Chapter 18

COLUMN STABILITY

18.1 Introduction; Discrete Rigid Bars

18.1.1 Single Bar System

1 Let us begin by considering a rigid bar connected to the support by a spring and axially loaded at the other end, Fig. 18.1. Taking moments about point A:

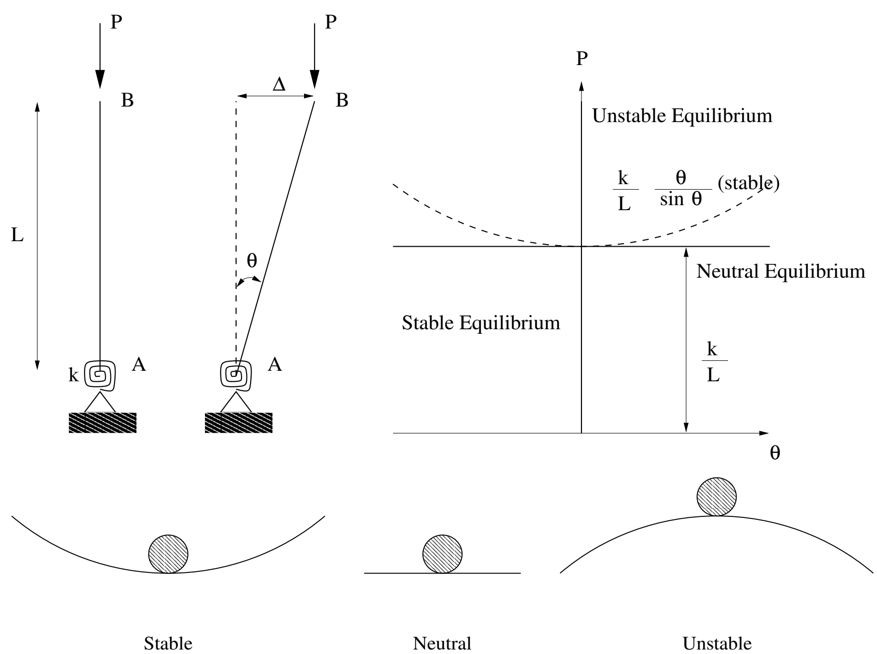

Figure 18.1: Stability of a Rigid Bar

$$
\begin{aligned}
\Sigma M_A &= P\Delta - k\theta = 0 && \text{(18.1-a)}\\
\Delta &= L\theta \ \ \text{for small rotation} && \text{(18.1-b)}\\
P\theta L - k\theta &= 0 && \text{(18.1-c)}\\
(P - \frac{k}{L}) &= 0 && \text{(18.1-d)}
\end{aligned}
$$

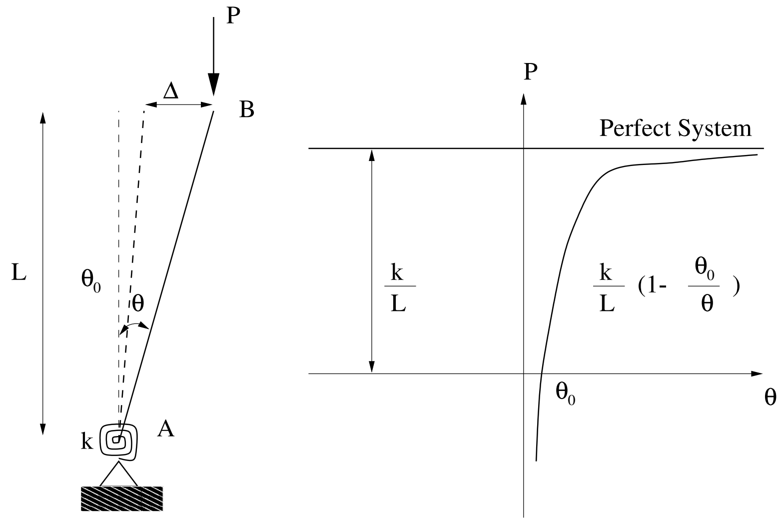

Figure 18.2: Stability of a Rigid Bar with Initial Imperfection

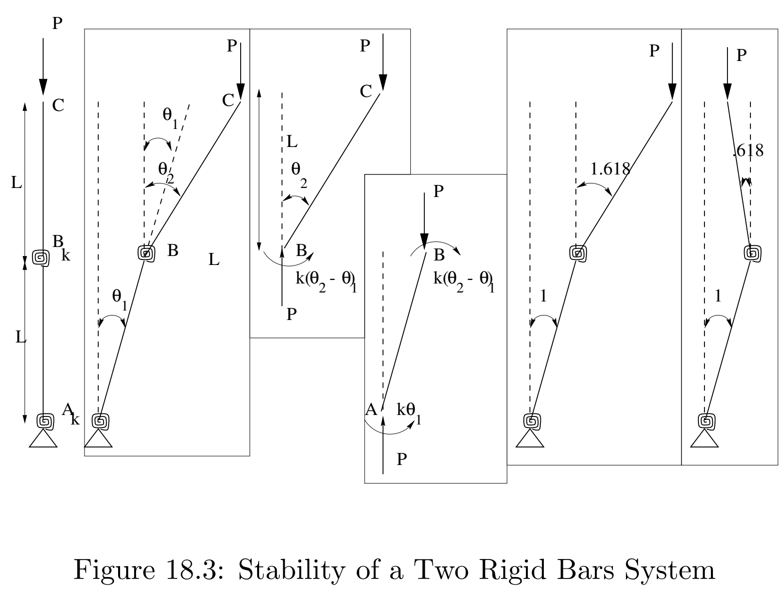

Figure 18.3: Stability of a Two Rigid Bars System

$$\begin{bmatrix} \frac{1-\sqrt{5}}{2} & -1 \\ -1 & 1-\frac{-1-\sqrt{5}}{2} \end{bmatrix} \begin{Bmatrix} \theta_1 \\ \theta_2 \end{Bmatrix} = \begin{Bmatrix} 0 \\ 0 \end{Bmatrix} \tag{18.12-c}$$

$$\begin{bmatrix} -0.618 & -1 \\ -1 & -1.618 \end{bmatrix} \begin{Bmatrix} \theta_1 \\ \theta_2 \end{Bmatrix} = \begin{Bmatrix} 0 \\ 0 \end{Bmatrix} \tag{18.12-d}$$

we now arbitrarily set $\theta_1 = 1$, then $\theta_2 = -1/1.618 = -0.618$, thus the second eigenmode is

$$\boxed{\begin{Bmatrix} \theta_1 \\ \theta_2 \end{Bmatrix} = \begin{Bmatrix} 1 \\ -0.618 \end{Bmatrix}} \tag{18.13}$$

18.1.3 ‡Analogy with Free Vibration

12 The problem just considered bears great ressemblence with the vibration of a two degree of freedom mass spring system, Fig. 18.4.

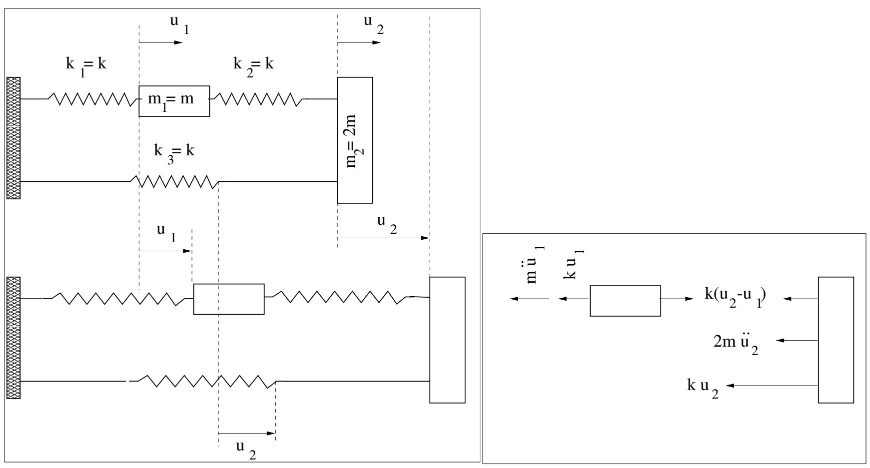

Figure 18.4: Two DOF Dynamic System

13 Each mass is subjected to an inertial force equals to the mass times the acceleration, and the spring force:

$$2m\ddot{\mathbf{u}}_2 + k\mathbf{u}_2 + k(\mathbf{u}_2 - \mathbf{u}_1) = 0 \tag{18.14-a}$$

$$m\ddot{\mathbf{u}}_1 + k\mathbf{u}_1 + k9\mathbf{u}_2 - \mathbf{u}_10 = 0 \tag{18.14-b}$$

or in matrix form

$$\underbrace{\begin{bmatrix} m & 0 \\ 0 & 2m \end{bmatrix}}_{\mathbf{M}} \underbrace{\begin{Bmatrix} \ddot{\mathbf{u}}_1 \\ \ddot{\mathbf{u}}_2 \end{Bmatrix}}_{\ddot{\mathbf{U}}} + \underbrace{\begin{bmatrix} 2k & -k \\ -k & 2k \end{bmatrix}}_{\mathbf{K}} \underbrace{\begin{Bmatrix} \mathbf{u}_1 \\ \mathbf{u}_2 \end{Bmatrix}}_{\mathbf{U}} \tag{18.15}$$

14 The characteristic equation is $|\mathbf{K} - \lambda\mathbf{M}|$ where $\lambda = \omega^2$, and ω is the natural frequency.

$$\begin{vmatrix} 2h-\lambda & -h \\ -h & 2h-2\lambda \end{vmatrix} = 0 \tag{18.16}$$