Hydraulic Engineering
Prof. Mohammad Saud Afzal
Department of Civil Engineering
Indian Institute of Technology Kharagpur

Lecture – 01
Basics of Fluid Mechanics – I

Welcome everyone. This is the first lecture of the course called hydraulic engineering and I will start with fluid properties. My name is Mohammad Saud Afzal. I am a faculty at department of Civil Engineering in IIT Kharagpur.

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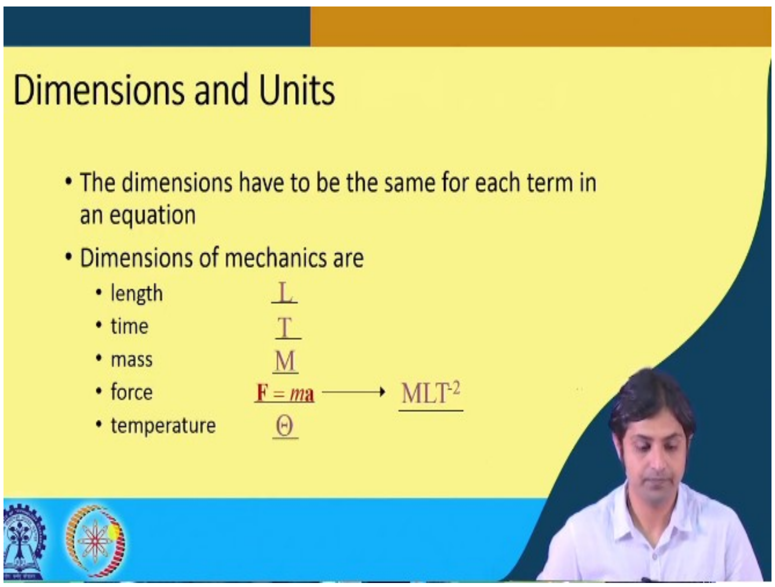

So to start, one of the most basic things that we need to know about the properties or the dimensions and units. As you have already read before in your class 10th and 12th the dimensions of each term on the 2 sides of an equation have to be the same. That is the most important requirement for the dimensional analysis. Some of the common dimensions of mechanics are length which is denoted by L, time which is denoted by T , mass which is denoted by M.

These are the 3 basic dimensions that we generally consider, force is given $F=ma$ and in terms of dimensions it can be written as MLT^{-2} because mass is M and acceleration is rate of change of velocity which gives LT^{-2}, temperature is denoted by Θ.

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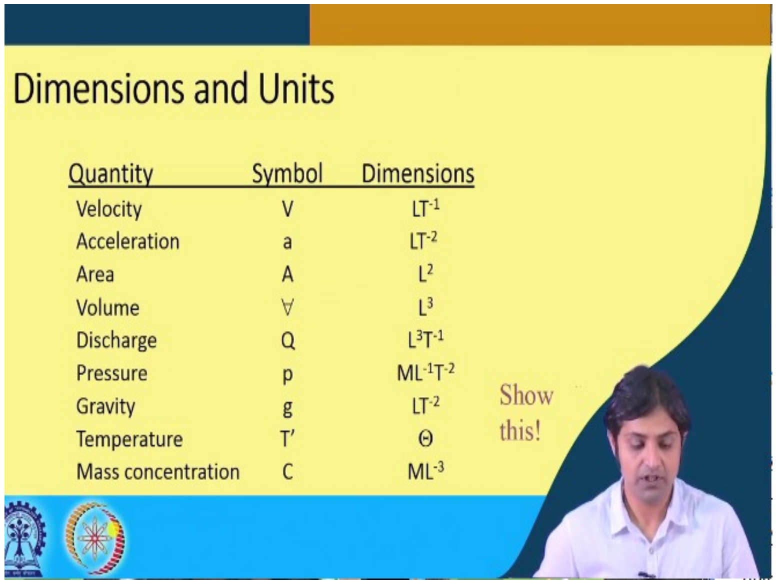

So, some of the important quantities symbols and their dimensions are given. Velocity is given by V , it is dimensions are LT^{-1} . Acceleration similarly, as we seen the last slide is LT^{-2}. Area is given by L^2, volume is L^3, the discharge is L^3T^{-1}, pressure is $ML^{-1}T^{-2}$ and gravity is LT^{-2}. So, you should be able to show some of these properties by doing this analysis by writing the basic equations. For example, what you can do is…for example, pressure is given by

$$P = \frac{F}{A}.$$

So, you can write down the dimensions of force you can break up force into mass into acceleration divided by area. So, mass basic dimensions you know, acceleration, you know and area also you know, so, for example, pressure comes out to be $M L^{-1}T^{-2}$.

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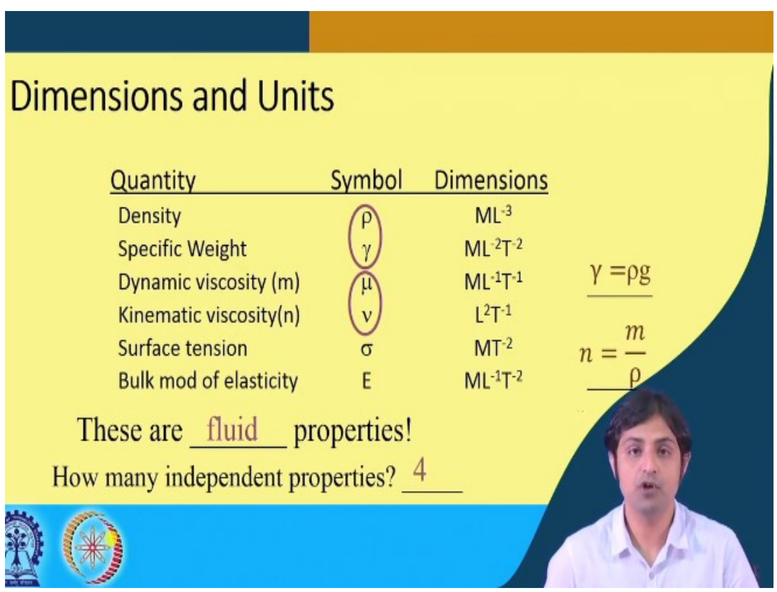

So, density is given by ρ, its dimension is ML^{-3} its specific weight is given as γ, we will come to that what specific weight is dynamic viscosity is the dimension, these are some of the more complex quantities for which we have simply written down the dimensions, but you can actually derive it, we will see, kinematic viscosity ν is given as L^2T^{-1}, surface tension σ is MT^{-2}, bulk modulus of elasticity E, is given as $ML^{-1} T^{-2}$.

So, these are actually fluid properties, these are some of the most important ones. As I was telling you I am going to tell what specific weight is, so specific weight can be related to the density by this equation, $\gamma=\rho g$, and similarly, we can relate kinematic viscosity and dynamic viscosity through density again. So, n here is ν , m is dynamic viscosity, m as can be seen in the slide.

So,

$$\nu = \frac{\mu}{\rho}$$

,

and now, as you can see since we can relate density and specific weight through ρ, and we can also relate dynamic viscosity and kinematic viscosity through ρ. So, there are how many independent properties one surface tension, second is bulk modulus of elasticity, one of the viscosities and you can either choose density as specific so, that means there are only 4 independent properties presented in this slide.

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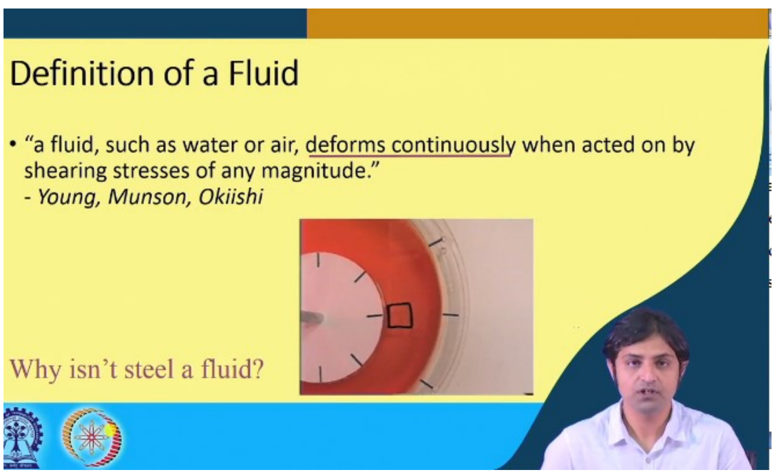

What is the definition of a fluid? According to Young, Munson and Okiishi in their book, 'a fluid such as water or air, deforms continuously when acted on by shearing stresses of any magnitude.' One of the important word here to note is "deforms continuously". So, that is what a fluid is. So, I have a question, why is not still a fluid? Because the steel does not deform continuously, if a small force is applied and by definition, if you apply whatever single force simple most force, the fluid will deform.

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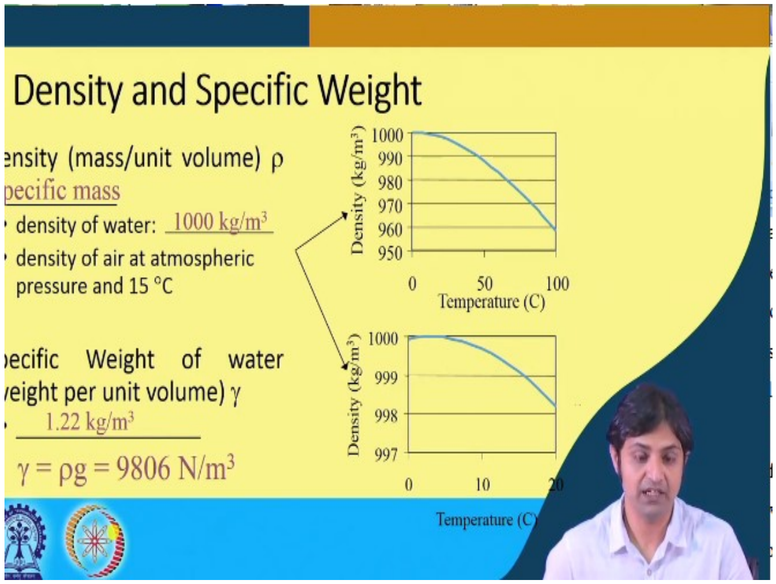

Now, going to some of the density and specific weight just to give you a brief revision from before, density is defined as mass per unit volume ρ. So, this is called specific mass, density of water is 1000 kilogram per meter cube, density of air at atmospheric pressure and 15^0 centigrade is given by this curve, this is a function of the temperature. This figure right now shows the variation of the density of air varying from 0 to 100.

And this figure now, it shows a zoomed image. So, what happens mostly between 0 and 20 the reason of doing this is, this is the temperature that we encounter mostly 0 to 20 degrees in nature, especially in this part of India, it goes up to 30-35 as well, but this gives the zoomed image, so that you are able to see in detail how the pressure variation is. A specific weight of water that is called γ is given as 1.22 kg/m^3 for air and specific weight of water is ρg 9806 N/m^3.

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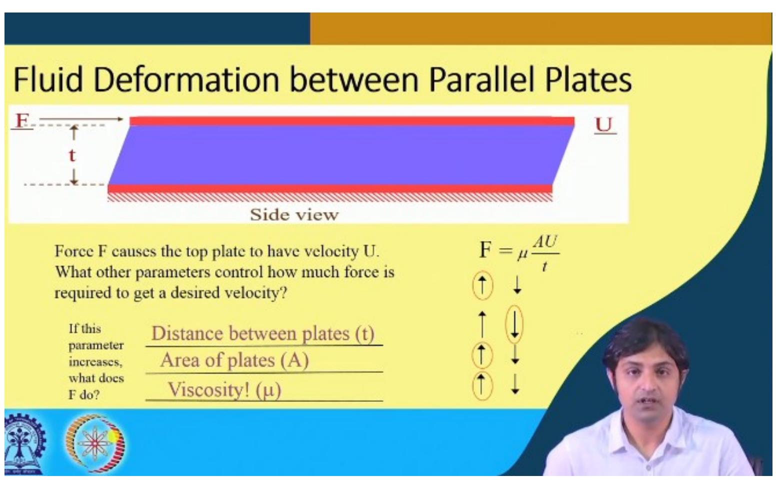

So, proceeding to our next topic of the fluid properties, it is important to note that we defined fluid as the something that deforms. So, we will see an example, where fluid deformation between parallel plates is done. So, this is the side view, where a force F is applied and the upper plate starts moving with velocity U and the thickness of the…the total thickness is given here denoted as t. So, as I told you force F causes the top plate to have velocity U.

Now, what are the parameters can control how much force is required to get a desired velocity, as has been seen from the experiment, distance between the plates which is denoted as t is quite an important parameter. Area of the plate A is also very important parameter viscosity ν is also important parameter. But what happens if these parameter increases and what does F do. So, F is found out to be directly proportional to μ area speed, but inversely proportional to the distance between the plates.

So, these arrows indicate one the left side that the parameters or the *F* increases on the right hand side denotes this decreases. So, for *F* to increase as you can see, on the left hand side the distance between the plates should go down or if the distance between the plates go down the force will increase or if increase the area of the plate the force goes up and if the viscosity of the fluid increases then the force increases.

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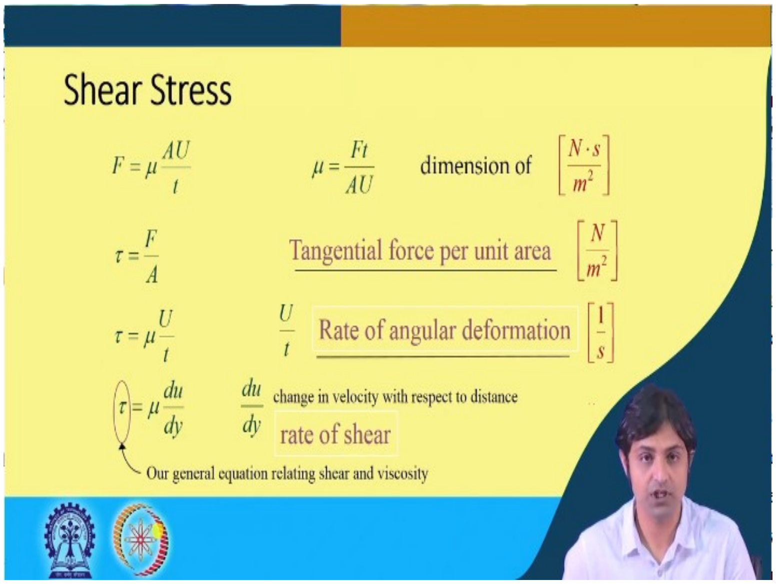

Shear stress, so force as we have seen in the last slide is given as

$$F = \mu \frac{AU}{t}$$

So, μ can be written as

$$\mu = \frac{Ft}{AU}$$

the dimensions of μ is Newton second per meter square, I think you should do this derivation after this lecture is finished at your own convenient time. Shear stress now is actually defined as $\tau = \frac{F}{A}$, and this is this tangential force per unit area and its dimension is N/m^2 or units is N/m^2.

Shear stress therefore, can be written as $\tau = \mu \frac{U}{t}$. What is $\frac{U}{t}$? U is the velocity, t is the distance between the plate, or we can also call it as a rate of angular deformation. So, the units of this is $\frac{1}{seconds}$, τ, therefore, can be written as $\tau = \mu \frac{du}{dy}$ as we have already said it is rate of angular deformation. What is $\frac{du}{dy}$? It is change in velocity with respect to the distance as in terms of differential equation. So, shear stress is our general equation that relates shear and viscosity. And this $\frac{du}{dy}$ also called rate of shear.

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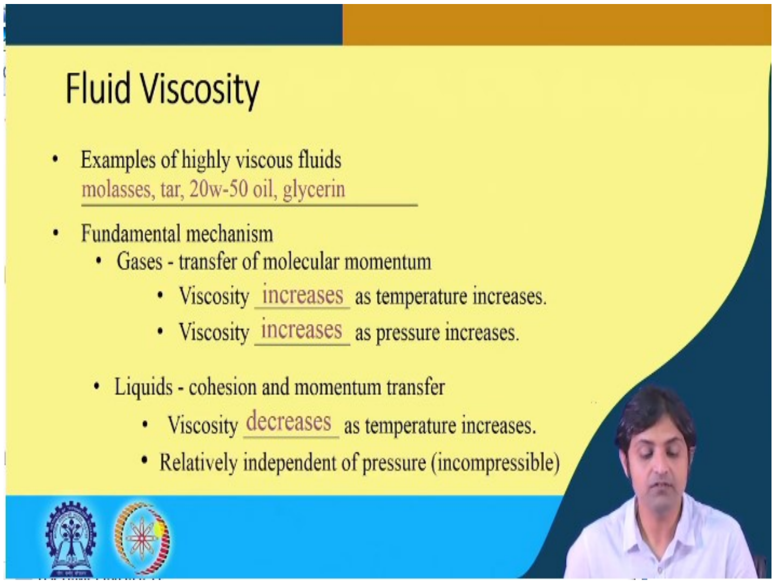

Coming to fluid viscosity. Some examples of highly viscous fluids are molasses, tar, if you have more concentration of oil or glycerin, these are some of the things which are highly viscous. So, what is the fundamental mechanism of viscosity in gases, it is due to the transfer of the molecular momentum. In case of gases, the viscosity increases as temperature increases, also the viscosity increases as the pressure also increases, liquids, it happens due to cohesion and momentum transfer and here the viscosity decreases as temperature increases.

It is very important to note that it is quite different from the gases where the viscosity increases on increasing the temperature. Also the viscosity in the liquid is relatively independent of pressure, for example, incompressible fluid.

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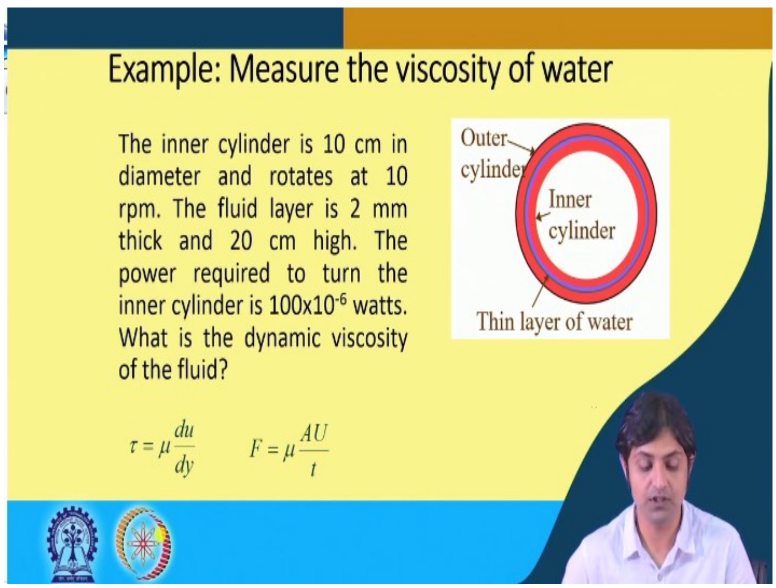

Now, one of the examples that we will talk about is used to measure the viscosity of the water. It is a very standard experiment. So, the inner cylinder in this particular case I have broken it down in form of a numerical as well. So, the inner cylinder here is 10 centimeters in diameter, there is another outer cylinder as well. So, the inner cylinder being 10 centimeters in diameter and it rotates at 10 rpm *rotations per minute*, the fluid layer in between those cylinders is 2 *mm* thick and the height at which it is 20 *cm* high

If it is given that the power required to turn the inner cylinder is $100 * 10^{-6}$ *watt*. What is the dynamic viscosity of the fluid? So, as you have seen one of the equations that can be used to find out the dynamic viscosities $\tau = \mu \frac{du}{dy}$ and $F = \mu \frac{AU}{t}$. These are the same equation one is it in the differential form other in the form which we normally write we require in this case as well.

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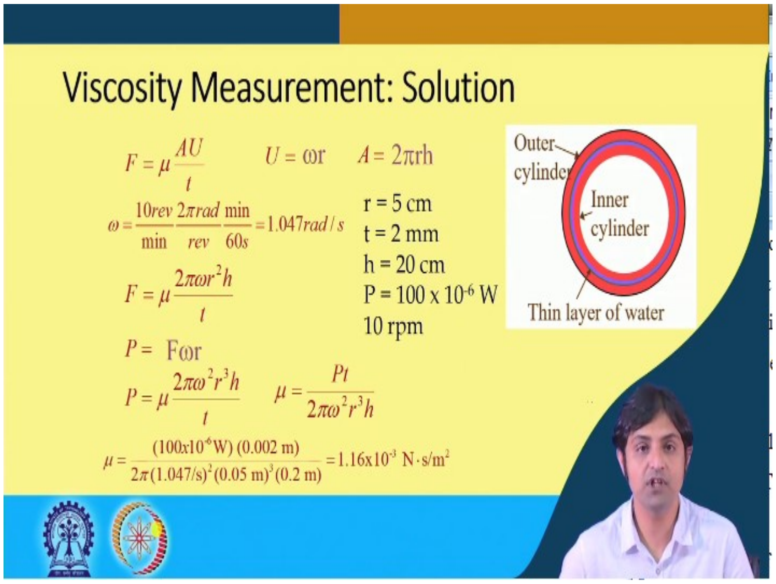

So, for to be able to solve this I have written all the quantities that I know, radius 5 *cm* of the inner cylinder, thickness 2 *mm*, fluid height is 20 *cm*, power is $100*10^{-6}$ *watt* and ω is 10 *rpm*. So, F is given as $F = \mu \frac{AU}{t}$, we need to know what *U* is, *U* in this case will be r $\times \omega$ or $\omega \times$ r. So, what actually is ω? $\omega = \frac{10 \times 2\pi}{60}$ if you do this calculation it will come out to be 1.047 radians per second.

ω can be directly obtained from this rpm value. Now, we also need to find out what an area is, area is A= $2\pi \times r \times h$. So, it will act up across all the surface of the cylinder, and we know the value of *r* we know the value of *h*. So, *F* is written as μ, *A* is written as $2\pi \times r \times h$, and *U* is written as r $\times \omega$, t is as it is. So, if we break it down this equation in terms of ω and r this is what we get $F = \mu \frac{(2\pi \times \omega \times r^2 \times h)}{t}$ the power is

$$P = F \times V$$

force into velocity or $F \times \omega \times r$.

So, power can be written using the equations of F and P given above

$$F = \mu \frac{(2\pi \times \omega^2 \times r^3 \times h)}{t}$$

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and using this equation if you take t on the other side and $2\pi \times \omega^2 \times r^3 \times h$ down you can obtain

$$\mu = F \frac{t}{(2\pi \times \omega^2 \times r^3 \times h)}$$

and on substituting the values P as $100*10^{-6}$ *watt* thickness as 2 *mm*, ω we already found out it was 1.047 and r^3, r is 5 centimeters or 0.05 *m*, h is 0.2 *m*, we can get $1.16*10^{-3} \frac{N.s}{m^2}$.

So, this is one of the standard ways of finding the viscosity of water or any fluid for that purpose. And that also demonstrate one of the examples in real life how are you going to solve for the value of the viscosity.

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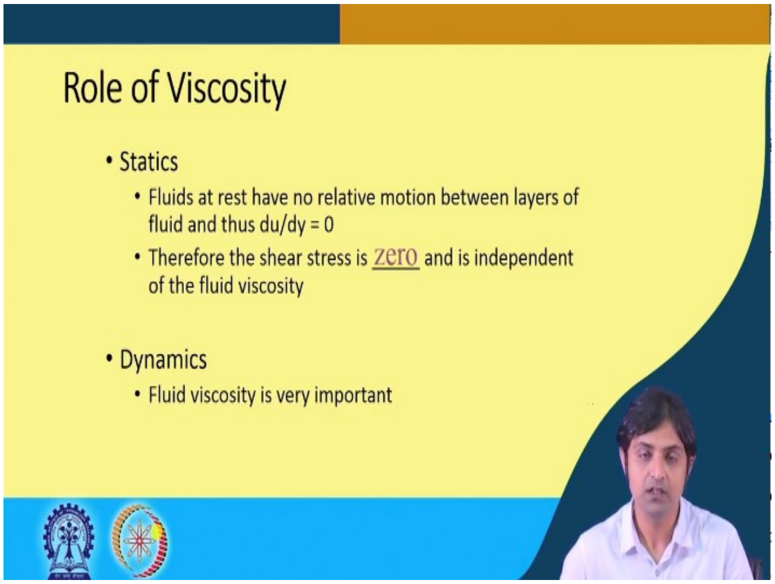

Now, what is the role of viscosity in statics? Its important to note that fluid at rest have no relative motion between layers of the fluid. Right? So, statics means rest. So, when the liquids do not move, there will be no velocity difference between those 2 fluids. So,

$\frac{du}{dy}$= 0. Therefore, the shear stress is 0. In case of fluid statics and is independent of the fluid viscosity. In case of dynamic fluid viscosity is very important because the water moves, different layers will have different velocity and thus will be the velocity gradient $\frac{du}{dy}$.

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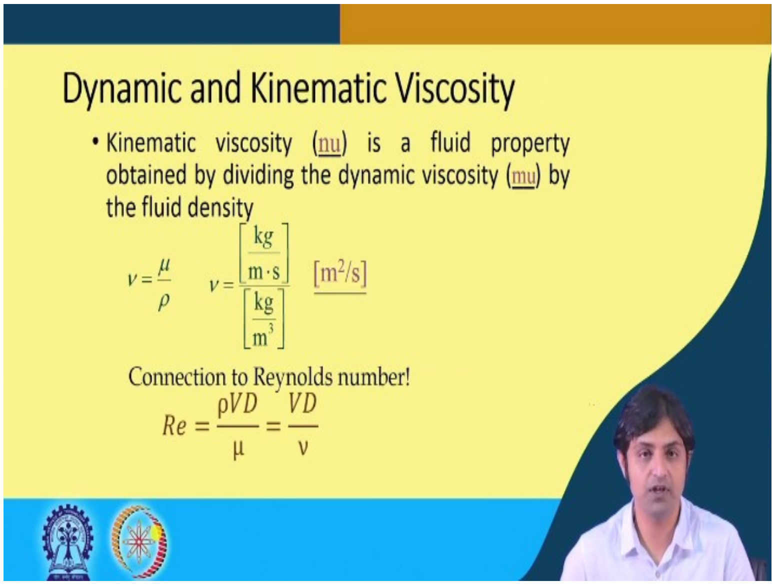

So, now, as we briefly touch dynamic and kinematic viscosity, kinematic viscosity also called ν is a fluid property obtained by dividing dynamic viscosity μ by fluid density. So, μ and ν are related by fluid density. As given below, $\nu = \frac{\mu}{\rho}$ and if we write it in terms of unit, we get ν as m^2/s, as we have seen in the slides at the beginning of this lecture. This also has a connection to Reynolds number. Reynolds number is a concept that we will study some weeks from now, but I think this is a fluid mechanics and hydraulics class so it is important to introduce the value. Reynolds number is $Re = \frac{\rho VD}{\mu}$. And since, as we can see from the above, that $\mu = \nu \times \rho$, we can simply write Reynolds number $Re = \frac{VD}{\nu}$ V is the velocity, D is characteristic dimension or in case of a particle of sphere it could be a diameter and nu is the kinematic viscosity of water.

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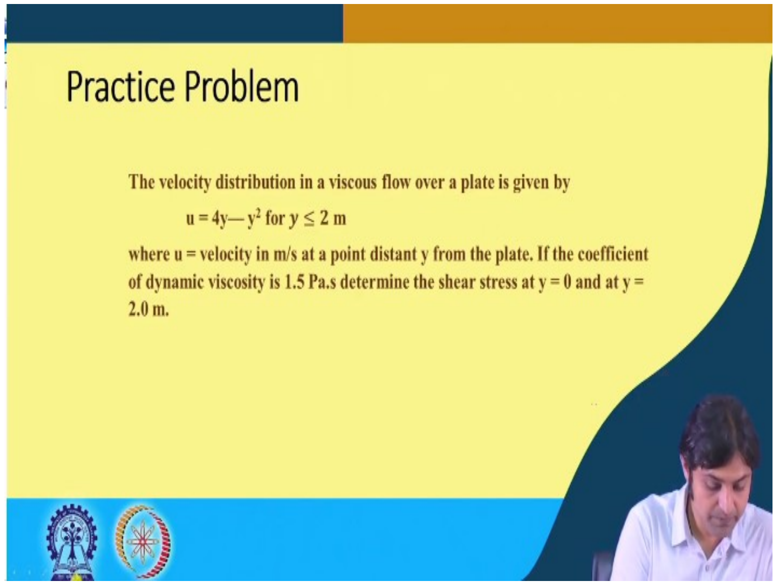

So, we are actually going to solve couple of problems now, where you will see how to, you know, be able to solve for the shear stress at under different conditions. So, the question says the velocity distribution in a viscous flow over a plate is given by $u = 4y - y^2$ for y which is less than 2 meters. Here, u is velocity in m/s at a point distance y from the plate if the coefficient of dynamic viscosity μ is 1.5 *Pa.sec* determine the shear stress at $y=0$ and advisable to 2 m. Okay. So, how are we going to solve this?

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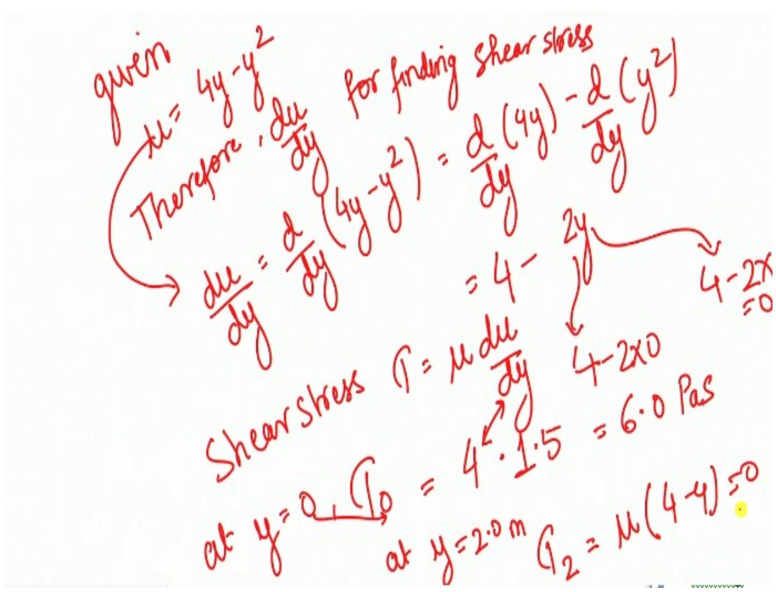

We will so what is given is $u = 4y - y^2$. Right? So, therefore, the first step is always finding $\frac{du}{dy}$ for finding shear stress, this is very important. So, using this equation here, if you do $\frac{du}{dy}$. So, going to the basic differential mathematics *4y* will become as, so $\frac{d}{dy}$ $(4y - y^2)$ will be $\frac{d}{dy}$, I am going very slow for this one because I am not sure if it is too obvious for you. So, this can be written as 4 because this $\frac{d}{dy}$ of 4y, 4 is a constant. So, this will turn out to be 4 and $\frac{d}{dy} = y^2$, 2 will come out. So, it will be *2y*. Okay?

Then we also know that shear stress $\tau = \mu \frac{du}{dy}$, we have already found out $\frac{du}{dy}$ and we already know what μ is. Right? So at y is equal to 0. So, at 0 what I do is I indicate shear stress at 0, as τ_0 , it will be given as. So, this will $\frac{du}{dy}$ will be 4, because *(4- 2*0)* multiplied by dynamic viscosity which was 1.5. So this comes out to be 6.0 *Pa.sec* . In other case, at y is equal to 2 *m*. If you do here *(4 - 2*2*) that comes to be 0. Therefore, $\tau_2 = \mu^* (4 - 4)$ that is 0. So, this is the first question that is there. So, now we can go and see.

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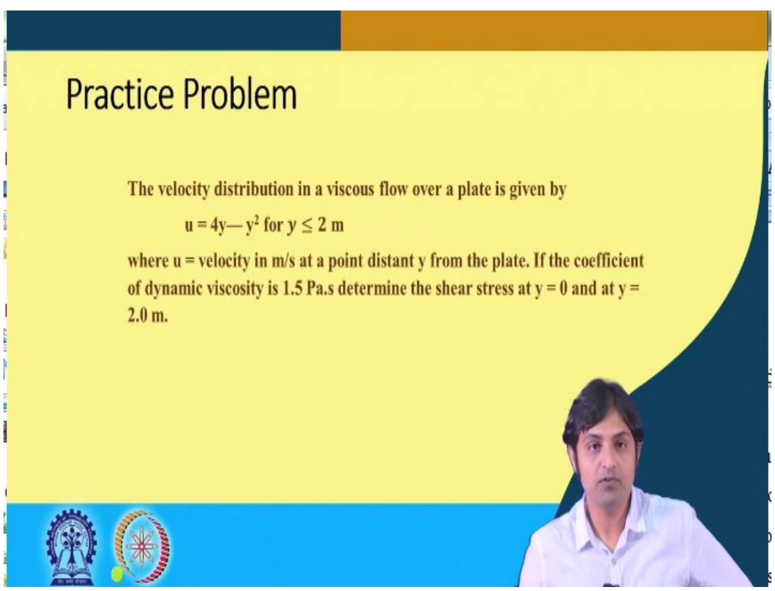

So, this problem we were able to solve. Now, there is one another problem related to this the first part.

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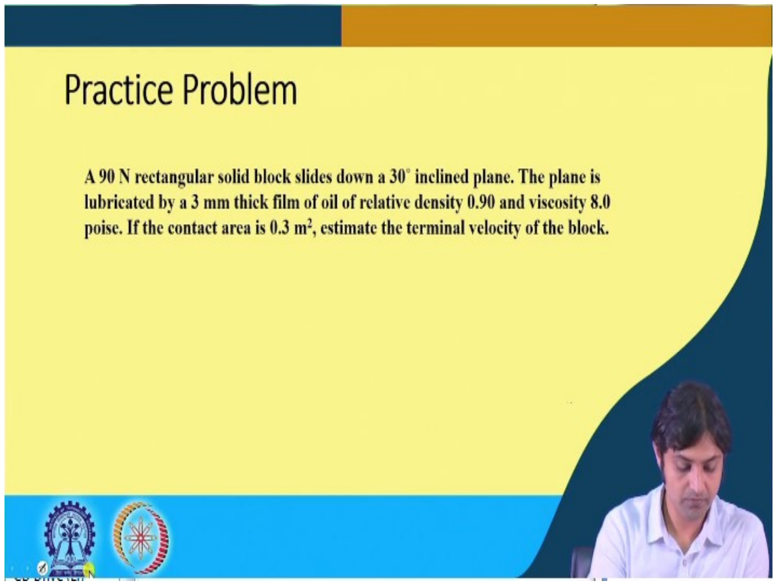

And that also we are going to solve. So, what it says is a 90 *N* rectangular solid block slides down a 30^o inclined plane, the plane is lubricated by a 30 *mm* thick film of oil of relative density 0.9 and viscosity 8.0 *poise*, if the contact area is 0.3 m^2, estimate the terminal velocity of the block. Here also as you can observe here the most important part here is the viscosity. Because of this viscosity there is going to be friction or shear stress and then the block will have some friction and therefore, because it will counter the weight of the gravity and which is generally taking the block down and then now, let us actually start how to you know, solve this problem. So, we are going to divide screen again.

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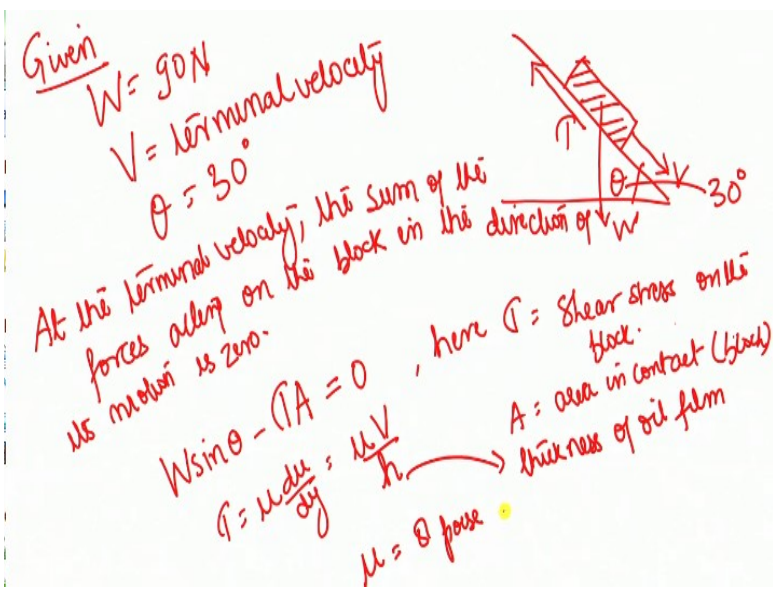

So, as last time it is important to always write down what are the things that are given. So, we are given that the weight of the block is 90 N . One of the other things that we can actually do is we can try to have a rough diagram, this is theta and this is the block, suppose this is moving with velocity V or terminal velocity which has been achieved. So, when it is moving down under the influence of the weight, there is a shear stress that is opposing the motion of the block and the weight will be acting W downwards.

So, you as you would recall you used to have something called free body diagram in your mechanics. This is very similar just to make the block. So, weight is given. Lets assume V is the terminal velocity, which is the value that is asked in the question and theta is given us 30^o. So this Θ is actually 30^o . So important to note that at the terminal velocity the sum of the forces acting on the block in the direction of its motion is 0. So, what happens at the phenomenon called terminal velocity? If there is the net force acting on the block is 0 then only there will be a terminal velocity. Otherwise, the block will accelerate or deaccelerate depending upon in which direction the force is larger. So, if we write down the force balance equation, so, we can see the weight is acting downwards. Right?

So, there will be a component of the weight acting in favor of the motion and that component is written as $W.sin\Theta$ and what is opposing the motion is the shear stress τ. Right? So, the force due to the shear stress is given by $\tau \times A$ and that should be equal to 0 in case of terminal velocity. Here τ is shear stress on the block, A is the area in contact area or the area of the block because the whole area is in the contact and τ is $\tau = \mu \frac{du}{dy}$ or μ if the terminal velocity is V and y is the thickness h $\tau = \mu \frac{V}{h}$ because that is the area that is in contact. So, h is thickness of oil film, μ is given as 8 *poise* or in terms of *Pa.sec* .

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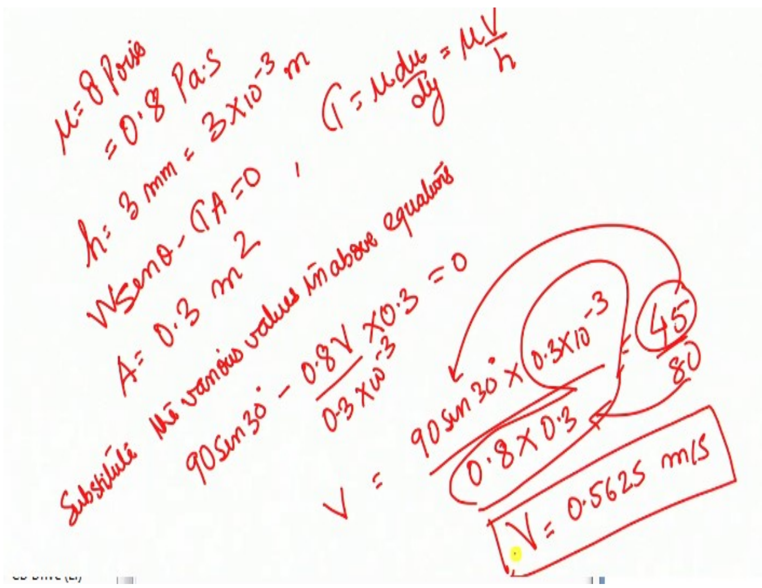

So resuming mu is 8 *poise*. And in terms of Pascal second is 0.8 *Pa.sec* simple conversion, h is given in the question as 3 *mm* or $3*10^{-3}\ m$. This is quite important to note down because now I have a new screen so what I am going to write down are the some of the important equations that we have seen in the previous white screen. So, one of the equations was

$W.sin\Theta - \tau \times A = 0$ and $\tau = \mu \frac{du}{dy} = \mu \frac{V}{h}$. Okay? Here, area is given as 0.3 m^2.

Now if we substitute the various values and above equations, so $90.sin30 - 0.8 \times \frac{V}{0.3\times10^{-3}} \times 0.3 = 0$. So what we get, so bring this side $V = \frac{90\times sin30 \times 0.3\times10^{-3}}{0.8\times0.3}$

or this becomes $\frac{45}{80}$, so this will be *0.5625 m/s*, we can verify this equation the calculation but $90 \times sin30$ = *45*. And if you do $\frac{0.8\times0.3}{0.3\times 10^{-3}}$, this will give you the value of *80*.

So how this has been done? So this 45 comes from here and using these 2 this comes here. So, I know it is, you can get confused, that is why I am trying to point it. So, this gives us the terminal velocity *0.5625 m/s* . Great. So, now we can proceed. So these were the 2 problems that we decided to solve for this particular lecture. Now, the next topic is going to be the fluid properties and this we will take up in the next class. Thank you.