Hydraulic Engineering
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Lecture- 22
Boundary Layer Theory (Contid..)

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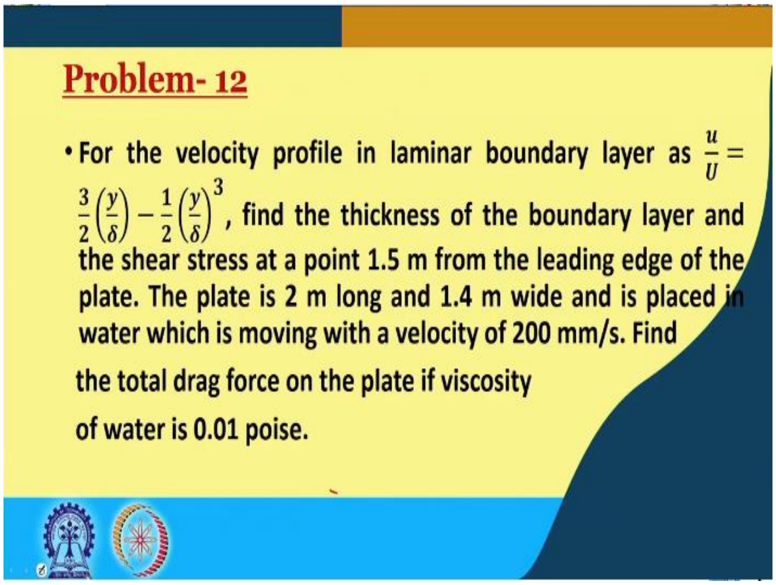

Welcome back. So, till now we have seen many problems on the laminar and turbulent boundary layer. This is a similar type of question that we did last time but instead this has some values in it. So, we have been given the thickness of the boundary layer, we have been given the length of the plate, we have been given how much wide is it and what is the U it is given, the viscosity is given. So what I am going to do is, we are going to solve this particular problem.

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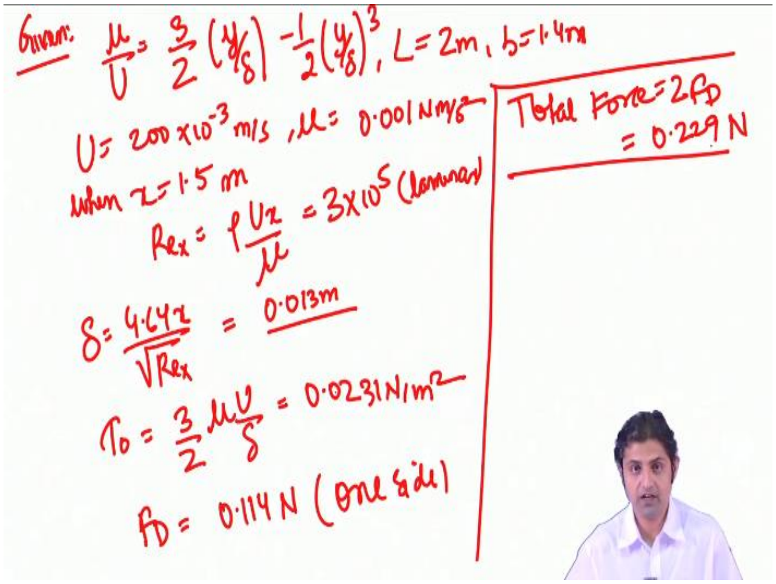

I mean, not in the entire detail but I will outline the procedure. So, velocity profile is given. So, I will definitely write what is given. So, u / U is given 3 / 2, so, this is the, sorry, 1 / 2, and we also have been given that the length of the plate is 2 meter, b is 1.4 meter and the U is also given as 200 into 10 to the power minus 3 meters per second and mu is also given as 0.001 Newton meter per second squared.

So, what do we do? When x is equal to 1.5 meter, Reynolds number at x is going to be ρ Ux / mu. So, after you substitute all these value, it will come 3 into 10 to the power 5. So, basically, it is laminar. And for laminar boundary layer delta, we know, 4.64 x under root Re at x and after substituting this x and Re what we get is 0.013 meter. So, this is the boundary layer thickness. So, boundary layer thickness and for that we have already have derived 3 / 2 mu U / Delta and after substituting in the values, you will get 0.0231 Newton per meter square. This is simply substituting in the values.

And similar, if we put in F D, the drag force that we have derived in the last thing, so it will come out to be 0.114 Newton. You should verify it actually. Just substitute in the values, because this was on one side. So, total is going to be, total force is 2 sides, 2F D, so, it is going to be 0.229 Newton. So, this is the same question but with numerical values

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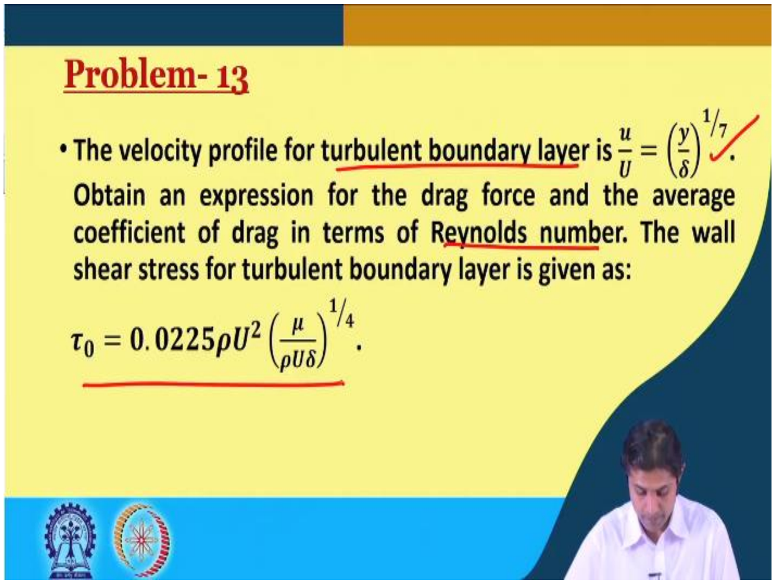

So, now we solve this question. This is typically an interesting one, a sort of a derivation of the, I mean, of the turbulent boundary layer von Karman analysis. So, this one I will go, that you are able to follow. So, let me first go and tell you what the question is, it says, that the velocity profile for turbulent boundary layer, so, the velocity profile for turbulent boundary layer is given by one by seventh power law. Obtain an expression for the drag force and the average coefficient of drag, in terms of Reynolds number.

Actually we have already seen those equations without going into too much detail, but here we can derive it. The wall shear stress for turbulent shear boundary layer is given as, tau 0 is given. So, how to attack this problem? We simply go white screen.

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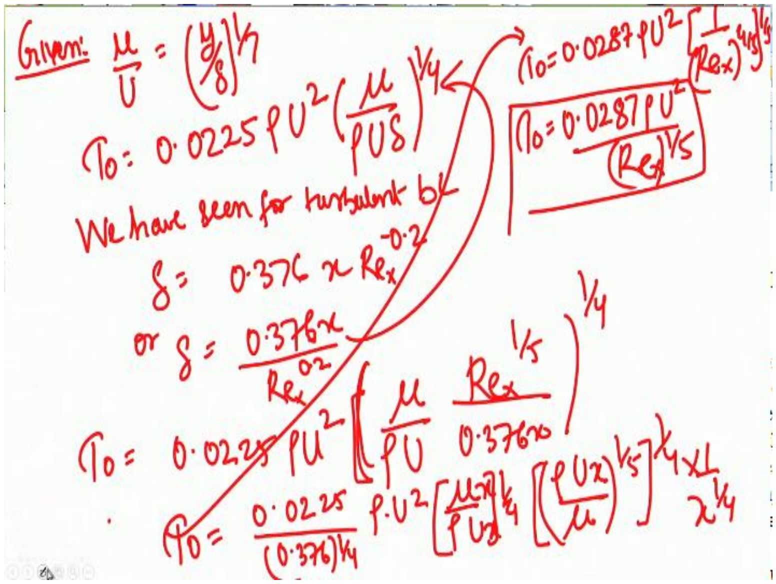

So, we write the things that are given. So, u / U is given as y / delta to the power 1 / 7, where tau 0 is 0.0225 ρ U square into mu / ρ U Delta to the power 1 / 4. So, we have seen, for turbulent boundary layer, delta is given as 0.376 x into Re at x to the power minus 0.2, or delta is given as 0 0.376x divided by Re x to the power 0.2. So, substituting this delta into the shear stress, if you substitute this, so, tau 0 will be 0.0225 ρ U square into mu / ρ U and in place of delta, we put this, so, delta will be 0.376x into Re x to the power 1 / 5 and to the power 1 / 4.

On rearranging, we can get, tau is equal to, so, there are some step,s which you will have to do, 0.0287. I will not miss those steps because that might be more. So, this will become 0.0225 divided by, this will come out, 0.376 to the power 1 / 4 ρ U square u / ρ U to the power 1 / 4 ρ U x / mu to the power 1 / 5 to the power 1 / 4 into 1 to the power x to the power 1 / 4.

So, if you multiply x here and x here and on rearrangement this expression actually can give you, tau 0 is equal to 0.0287 ρ U square into 1 / Re to the power x 4 / 5 to the power 1 / 5, or tau 0 = 0.0287 ρ U square divided by Re to the power x into 1 / 5.

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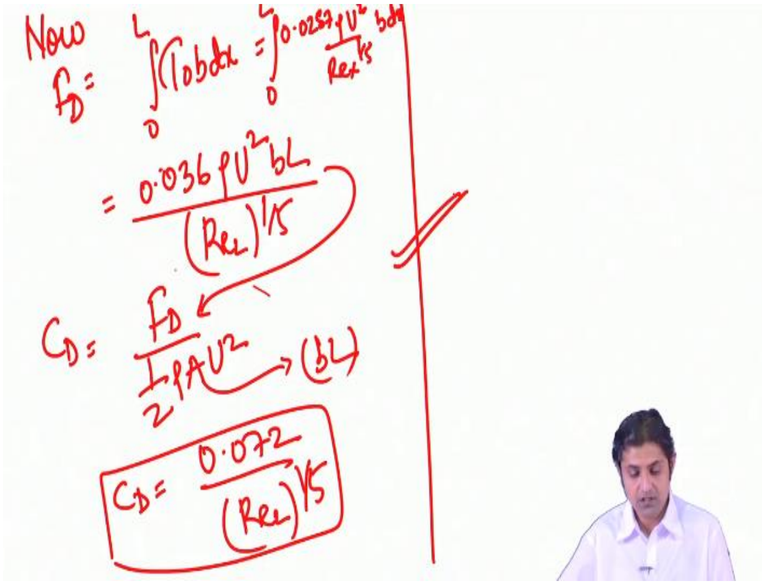

So, now, F D is equal to integral 0 to L tau 0 b dx. If you substitute this and integrate it, we are going to get, we already have this tau 0, in terms of Reynolds number to the power 1 / 5, so, I will just to be, you know, to make it complete, 0 to L tau 0, can be written as, 0.0287 ρ U square divided by Re x to the power 1 / 5 b dx. Now, Re should be written, in terms of Ux, I mean, the usual numbers and FD on simplification, will come out to be 0.036 into ρ U square b L divided by Re L to the power 1 / 5.

And C D in the end, will be F D divided by half ρ AU square, F D will be this thing, A is going to be b into L, for example. Therefore, C D is going to be 0.072 divided by Re to the power L to the power 1 / 5. Same way of solution, so, no difference. So, if you get a velocity profile you can proceed it in a similar way, given that for a turbulent flow the shear stress profile is given.

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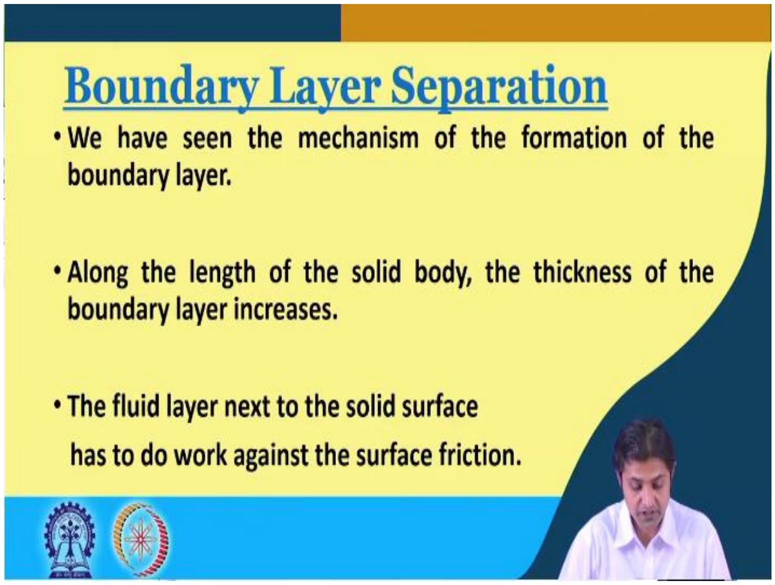

So, now, we have to study about something called the boundary layer separation. What is boundary layer separation? So, we have seen, the mechanism of the formation of the boundary layer, as such. So, along the length of the solid body, the thickness of the boundary layer increases, that we have already seen. And the fluid layer next to the solid surface has to do work against the surface friction, that is true.

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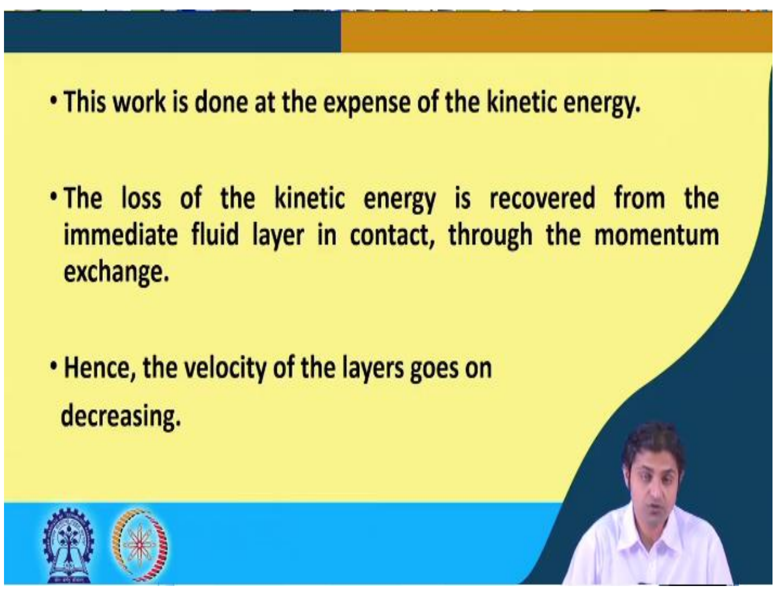

And this work is done at the expense of the kinetic energy. The loss of the kinetic energy is recovered from the immediate fluid layer in contact; through the momentum exchange. Hence, the velocity of the layer goes on decreasing.

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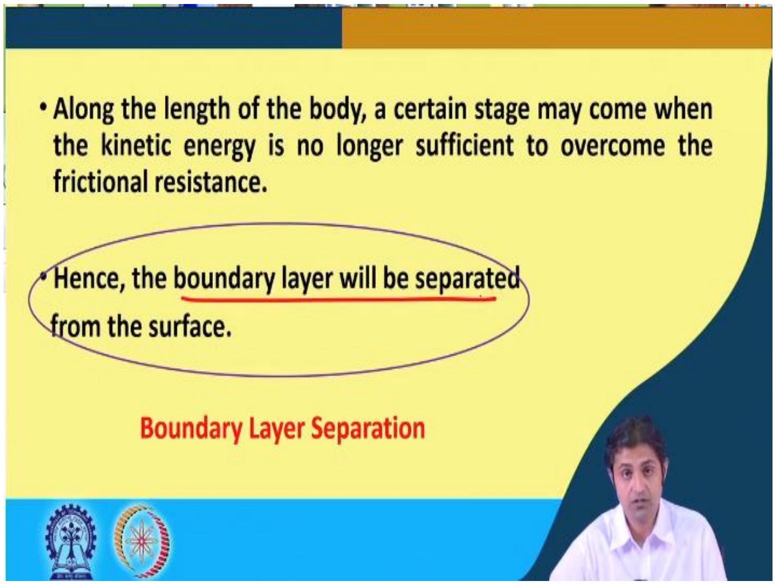

So, along the length of the body, a certain stage may come when the kinetic energy is no longer sufficient to overcome the frictional resistance. I mean, the kinetic energy transport from the upper layer to the below layer, for example, is no longer sufficient to overcome the frictional resistance. In that case, the boundary layer will be separated from the surface and this is called the boundary layer separation.

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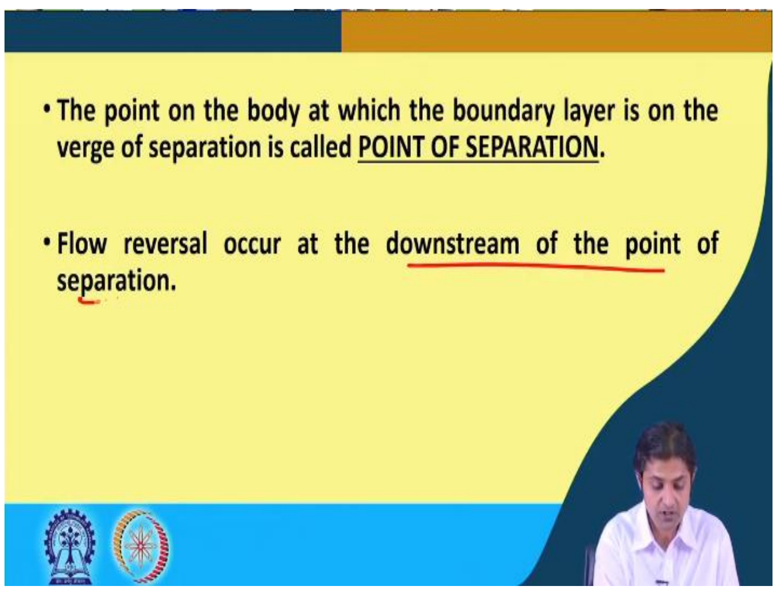

The point on the body at which the boundary layer is on the verge of separation is called the point of separation. The flow reversal can occur at the downstream point of the separation.

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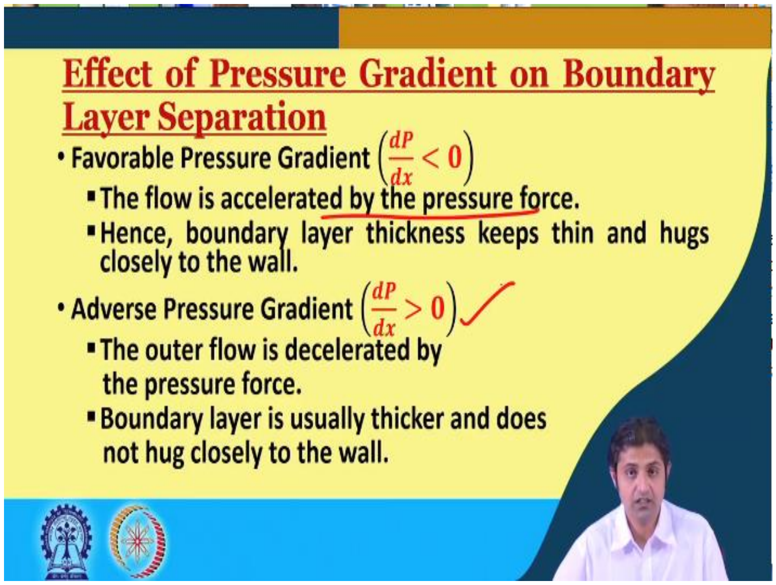

So, this is the, this is what this phenomena is, I mean, this is what exactly this phenomena. Now, we are going to see what the effect of pressure gradient boundary layer separation is. So, what is a favorable pressure gradient? That the pressure dP/dx is less than 0. So, pressure at one point, if it is higher than the second point, then the flow will occur. So, that is the favorable pressure gradient. In this case, the flow is accelerated by the pressure forces. Hence, the boundary layer thickness keeps thin and hugs closely to the wall in the favorable pressure gradient.

However, in adverse pressure gradient, where, dP/dx is greater than 0, the outer flow is deaccelerated by the pressure forces, because the pressure forces will act in the opposite direction because of the adverse pressure gradient. In that case, the boundary layer is usually thicker and does not remain very close to the wall.

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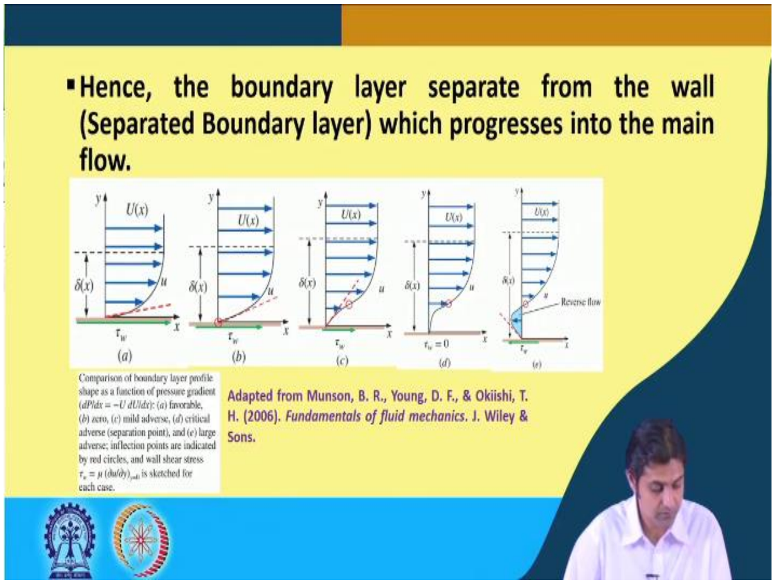

Therefore, the boundary layer separate from the wall which progresses into the main flow and this is the, so, this is the flow reversal, you know. You see, this is the point of separation. So, comparison of boundary layer profile shape as a function of pressure gradient dP dx is minus U du dx. a is a favorable case, you see, b is 0, this one, c is mild adverse because this is a pressure gradient, a different pressure gradient, you see, d is critical and the fourth is large adverse, you see, the pressure gradient dP dx.

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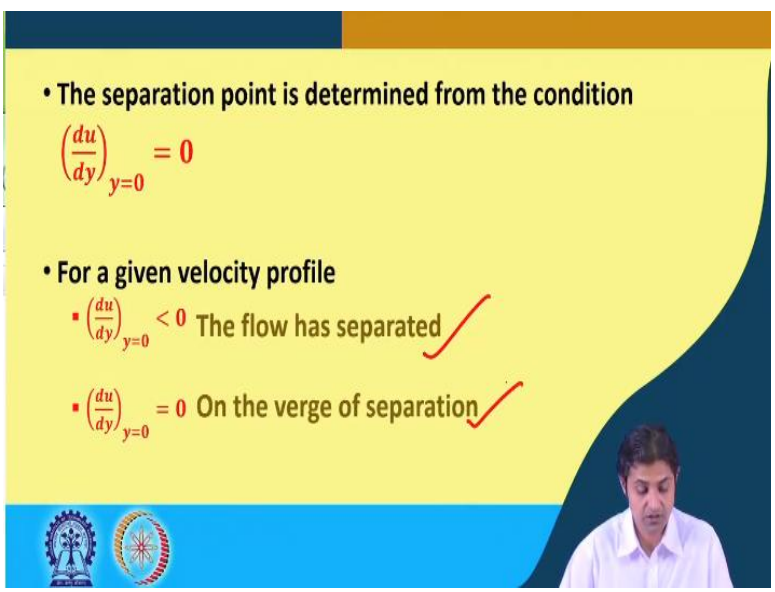

So, the separation is determined from this, where the separation point is going to be is determined from the condition du / dy at y is equal to 0 is equal to 0. So, for a given velocity profile, if du / dy at y is equal to 0 is less than 0, this means, that the flow has already separated. And if it is equal to 0, it is in the verge of separation.

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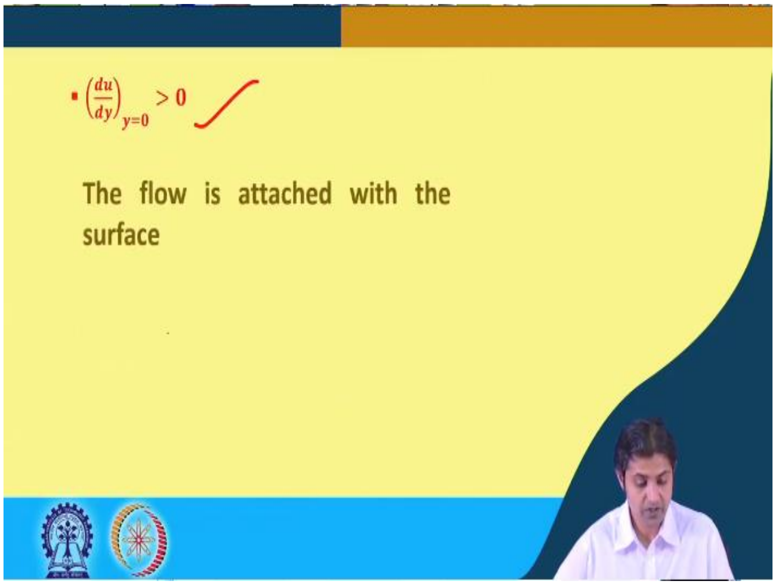

And if it is greater than 0, this means, it will not separate; the flow is attached with the surface. Because du / dy at y is equal to 0, the shear stress, that is the main thing. You know, if it is less than 0, that means, flow has already separated because.

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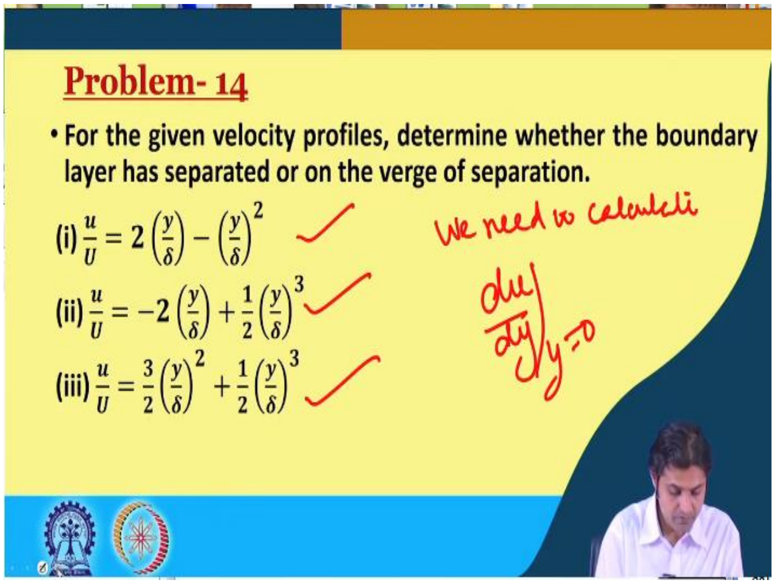

So, now, we are going to solve 2 problems related to that. One is, for the given velocity profiles, determine whether the boundary layer has separated or on the verge of separation. So, these are the 3 velocity profiles. Very simple, in this velocity profile, we need to calculate, du / dy at y is equal to 0. Simply, if it is negative it has separated if it is not then, now if it is 0, then it is on the verge of separation.

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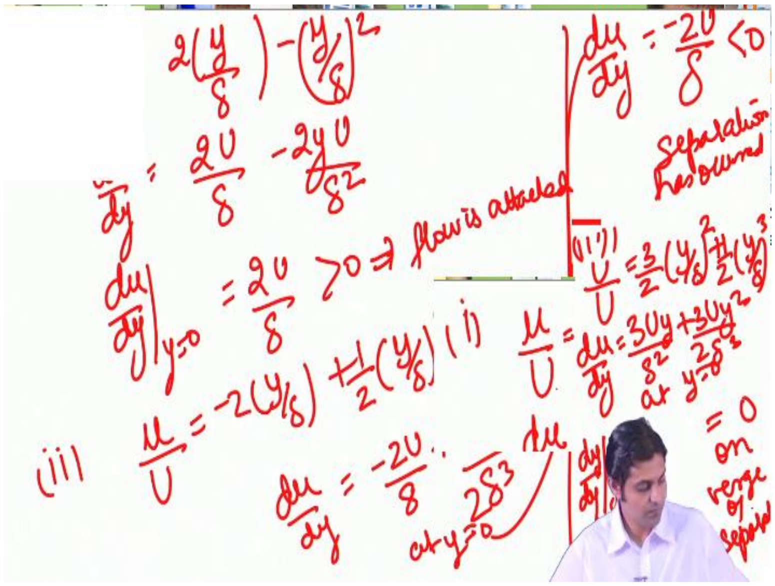

So, we are going to have this white screen. So, let us say, for part one actually, u / U is equal to 2 into y / delta - y / delta whole square. So, let us do, du / dy, so, this is going to be, 2U / delta - 2y U / delta square and du / dy at y is equal to 0, will give us, 2U / delta, which is greater than 0, that means, flow is attached.

The second velocity profile, where u was, u / U is - 2 y / delta + half y / delta whole cube. Here, du / dy is going to be, minus 2U / delta + 3U y square / 2 delta cube at y is equal to 0 du / dy is going to be, minus 2U / delta, which is less than 0, that means, separation has occurred. In the third problem, when you do, third problem has u / U as 3 / 2 y / delta whole square + 1 / 2 y / delta whole cube. So, this du / dy is going to be, 3U y / delta square + 3U y square / 2 delta cube. So, that means, at y is equal to 0 du / dy at y is equal to 0 is 0. So, this is equal to 0, means, on verge of separation.

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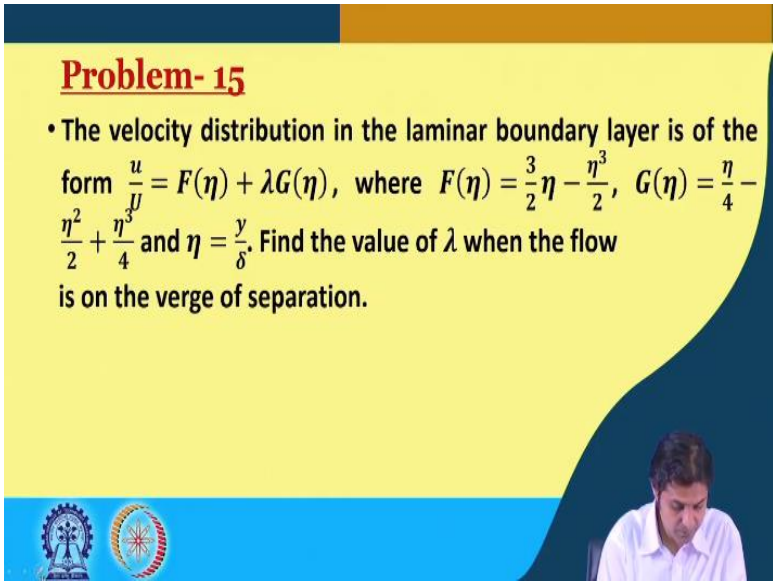

So, now, one more problem. So, the velocity distribution in the laminar boundary layer is of the form u / U is equal to F of ETA + lambda G of ETA, where F of ETA is 3 / 2 ETA - ETA cube / 2, and G of ETA is ETA / 4 - ETA square / 2 + ETA cube / 4 and ETA is y / delta. Find the value of lambda, when the flow is on the verge of separation. So, this is an interesting problem.

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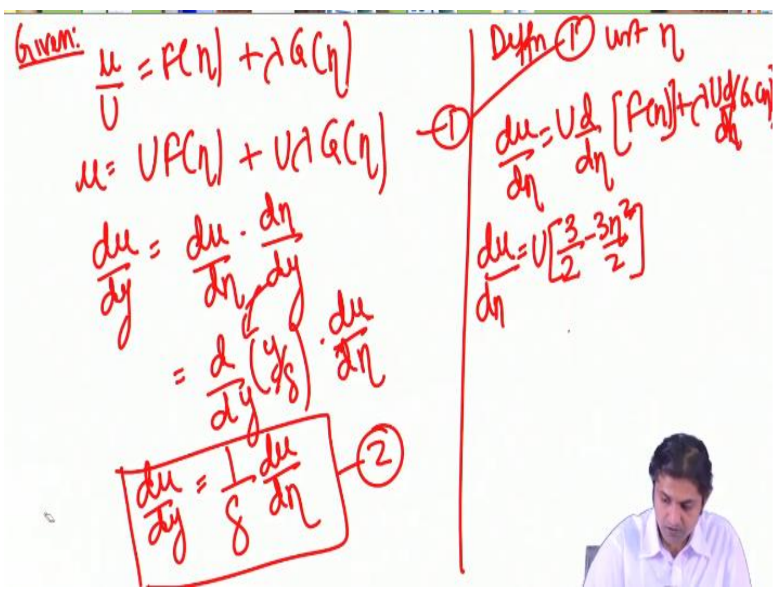

So, what is given to us, u / U is equal to F of ETA + lambda G of ETA, where F ETA and G ETA is given. So, from the profile, u can be written as, U F of ETA + U lambda G of ETA. This is equation number one. So, du / dy can be written as, du / d ETA into d ETA / dy, or du / d ETA, can be written as d / d. So, d ETA / dy can be actually, this can be written as d / dy of y / lambda into du / d ETA, this is here, or du / dy can be simply written as 1 / lambda into du / d ETA.

This is without utilizing that and this is equation number 2. If we differentiate this one, with respect to ETA, what we get is, du / d ETA is equal to Ud d ETA of F of ETA + lambda Ud d ETA of G of ETA. And this will on putting the values, this will give us, du / d ETA is equal to, so, U 3 / 2 - 3 ETA square / 2 3 ETA square.

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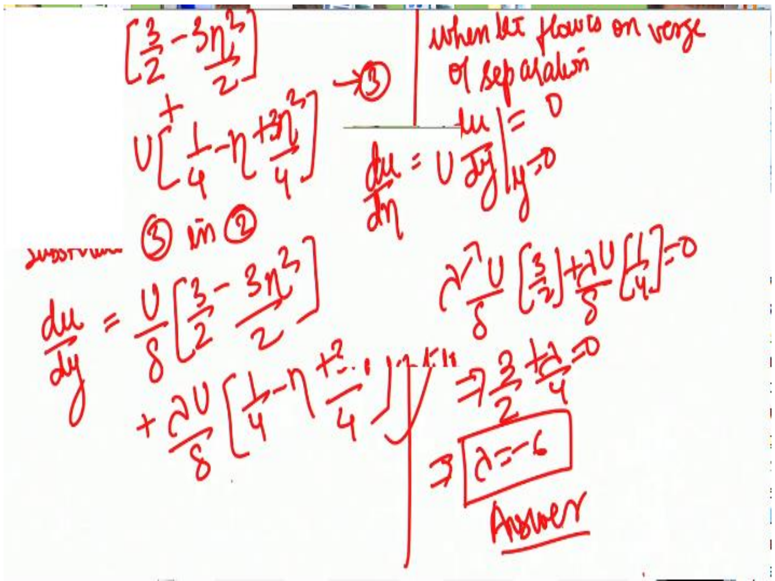

So, I will just write down again. This du / d ETA, finally comes out to be, U 3 / 2 - 3 ETA square / 2 + lambda U into 1 / 4 - ETA + 3 ETA square / 4 and this equation we call equation number 3. Substitute 3 in 2, and then we get, du / dy is equal to U / lambda 3 / 2 - 3 ETA square / 2 + lambda U / Delta into 1 / 4 - ETA + 3. So, when the flow is on verge of separation, this means, du / dy is equal to at y is equal to 0 is 0.

So, what we do? We put it equal to 0 and what we obtain is, u / Delta into 3 / 2 + lambda U by, there are some other steps, which I think you must do it, 0, or U / Delta you take common. This will give us, 3 / 2 + lambda / 4 is equal to 0, which implies, lambda is equal to minus 6. So, for lambda is equal to minus 6, the flow is on the verge of separation and this is the answer to this question.

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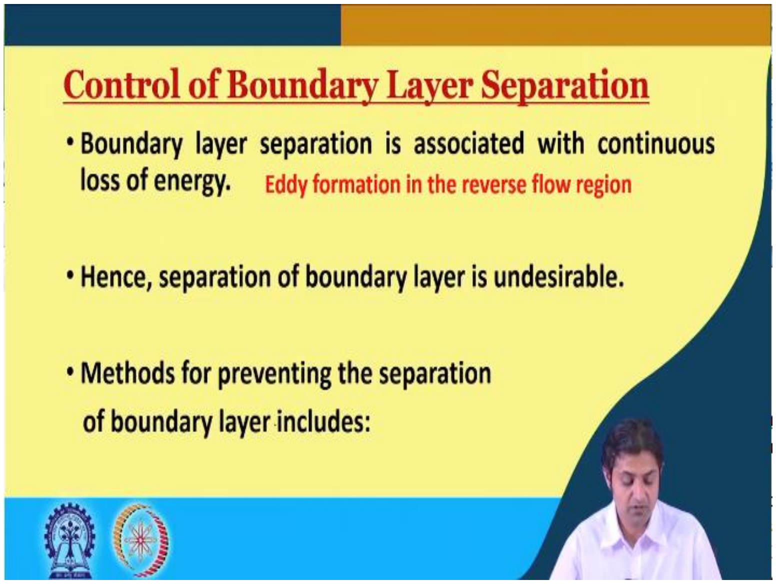

So, now, the control of boundary layer separation, we are proceeding towards the end. So, how this boundary layer separation can be controlled? So, boundary layer separation is associated with continuous loss of energy. So, that means, eddy formation is there in the reverse flow region. Hence, separation of boundary is undesirable and therefore it needs to be stopped. And some methods for preventing the separation of boundary layer include:

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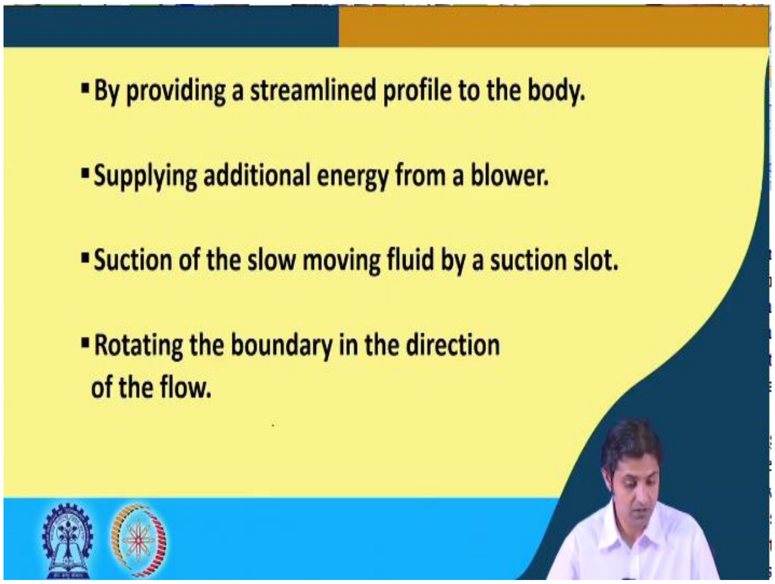

These are only some. So, if you provide a streamlined profile to the body or we can supply additional energy from a blower, suction of the slow moving fluid by a suction slot and rotating the boundary in the direction of the flow are these some of the measures using which the boundary layer separation can be stopped. So, these are the references, as usual for the actually entire hydraulic engineering course. And this also concludes our topic on boundary layer analysis. Thank you so much. I will see you in the next lecture. Bye.