

**Hydraulic Engineering**  
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**Lecture – 35**  
**Non-Uniform Flow and Hydraulic Jump (Contd.)**

Welcome back students to this lecture. Last time we left off by finishing the classification of different type of slopes; mild slope, steep slope, critical slope, horizontal bed and adverse slope.

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**Problem- 1**

- Find the rate of change of depth of water in a rectangular channel 10 m wide and 1.5 m deep, when the water is flowing with a velocity of 1 m/s. The flow of water through the channel of bed slope 1 in 4000, is regulated in such a way that energy line is having a slope of 0.00004.

Solution:       $b = 10 \text{ m}$      $y = 1.5 \text{ m}$        $V = 1 \text{ m/s}$   
                     $S_0 = 1/4000$        $S_f = 0.00004$

The slide also features two circular logos at the bottom left and a small video inset of a man in a white shirt at the bottom right.

And now, we are going to solve some questions, couple of problems on this topic. So, the question is; find the rate of change of depth of water in a rectangular channel, which is 10 meter wide and 1.5 meter deep, when the water is flowing with a velocity of 1 meters per second. So, the channel is 10 meter wide and 1.5 meter deep and the velocity of the water that is flowing is 1 meters per second.

The flow of water through the channel of bed slope 1 in 4000, is regulated in such a way that the energy line is having a slope of 0.00004. See, we have been given  $S_0$ , we have been given  $S_f$  and other things, you know, the velocity and so what we have to find is, find the rate of change of depth of water, so we have been asked to calculate  $dy$  by  $dx$ . So, now you understand, most of the other things are given, very, you know, in the last lecture what we have studied we are going to use that. So, the best is to go to the whiteboard.

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Soln Given  $b = 10 \text{ m}$ ,  $y = 1.5 \text{ m}$ ,  $V = 1 \text{ m/s}$   
 $S_0 = 1/4000$ ,  $S_f = 0.00004$   
 $A = bxy = 10 \times 1.5 = 15 \text{ m}^2$   
 $T = b = 10 \text{ m}$   
 $Q = A \times V = 15 \times 1 = 15 \text{ m}^3/\text{s}$   
 $\frac{dy}{dx} = ?$   
*This is in fact a gradually varied flow*

$\frac{dy}{dx} = \frac{S_0 - S_f}{1 - \frac{Q^2 T}{g A^3}}$   
 $\frac{dy}{dx} = \frac{1/4000 - 0.00004}{1 - \frac{15^2 \times 10}{9.8 \times 15^3}}$   
 $\frac{dy}{dx} = 2.25 \times 10^{-4}$   $\rightarrow$  Very small  
 $\frac{dy}{dx}$  is very very small  $\ll 1$   
*This is in fact a*

So, we are going to see the solution. So, what things we have already been given? Given is,  $b$  is given as 10 meter, depth we have been given 1.5 meter, we also have been given velocity of the flow 1 metres per second, bed slope also has been mentioned; 1 in 4000, so 1 by 4000 is the bed slope, we also have been given the slope of,  $S_f$  is also given, that is, 0.00004.

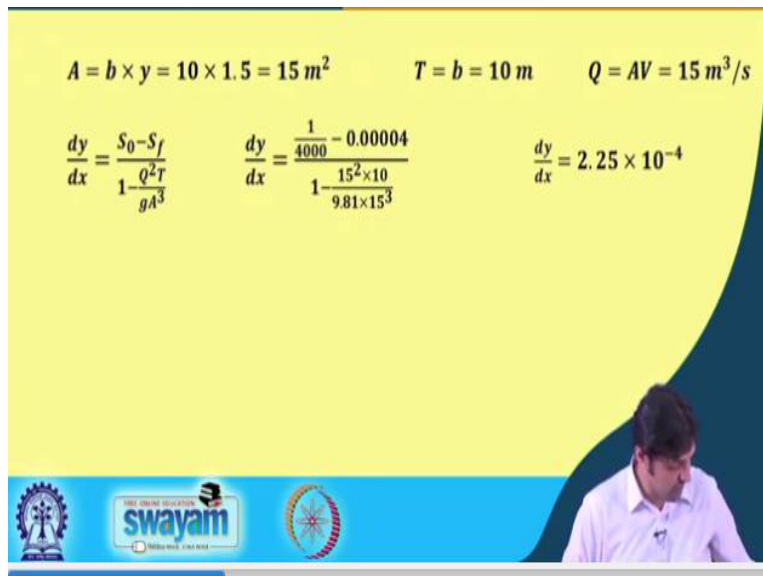
So, this is quite a simple thing. First, what we need to do? We need to calculate the area of the flow and that is nothing but  $b$  into  $y$ , so 10 into 1.5 and that is going to be 15 meter square. So, we solve, we will go very systematically,  $T$  is the top width is nothing, top width is, you know,  $b$ , that is, 10 meter, discharge is area into velocity, area is 15 meter into velocity is 1 meters per second, so that is 15 meter cube per second.

So, we have been given  $y$ , we have been given  $b$ , we have been given when, so we have been given  $V$ ,  $S$  and  $S_f$ , so we calculated  $A$  as 15 meter square, top width is anyways given already 10 meter and  $Q$  is we calculated 15 meters cube per second. So, what is the equation in the gradually varied flow or in any type of flow that we have derived? We have derived that  $dy$  by  $dx$ , this is the equation that relates  $dy$  by  $dx$  to the some number.

So, we have been asked to calculate  $dy$  by  $dx$ , so this is the thing that we need to find and the equation was  $dy$  by  $dx$  is equal to  $S_0 - S_f$  divided by  $1 - Q$  square multiplied by  $T$  divided by  $g$  into  $A$  cube. So,  $dy$  by  $dx$  is going to be,  $S_0$  is 1 by 4000,  $S_f$  is 0.00004, so  $1 - Q$  is 15, so it will be 15 square into,  $T$  is 10, divided by,  $g$  is 9.8 and then  $A$  cube, area is also 15, so 15 cube.

So, if you calculate that  $dy$  by  $dx$  will come out to be 2.25 into 10 to the power  $-4$ . Will this have any unit? No unit. So, if you see, the slope  $dy$  by  $dx$  is very, very small, we can say much less than 1, so this is in fact. So, I will write here, this is in fact a gradually varied flow. So, we go back, see this is, so  $dy$  by  $dx$  came out to be, sorry, 2.25 into 10 to the power  $-4$ . So, I will erase this and I have included the solution here as well, so that, so same thing.

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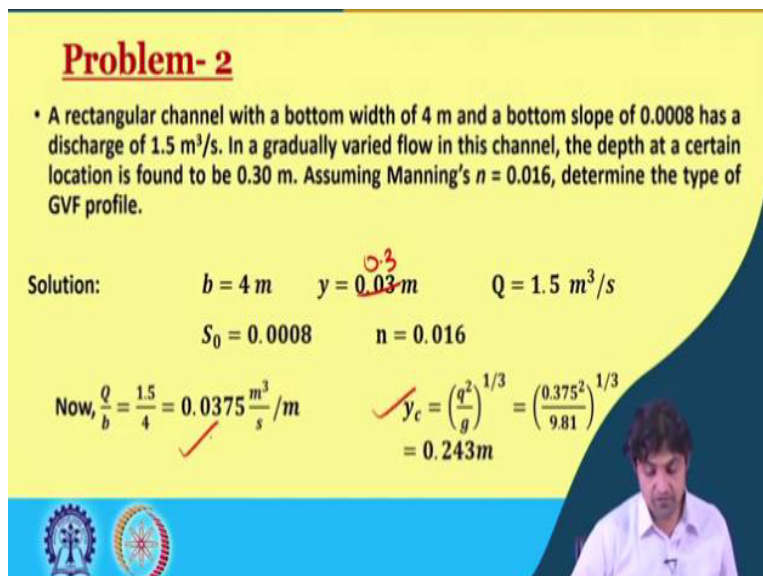


$$A = b \times y = 10 \times 1.5 = 15 \text{ m}^2 \quad T = b = 10 \text{ m} \quad Q = AV = 15 \text{ m}^3/\text{s}$$

$$\frac{dy}{dx} = \frac{S_0 - S_f}{1 - \frac{Q^2 T}{g A^3}} \quad \frac{dy}{dx} = \frac{\frac{1}{4000} - 0.00004}{1 - \frac{15^2 \times 10}{9.81 \times 15^3}} \quad \frac{dy}{dx} = 2.25 \times 10^{-4}$$

These were the things that were given, area, the same thing which we have worked out in the screen. So, the same formula  $dy$  by  $dx$  is equal to  $S_0 - S_f$  and this is what we are going to get. So, this was the final answer which we have seen on the white screen.

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**Problem- 2**

- A rectangular channel with a bottom width of 4 m and a bottom slope of 0.0008 has a discharge of 1.5 m<sup>3</sup>/s. In a gradually varied flow in this channel, the depth at a certain location is found to be 0.30 m. Assuming Manning's  $n = 0.016$ , determine the type of GVF profile.

**Solution:**

$$b = 4 \text{ m} \quad y = 0.3 \text{ m} \quad Q = 1.5 \text{ m}^3/\text{s}$$

$$S_0 = 0.0008 \quad n = 0.016$$

Now,  $\frac{Q}{b} = \frac{1.5}{4} = 0.375 \frac{\text{m}^3}{\text{s}}/\text{m}$

$$y_c = \left( \frac{q^2}{g} \right)^{1/3} = \left( \frac{0.375^2}{9.81} \right)^{1/3} = 0.243 \text{ m}$$

So, now we are going to look at another problem. So, the question is, a rectangular channel with a bottom width of 4 meter and a bottom slope of 0.0008 has a discharge of 1.5 meters

cube per second. In a gradually varied flow in this channel, the depth at certain location is found to be 0.30 meter. Assuming the Manning's n is equal to 0.016, determine the type of gradually varied profile.

So, see, we have been given, the bottom width 4 meter, here, slope  $S_0$  is given,  $Q$  has also been given and we have been given  $y$  as well, Manning's number is given. So what type of the gradually varied profile will it be? So, that we are going to see, and it will be a very, very good practice for you, when you start, you know, applying the things that you have learned in this particular question, about the mild slope, steep slope and adverse slope, critical slope and the horizontal bed. So, as I will go to the white screen.

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**Soln:**  
 Given  $b = 4\text{ m}$ ,  $y = 0.3\text{ m}$   
 $Q = 1.5\text{ m}^3/\text{s}$ ,  $S_0 = 0.0008$   
 $n = 0.016$   
 Now  $\frac{Q}{b} = \frac{1.5}{4} = 0.375\text{ m}^3/\text{s/m} = q$   
 $y_c = \left(\frac{q^2}{g}\right)^{1/3} = \left(\frac{0.375^2}{9.81}\right)^{1/3}$   
 $y_c = 0.243\text{ m}$   
 $S_c = \frac{14n^2q^3}{S_0} = 0.0008$   
 $R = \frac{4y_0}{4+2y_0}$   
 $1.5 = \frac{1}{0.016} \times 4y_0 \times \left(\frac{4y_0}{4+2y_0}\right)^{2/3} \times (0.0008)^{1/2}$   
 $\Rightarrow y_0^{2/3} = 0.0842$   
 $(4+2y_0)^{2/3} = 0.0842$   
 $y_0 = 0.426\text{ m}$   
 $y_0 > y_c \rightarrow \text{M slope channel}$   
 $y_0 > y > y_c \rightarrow \text{M2 curve}$   
 Diagram: A channel cross-section with width  $b$  and depth  $y$ .

So, to start, this Soln means solution, so what are the things that are given to us, you know, so I will write. So, given is,  $b$  is equal to 4 meter,  $y$  we have been given as 0.30 meter, this  $y$  is given,  $Q$  has been given as 1.5 meter cube per second,  $S_0$  is also given 0.0008 and Manning's roughness is also given, 0.016. So, now, capital  $Q$  by  $b$  is 1.5 divided by 4 is 0.375 meter cube per second per meter, so this is small  $q$ .

So, in the normal, I mean, you remember, the way we calculated the critical depth  $y_c$  was  $q^2$  by  $g$  to the power 1 by 3, if you remember the formula from our lectures last week. So, this is going to be 0.375 whole square by,  $g$  is 9.81, and whole to the power 1 by 3. If you do this calculation using your calculator or, you know, by hand, this is going to be 0.243 meter. So, critical depth which we have got is 0.243 meters.

What is the other thing that we need to find? So, we can use the Manning's equation and see,  $1 \text{ by } n A \text{ into } R \text{ to the power } 2 \text{ by } 3 \text{ into } S_0 \text{ to the power } 1 / 2$ . So, we see, area we already know, so if in this is a rectangular channel, so this is the  $y$  and this is  $b$ , so area is  $b y_0$  already, because if the, if we also need to calculate the normal depth, so area is going to be  $b y_0$ .

So, using, and these hydraulic radius will be  $A \text{ by } P$ , so here it is actually  $4 y_0$ , so a radius, hydraulic radius will be  $4 y_0 \text{ divided by } 4 + 2 y_0$ . So, if we use all these value in this equation here, so  $Q$  is 1.5 given, is equal to  $1 \text{ by } 0.016 \text{ is } n$ ,  $A$  is  $4 \text{ into } y_0$ , radius, hydraulic radius is  $4 y_0 \text{ divided by } 4 + 2 y_0$  and this whole to the power  $2 \text{ by } 3$  and then multiplied by  $S_0$  to the power  $1/2$ ,  $S_0$  is given 0.0008 to the power  $1/2$ .

So, this can, will actually give us,  $y_0 \text{ to the power } 2 \text{ by } 3 \text{ divided by } 4 + 2 y_0 \text{ to the power } 2 \text{ by } 3$  is equal to 0.0842. So, if we solve this, you can use any method and you will, we will obtain  $y_0$  as 0.426 meter. So, the normal depth we have calculated using Manning's equation. Is that true? Because that was the thing that we started with, so  $y_c$  came out as 0.243 and the normal depth came out to be 0.426.

Now, if you see, we were talking that if there is, you know, if there is a horizontal bed there is not going to be any normal depth. And see why we were saying that? We were saying that because if  $S_0$  is equal to 0, then this  $Q$ , there is no existence of  $Q$  as such according to this equation. Also, when the slope was adverse, so  $S_0$  is less than 0 then, this means,  $S_0$  to the power  $1/2$  is a real, is an imaginary number.

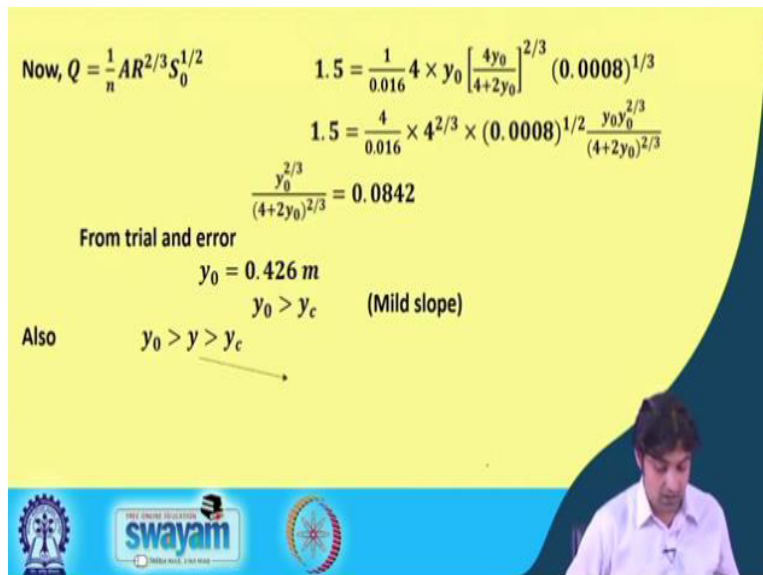
So, concentrating  $y_c$  is 0.243 and  $y_0$  is 0.426, so first thing, since normal depth is greater than  $y_c$ , this is going to be an M slope, first of all, M slope channel. Now, the water depth is actually 0 point, so actually water depth is not 0.03, which I wrote, it was 0.3 meter, here. So, we see this, the  $y_0$  is greatest, greater than  $y$  and so this is critical depth, this was 0.426, this was 0.3 and this was,  $y_c$  was 0.243.

So, which type of curve? This is an M slope plus, since  $y$  lies between  $y_0$  and  $y_c$ , it is going to be M2 curve. So, this is a good example, a very simple and good example, indicating how you have to find the type of the GVF profile. So, this came actually to be M2 curve. I will

just erase this. So, this is actually 0.3 meter, because the depth is 0.3 meter, so this is a typing error.

So, similarly we have this again a typed solution available. I think I will just correct this one when I provide you with the course material, so these are the things that were given. So, similarly we have calculated small q and critical depth is calculated like this, you see, this is small q has been calculated, the same way it came out to be 0.243 meter.

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Now,  $Q = \frac{1}{n} AR^{2/3} S_0^{1/2}$

$$1.5 = \frac{1}{0.016} 4 \times y_0 \left[ \frac{4y_0}{4+2y_0} \right]^{2/3} (0.0008)^{1/2}$$

$$1.5 = \frac{4}{0.016} \times 4^{2/3} \times (0.0008)^{1/2} \frac{y_0^{2/3}}{(4+2y_0)^{2/3}}$$

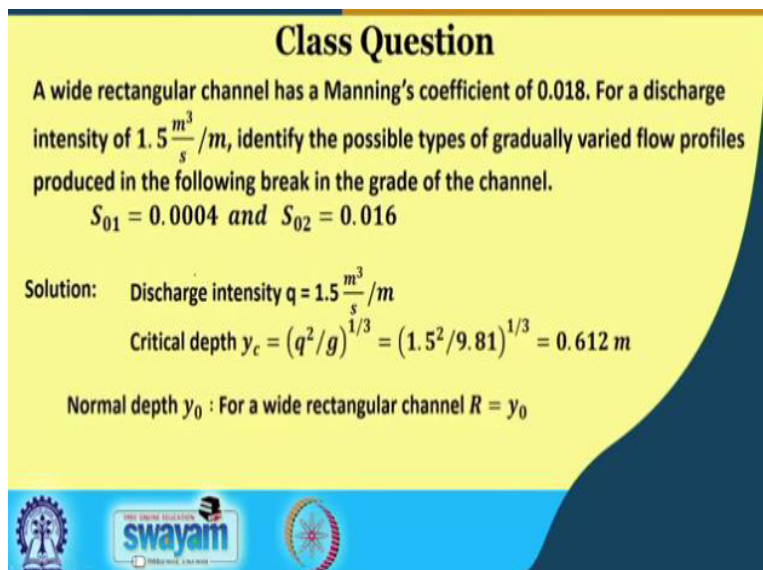
$$\frac{y_0^{2/3}}{(4+2y_0)^{2/3}} = 0.0842$$

From trial and error  
 $y_0 = 0.426 \text{ m}$   
 $y_0 > y_c$  (Mild slope)

Also  $y_0 > y > y_c$

So, now using the Manning's equation, we estimated the normal depth  $y_0$  and this  $y_0$  came out to be 0.426. Since, it is  $y_0$  is greater than  $y_c$ , it is a mild slope and  $y$  is greater than  $y_0$ , let us say, it is called M2 curve.

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### Class Question

A wide rectangular channel has a Manning's coefficient of 0.018. For a discharge intensity of  $1.5 \frac{\text{m}^3}{\text{s}/\text{m}}$ , identify the possible types of gradually varied flow profiles produced in the following break in the grade of the channel.

$S_{01} = 0.0004$  and  $S_{02} = 0.016$

Solution: Discharge intensity  $q = 1.5 \frac{\text{m}^3}{\text{s}/\text{m}}$

Critical depth  $y_c = (q^2/g)^{1/3} = (1.5^2/9.81)^{1/3} = 0.612 \text{ m}$

Normal depth  $y_0$  : For a wide rectangular channel  $R = y_0$



So, we have another question; a wide rectangular channel has a Manning's coefficient of 0.018. For a discharge intensity of 1.5 meters cube per second per meter, identify the possible values of gradually varied flow profiles produced in the following break in the grade of the channel. S01 is 0.0004 and S2 is going to be 0.016. So, this actually is a good question, so we will, so because in this topic we have less number of questions, so I think it is better to solve it here, white screen.

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Soln:

Discharge intensity  $q = 1.5 \text{ m}^3/\text{s}/\text{m}$

$$y_c = \left( \frac{q^2}{g} \right)^{1/3} = \left( \frac{1.5^2}{9.8} \right)^{1/3} = 0.621 \text{ m}$$

Normal dept. for wide rectangular  $R = y_0$

Type of grade change: sub to steep

$$q = \frac{1}{n} y_0 y_0^{2/3} S_0^{1/2}$$

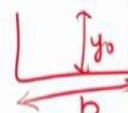
$$y_0 = \left( \frac{nq}{\sqrt{S_0}} \right)^{3/5} \Rightarrow y_0 = \left[ \frac{0.018 \times 1.5}{\sqrt{S_0}} \right]^{3/5}$$

Slope	$y_0$
0.0004	1.192
0.016	0.396

multislope

$y_c$	$y_0$	$y_{0c}(\text{m})$
0.621	1.192	0.396

Section in  $y_0 < y_c$



$$R = \frac{by_0}{b + 2y_0} \approx \frac{by_0}{b}$$

So, the solution shall go like this, so discharge intensity is small  $q$  is equal to 1.5 meter cube per second per meter, so  $y_c$  we will find by small  $q$  square by  $g$  to the power 1 by 3, very simple formula, must be remembered actually, by 9.8 to the power 1 by 3 and that is going to be 0.621 meter. So, for normal depth what do we use? Manning's equation, so for wide rectangular channel, so it is, so wide, so hydraulic radius becomes  $y_0$  itself, for a wide rectangular channel.

So, why? You have you must have a question, why. So, hydraulic radius is  $A$  by  $P$ , so  $by_0$  is  $A$ ,  $P$  is  $b + 2y_0$ , if it is wide, that  $b$  is very much, much greater than  $y$ . So,  $b + 2y_0$  will more or, because it is very wide, so this will be,  $b + 2y_0$  will be approximately equal to  $b$ , so this  $b$  and  $b$  gets cancelled. That is why we say normal depth therefore, normal depth for wide rectangular channel, the radius, the hydraulic radius is normal depth itself.

So, now, we use this formula,  $q$  is equal to  $1$  by  $n$   $y_0$  into  $y_0$  to the power  $2$  by  $3$  into  $S_0$  to the power  $1/2$ , you can also use that formula, capital  $Q$ , but I find it by using small  $q$  much better, so we can do, so  $y_0$  can be written as  $n$  into  $q$  divided by under root  $S_0$  to the power  $3$

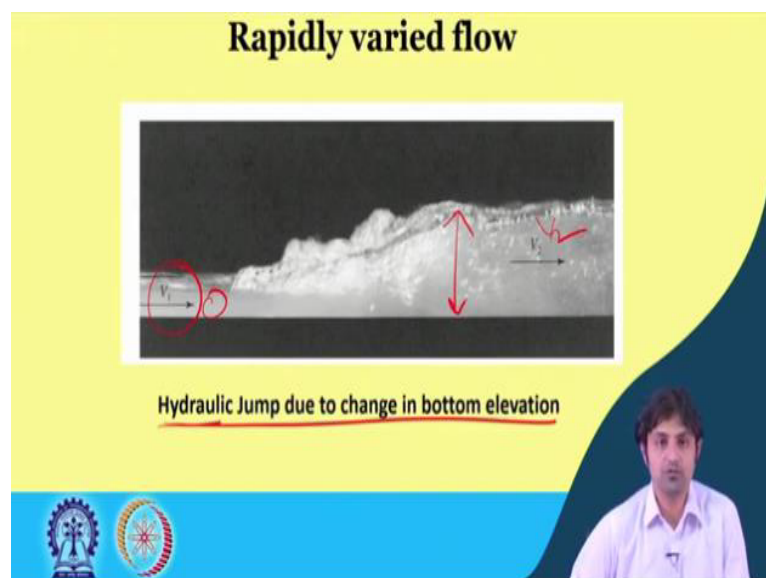
by 5, so this implies  $y_0$  is going to be,  $n$  is we just substitute in the value 0.018, small  $q$  was given as 1.5 and this under root  $S_0$ .

So, the question is, so actually, we can draw a table here that says slope and  $y_0$ , so 1, 2, 3 and 4, so there are 3, you know, there are slope 0.0004, 0.016 and then there is a critical slope, because this is the critical, so for this  $S_0$ , the normal depth if we put, we can get, this is the 1.197 and if you put  $S_0$  as 0.016, we are going to get 0.396 and  $y_c$  is 0.612, there are 2 normal depths  $y_{01}$  and  $y_{02}$  here, here.

So, one we put 1.197, the other is going to be 0.396, so this is what is written. So, if you go from 1 to 2, you know, since  $y_0$  here, this one, is greater than  $y_c$ , this is mild slope, first one. But in the second case, the  $y_0$  is less than, second case  $y_0$  is less than  $y_c$ , so basically this is the steep slope. So, the resulting M2 curve on mild slope and S2 curve on steep slope.

Because the water depth is also, you know, it is here, so the type of grade change is mild to steep. So, basically, until this, is fine. So, we have included the solution again here for your convenience.

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So, now we start with rapidly varied flow and then it is almost to the end, but we will start with some concept of the rapidly varied flow. So, you see the figure here and what do you notice? See, if you see, there is something that is coming with a velocity  $V$  here and somehow this height is larger, I mean, this seems that there is something which is different

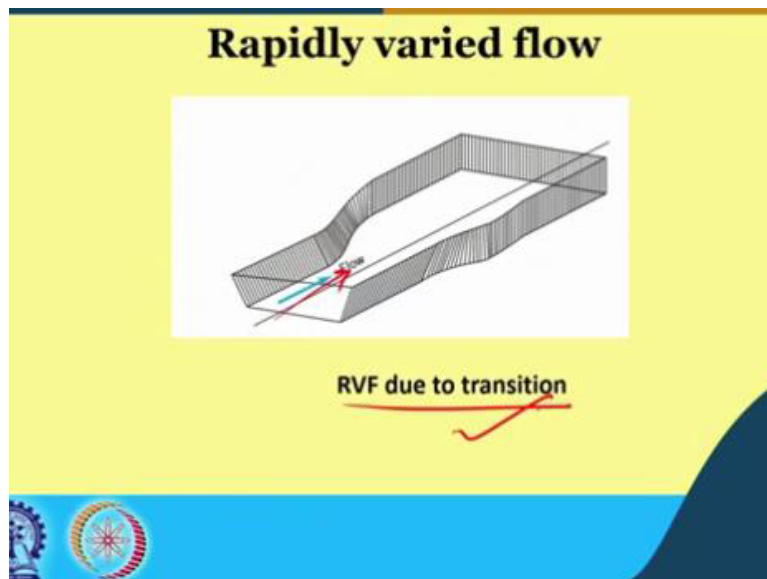


from this area, so the velocity becomes  $V_2$ , maybe the height has increased or there is a bump or something.

So, in that case, the flow becomes, so it varies very rapidly. Suddenly, if there is something, the rapid change in the water level elevation or some water level elevation, it is called rapidly varied flow. In this particular case, this there is a hydraulic jump due to the change in bottom elevation. We are able to source, we are able to say that because we know it from before. This is the photograph from probably one of the experiments.

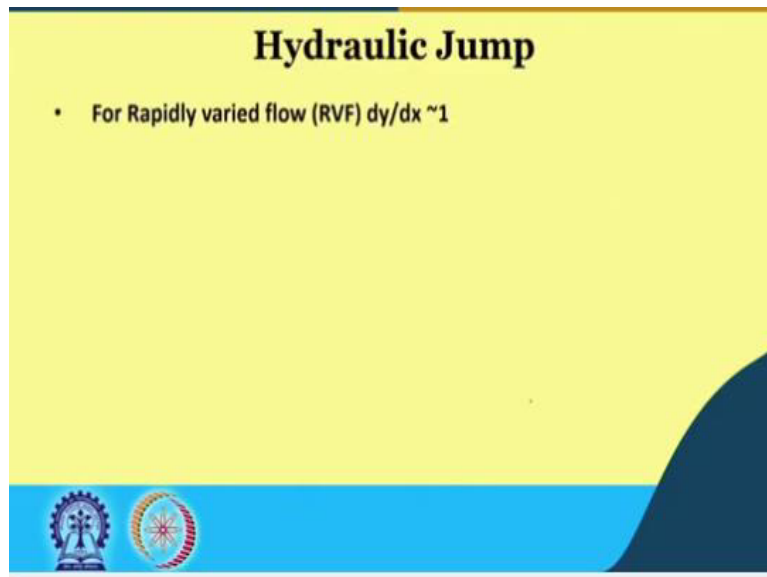
So, that was a fair introduction to rapidly varied flow. You now, understand what, you know, hydraulic jump is one type of rapidly varied flow.

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So one; if you see a flow like this, if there is a flow coming in this direction and the area suddenly increases that is the rapidly varied flow that happens due to the transition. So, first was due to the height, the second one was due to the, this one is due to the transition, so changing the area, transitioning from smaller area to larger area, for example.

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So, now, the most famous type of rapidly varied flow is called hydraulic jump. So, this is the concept that becomes the core of our next 2 lectures and we will start the next lecture and with this particular concept, hydraulic jump, go through a small derivation and solve some problems which will give you a better understanding of hydraulic jump. So, this is all for now. I will see you in the next lecture. Thank you.