





Uniform and Non-uniform Flows

- **Uniform Flow:** The flow is defined as uniform flow when in the flow field the velocity and other hydrodynamic parameters do not change from point to point at any instant of time.
- For a uniform flow, the velocity is a function of time only, which can be expressed in Eulerian description as

$$\vec{V} = V(t)$$


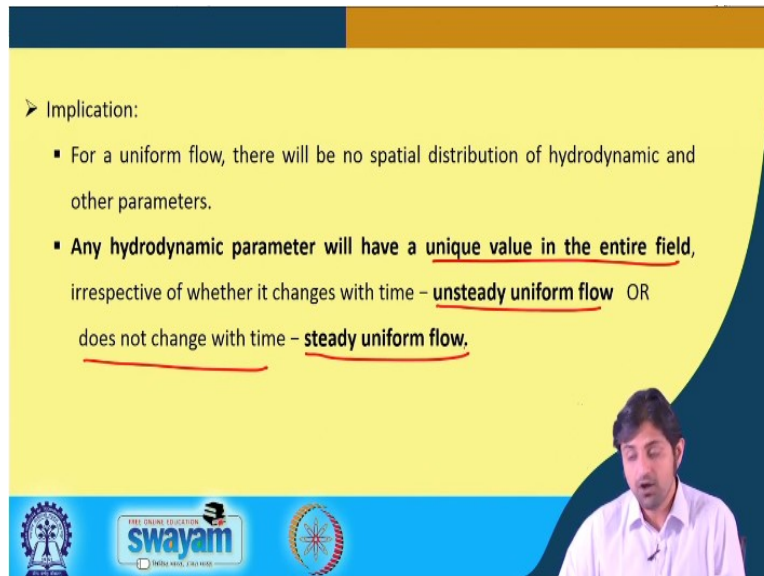
So, we have seen steady and unsteady flow. Now, we are going to see uniform and non uniform flows, the definition of the uniform and non uniform flow. Uniform flow, the flow is defined as uniform flow when the flow field that is the velocity and the other hydrodynamic parameters do not change from point to point. So, in a steady flow it was not changing with respect to time, here, these properties should not be changing with respect to the space at any instant of time.

So, uniform flow is the one, where the fluid properties do not change with respect to the space. For a uniform flow, the velocity is a function of time only that is it. Which can be expressed in Eulerian description as $\vec{V} = V(t)$. So, $v \neq v(x, y, z, t)$, but only $v = v(t)$. So, these x, y, z components vanish when it is a uniform flow.

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➤ Implication:

- For a uniform flow, there will be no spatial distribution of hydrodynamic and other parameters.
- Any hydrodynamic parameter will have a unique value in the entire field, irrespective of whether it changes with time – unsteady uniform flow OR does not change with time – steady uniform flow.



Implications for a uniform flow; there will be no spatial distribution of hydrodynamic and other parameters, as we told, because the parameters really does not depend upon the location. Any hydrodynamic parameter will have a unique value in the entire field, irrespective of whether it changes with time. Therefore, now, we have two parameters here is space and time, time was with if something depends upon time, it will be unsteady, if something does not depend upon time it will be steady.

So, combining the steady and unsteady with uniform and non uniform we can have either unsteady uniform flow, which means the velocity will be the only a function of time, or actually it might not even change with time as well, this type of flow can be called a steady uniform flow. So, the flow properties are neither going to change in space or with respect to time. Good.

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➤ **Non-Uniform Flow:** When the velocity and other hydrodynamic parameters changes from one point to another the flow is defined as **non-uniform**.

➤ Important points:

- For a non-uniform flow, the changes with position may be found either in the direction of flow or in directions perpendicular to it.
- Non-uniformity in a direction perpendicular to the flow is always encountered near solid boundaries past which the fluid flows.

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Now, non-uniform flow, when the velocity and other hydrodynamic parameters changes from one point to the other, which is exact opposite of uniform flow. So, the properties in uniform flow were not dependent on the position of the particle or from in these respect to the space, but non-uniform flow it will change from one point to the other and such flows are called non uniform flows. These are the some of the terms that you will be you will be encountering again when we start doing the open channel flow for example, that is why it is important to go through it once again.

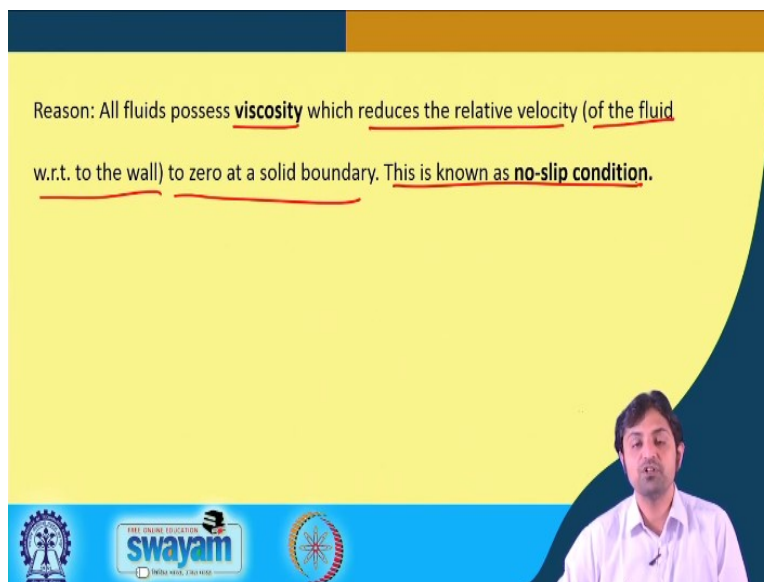
Now, important points are for a non uniform flow, the changes with position may be found either in the direction of flow or in the direction perpendicular to it. So, we have said that, for a non uniform flow, the particle properties or the fluid part fluid particle properties changes with position. So, position or the direction can be infinite, but generally, it is a customary in fluid mechanics to calculate the changes in two directions. One direction is the flow direction.

Flow direction means for example, the river flows say in one direction x direction, then we will calculate the changes of the properties in the direction of the flow. In our case, it is x direction and the direction perpendicular to it. So, suppose, there is a river flowing like this and our coordinate system is x and y , for example, this is two-dimensional space. So, the best way to calculate the changes is in this direction and this direction perpendicular. Good.

So, non uniformity in a direction perpendicular to the flow is always encountered near solid boundaries past which the fluid flows. We will come across it later in our lectures, that how near solid boundaries this velocity component changes and that is one of the main reasons why near the solid boundaries we need to, you know, calculate the changes and thus they introduce non uniformity in the direction perpendicular to the flow.

So, if suppose, an example is there is a plate the fluid will be flowing like this. So, there will be velocity here, but this plate is at rest, we will come to it later as well, but since the fluid is flowing across this, then this is at rest and the particle adjacent to it will try to be at rest as well or the particle nearest to this plate will be at rest. Here, the velocity is suppose, V of the river I mean the stream velocity here, is zero. So, as you can see during this distance the velocity goes from V to zero. Good. Alright then. Nice.

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Reason: All fluids possess viscosity which reduces the relative velocity (of the fluid w.r.t. to the wall) to zero at a solid boundary. This is known as no-slip condition.


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Reason, as I was telling you before for the, I mean, in the earlier point that we said, all fluids possess viscosity, sorry I will use the pen, all fluid possess viscosity. Viscosity in a layman's term or in a old term can be said as friction in fluids for example, and because of the friction, this reduces the relative velocity of the fluid, with respect, to the wall to zero at solid boundary as I was telling you and this condition is called as a no-slip condition and because of the no-slip condition, there is a non uniformity of the flow in the y direction.

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Streamline

- In a fluid flow, a continuous line so drawn that it is tangential to the velocity vector at every point is known as a **streamline**.
- If the velocity vector $\mathbf{V} = u\hat{i} + v\hat{j} + w\hat{k}$
- Then the differential equation of a streamline is given by

$$\frac{dx}{u} = \frac{dy}{v} = \frac{dz}{w}$$


Now, what is a stream line? In a fluid flow, a continuous line so drawn that it is tangential to the velocity vector at every point is known as stream line. So, what is the streamline it is a continuous line such that this line is tangential to the velocity vector at every point. So, if you draw such a hypothetical line this is called a stream line. Suppose, if the velocity vector is given by in \hat{i} direction as u , in \hat{j} direction as v and in \hat{k} direction as w , better to write it like this, $u\hat{i} + v\hat{j} + w\hat{k}$.

Then the differential equation of the stream line, this you have already done before but I will straight away go ahead and write the equation is $\frac{dx}{u} = \frac{dy}{v} = \frac{dz}{w}$. So, this is the equation of the stream line. Alright.

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Practice Problem

In a flow the velocity vector is given by $V = 3xi + 4yj - 7zk$. Determine the equation of the streamline passing through a point $M = (1, 4, 5)$.

Solution:

The equation of the streamline is

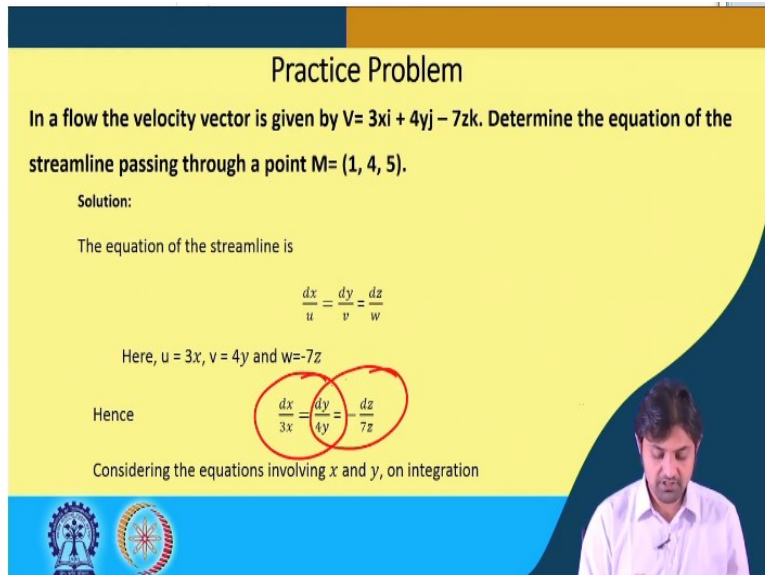
$$\frac{dx}{u} = \frac{dy}{v} = \frac{dz}{w}$$

Here, $u = 3x$, $v = 4y$ and $w = -7z$

Hence

$$\frac{dx}{3x} = \frac{dy}{4y} = \frac{dz}{-7z}$$

Considering the equations involving x and y , on integration



So, now, we will end this lecture with this practice problem, first, we will finish this practice problem and then finish it. The question is, in a flow the velocity vector is given by $3x\hat{i} + 4y\hat{j} - 7z\hat{k}$. This means the u component is $3x$, y component is $4y$, and z component is minus, I will just do the eraser, $-7z$. Now, the question is, determine the equation of this streamline passing through a point M and where M is given by $(1, 4, 5)$, the procedure is very simple.

We have done this equation before, the equation was the equation of the streamline is given by $\frac{dx}{u} = \frac{dy}{v} = \frac{dz}{w}$, this we have seen in the last slide. This is the most important equation and we already know u , for example. So, we can write as I wrote before, u is $3x$, v is $4y$, w is $-7z$, very it is very important to note that we must be writing these components whatever things we know from before when we start solving a problem we should be writing it down first.

For example, $u = 3x$ we have written and so on. So, this equation can be written as $\frac{dx}{3x} = \frac{dy}{4y} = \frac{dz}{-7z}$ because Z was $-7z$. Let, me take away this ink. Now, the procedure is we will

solve this one at a time, $\frac{dx}{3x} = \frac{dy}{4y}$ and secondly we are going to solve this two.

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$\frac{1}{3} \ln x = \frac{1}{4} \ln y + \ln C'_1$ Where, C'_1 = a constant
 Or, $y = C_1 x^{\frac{4}{3}}$ Where C_1 is another constant.

Similarly, by considering equations with x and z and on integration
 $\frac{1}{3} \ln x = -\frac{1}{7} \ln z + \ln C'_2$ Where, C'_2 = a constant
 $z = \frac{C_2}{x^{\frac{3}{7}}}$ Where, C_2 is another constant.

Putting the coordinates of the point M (1, 4, 5). $C_1 = \frac{4}{(1)^{4/3}} = 4$ and $C_2 = 5 \times (1)^{7/3} = 5$

The streamline passing through M is given by
 $y = 4x^{4/3}$ and $z = \frac{5}{x^{7/3}}$

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And when we integrate the first equation will can be written as $\frac{\ln x}{3} = \frac{\ln y}{4} + \ln C'_1$, where C'_1 is a constant. And how do we obtain this C'_1 that will talk about it next. So, in other word, we can simply write because all these are ln we can write, $y = C_1 x^{\frac{4}{3}}$. And on integrating the second equation as I told this one here, $\frac{dy}{4y} = -\frac{dz}{7z}$.

So, either we can do integrate this equation the second, or we can also use this equation these two equations together. So, in this practice problem we have taken the x and z. So, this gives

$\frac{\ln x}{3} = -\frac{\ln z}{7} + \ln C'_2$, where C'_2 is again a constant. So, we can get simply $z = \frac{C_2}{x^{\frac{3}{7}}}$. So, these are the two equations that we got, correct. Now we need to obtain C_1 and C_2 , what is the way we have said?

Find the equation of the stream line which passes through a point M, where the coordinate of M was given, x was 1, y was 4, and z was 5. This also means that this (1, 4, 5) should also be satisfying this equation and this equation, and on substituting in y in this equation, we can get C_1 because y here is 4, so, that is, 4 and x was 1, so we have put 1 here, in this way, we get C_1 is $C_1 = 4$.

Similarly, using 1, 4, 5, and putting it into z equation here again, C_2 can be written as $\sqrt[7]{zx^{\frac{4}{3}}}$. z is 5, right, so, this is 5 here, and x was 1 again, and this gives $C_2 = 5$. So, the stream line passing through M can be given by $y = 4x^{\frac{4}{3}}$, and $z = \frac{5}{x^{\frac{4}{3}}}$. So, this is the problem where we saw how to solve the streamline equation. And this is enough for today. So, we will resume in the next lecture. Thank you so much for watching.