. That is what they write it which is easy to interpret it as compared to this test because this is same unit, same concept but is a volumetric fluxes.

Whatever comes from BA to BB that will be equal. It is not a big issue. or you know it qA equal to qB the same concept I am talking on in terms of volumetric fluxes okay that is all in terms of volumetric fluxes I am talking about. Now if you look it there is a reasons where the streamlines are closures there is a reasons where the streamlines are far away okay. So if it is a far away the same volumetric fluxes are coming it so you can interpret it the velocity variability at these locations will be the more, it will be higher velocity here, here we will have the lesser velocity, it will have a lesser velocity. So the streamlines patterns are indicates that where we have the more velocity, okay or where we have the acceleration de-accelerated situations where like the as this narrows the area so definitely the same volumetric flux will go through this length is reduced so you will have more the velocity.

Here the length is increasing between these two streamlines. So velocity will be decreased. So it is indicating it whether the flow is accelerating or the de-accelerating. So understanding the streamlines, you can find out what would be expected velocity variations as well as whether we are getting a accelerated or de-accelerated regions. accelerated or the deaccelerated regions. Like if the streamlines are the parallels, so as it expected it that there is nothing, if there is two streamlines are parallel, so there will no change of the velocity fields.

That is what we can expect it. If it is a parallel, whatever the flow velocity will be there, that velocity will go there. So there is no acceleration, de-accelerations and the parallel. But if there is a distance between two streamlines and increases or decreases that what will be show it or the closure or the far away that what will indicate us that how the velocity restrictions will change it, how we can to know it in case of the higher velocity to lower velocity de-accelerators are happening it or the reverse accelerated zones we can identify if you try to understand the streamlines better. That is what we introduce you this virtual fluid balls to draw the streamlines then at least understand the fluid flow problems. Now let me commit that is what I am talking about that we have a volumetric flux what is going in through this stream to conjugative streamlines that is what will be coming to these ones.

flow can cross, no flow can cross a streamline. It is combined with these two streamlines. That is what I explain it. The same concept is here. To cut sectors that slice should start at the streamline A and B.

That is what is explaining to here. If you look at that, this is the flow rate or volume per unit times we can find out the same throughout this cross section slice that is what technically represented is that and I just explaining you that how we can do it. Now in mathematically also we can define it that find out what is the discharge okay how much of discharge is going it if I am assuming a velocity variations. For example as I define it the same concept I can talk about here and this is my control volume. This is my control volume, this is the two streamline S1 and S2 and on these streamlines this is the sections A, sections B and I always can consider the velocity field like you can consider very complex surface that is not a big issue.

Only the problem is that you have to do the surface integrals. If you are good in mathematics, you can do it. If you do not have a good in mathematics, there are a lot of tools, mathematical tools today are available in symbolic mathematics presentations and all. You need not to do one surface integrals. You can do through There are a lot of tools, mathematical tools are there.

You can do a surface integrations. That is not a big issue. But let me consider it very conceptually what we are looking at. There is a velocity variations. I need to compute how much of max flux we are going out.

So I have this. Over these streamlines, I have normal vectors. I now have this velocity field. So I can compose it all these components, if this is the ds, dx, dy and I have a velocity field and this. So, I can easily find out for a smaller, infinitely small dx slice, the normal vectors will be these coordinates. Just look at this geometry, define the n-vector component in x and y. and component that so we can get the normal vectors okay. You can get the normal vectors and you know it I am looking if velocity the change of volume rate of per unit time that is what is the discharge okay do not have to much complicated here.

So, it is a discharge for a infinitely small control surface if I integrate it that means if I look at v dot n dot eda okay you know it q is equal to simple v into a okay. So, if you look at how to estimate it the volumetric flux. We do a get it the velocity component into the dA component that is what we get it which we this is the velocity projections and on this this is the dot products okay that is what again I am predicting. This is the dot product of these two velocity field to get it what is the velocity components into the dA which is a simple equations what we call it q equal to A into V okay that same equations but this is what in terms of velocity factors but in this is a called it is one dimensional continuity equations. So, the same concept we are using it do not have to much but if I use it the normal position vectors and all and if I consider dA is equal to dS and these derivations if I continue it.

I will get it, I know this dv is equal to the d phi, so that is very interestingly that is what is equal to me this value, sorry, d psi. What do you mean by that? that if you have a value let me assume it phi is equal to 3 phi equal to the 2 the difference of these two that means d phi will come 3 minus 2 that is what is 1 that what will be indicating for us the what is a volumetric flux is passing through it. if I integrating it for the total surface again I am getting the same concept phi 2 minus phi 1. So that means when you draw a stream functions the difference of these two stream functions give us the Q value that may Q is equal to phi 2 minus phi 1.

So that is what is the constant values what we get it, 2 minus 3 is equal to 1. That is what is indicating for me this is what the volumetric flux what is passing through these two streamlines. So that is the indicating for us the conservation of mass across that volumetric flow into control volume through the slice Nb which is I try to explain through a control volume concept. But conceptually we try to understand it when I draw the streamlines with a different constant values. the difference of these two concerned values are indicating me what is the Q value the discharge is passing through it for a incompressible flow. But for a compressible flow also I can get it the difference of this T2 is we will talk about how much of mass flow is going through it.

So, The value indicates the difference of this is indicates for how much of mass flows is going through it. The expansion and contraction zones indicates is whether the velocity is accelerated or deaccelerated, the velocities are increasing trend or decreasing. far away or closer by that what you can interpret it with the streamlines. So as we introduce this F 16 fight of plane the streamlines or the rotating body or streamline patterns very complex flow problems you can understand from the streamlines how the flow is behaving it in terms of accelerates and deaccelerated zones in terms of how much mass flux are going on, how much mass flux are going on between two streamline functions. Here I will be give it examples of numerical examples of very simple problems is in generally comes in a get exams or engineering service exams.

Mostly they are confined with a two-dimensional continuity equations, mass conservation equations which most is based on the basic definitions we can solve these problems. Let I solve three simple problems how we can really solve these. These are the problems comes in the gate or engineering service levels. Let you have this a two-dimensional the steady problems in compressible flow in xy planes which define a stream functions as these functions. So, and a, b, c are constants and that values are given to us to verify this flow field does it satisfy the continuity equations.

I think this is quite easy that stream functions are given it which is a cubic functions okay. The stream function just to interpret it slightly is a given a cubic functions okay. in terms of a functions of x and y and we are trying to look it and a, b, c values are given. We have tried to look it whether these functions satisfy these two-dimensional steady continuity equations. The basic equations again I will come to the generic equations do not come to that.

Velocity divergence is equal to 0, that is what you will come with. I am writing in 3-dimensionals just for and being a 2-dimensional this is equal to 0 okay. It is nothing component like this okay. Do not have to only remember these equations okay. That is what and you should know how to expand that which is dot product of 2 vectors okay.

So if you look at that part so we get these functions. As I explain it without just thinking it first you should write what should be the u? u will be the gradient of stream functions in the y directions that is what that means physically if you interpreted it you draw the stream functions get a gradient of the stream function to y directions that what will give us the U component. That is things you should remember this and these equations okay. If this is the this otherwise just a mirror equations so VU will be the minus of this component. So what I will do it? since the stream functions is given to me, I can get the partial derivative of stream functions with respect to y and x, get this u component, v component, get this partial derivative component of u and v.

If it comes to 0, in this case it comes to 0, it is satisfied it okay. It is just satisfied it here. So that is what is coming here. Always you can ask me that why do you have this abc value? You can just substitute this abc value to get it this part because anyway the second derivative of these components will be respect to the x will be the 0. This does not have the functions of y, again you derivative is equal to 0.

So, it is need not to have a ABC value. I do not know why this ABC values are given. It is need not to have a ABC values because if you do a first find out u and b, again do the partial derivative, you know it if they do not have a functions of the y as well as the x so it will come to be 0. That is the reasons you do not need a VC value but it has given here that is what we make the problems is complicated unnecessarily that is what it happens. Let me commit to the examples of gate 2010 questions which is give a stream functions find out the velocity factors. okay it is a quite easy.

So it is given the stream functions you have to find out the velocity factors okay which will be velocity factors. That means you need to remember it okay that is what you say get. So U scalar component is a gradient of the y directions, v is a minus of gradient of the x directions. So you do partial derivative stream functions given, you give a just do a partial derivative of this. Now then you write v is equal to ui vj that is what.

I think these are very simple problems which we just substitute the things and can get the things. Only you should physically understand streamline functions which is otherwise it is quite easy to do a partial derivative that is what and you just substitute it get this velocity factors. That was the gate 2010 questions. another very interesting problems it is not that difficult it is a 90 degree corners okay. Let me come back to that we have the streamline of a steady ideal fluid flows okay 90 degree corners defined by the functions 12 x y okay 12 x y you can get the unit of stream functions is meter square per second which is a indicating of accelerations close to the accelerations.

That is what you can get it how I am getting it meter square per seconds. It is very easy u is equal to same d phi by dx u is meter per second multiplied by the meters comes about meter square per second. So we do not have to so that is what is again I am telling you that stream functions the unit is meter square per second is x raised to infinity. So if you have the 90 degree band you have the flow, flow is like this these are the streamlines, these are the streamlines. these are the the streamline patterns we will go with. So in these problems this is not looking the streamline patterns it is just looking it that if you at a point x equal to 2 meters okay this is x directions this is the y directions 2 meters y equal to 4 meters you get it what will be the velocity at these points.

that is very easy. So we will follow basic equations definitions as I repeatedly telling about you the basic definitions how to remember it. Estimate substitute these coordinates get this velocity component scalar velocity component then you compute the resultant velocity which you will have either here the u and the v component small and u v then you get the velocity components okay that is what is here it is only magnitude is given. So you can always get this the directions of the velocity factors as you know the u and v components. So it is quite easy problems comes in most of the gate or engineering service exams but please try to remember the definitions and all. Let me I summarize these lectures as you have to remember it not to remember it it has to I repeatedly tackling that you have to remember the basic equations of mass conservation equations for steady flow that is what the divergence of this mass flux you can remember it this basic part as I repeated so many times and this is the mirror part and if you are looking for the stream functions of cylindrical coordinate systems or if you try to interpret it this Q it will be the phi 2 minus phi 1 as I explained you the thoroughly.

So, with this I just want to conclude this lectures by giving thanks to my PhD research scholar groups who really helped us to prepare so nice presentations for you. Thank you. Thank you.