Hydraulic Engineering
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Lecture-24
Dimensional Analysis and Hydraulic Similitude (Contd.,)

Welcome back. So, beginning this lecture, we are going to see this problem of pipe flow and this is the exact same point where we stopped our last lecture.

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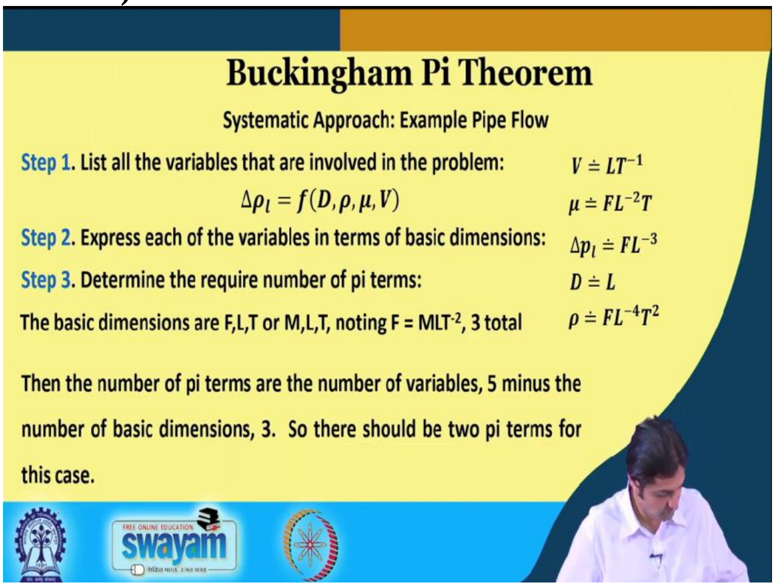

So, there are several steps, I have mentioned this to you before that if you follow these steps carefully, you will be able to tackle all the problems that are related to dimensional analysis. So, the step 1 is, you have to list all the variables that are involved in the problem. In our case, we know that listing the variables was, one is pressure per unit length, something that needs to be find out. Then there is a diameter D, there is the density ρ, then there is viscosity μ and the velocity V. So, first step we have done. We have listed all the variables that are involved in the problem.

Second step is, we have to express each of these variables in terms of basic dimensions, very simple to do. So, we have to write down the dimensions of all these 5 variables. So, velocity is LT^{-1} here, μ is $FL^{-2}T$, so, delta pl is FL^{-3}. These dimensions, if you recall, we had written some slides ago when we were trying to explain it through experimental procedure and the D is L. Whatever remaining it is ρ, it is $FL^{-4}T^{2}$. So, this is step 2.

After that, we have to determine the unique number of pi term. So, what we have seen? The unique number of pi terms. How? So, going to tell you now, first I will tell you and then do all the. So, first we have to know what is k, k is total number of variables. So, in our case, 1, 2, 3, 4, 5, or we can count here, as well, 1, 2, 3, 4, 5. So, k was 5. And what our miniμm numbers of reference dimensions that are there? So, that you can see by looking at the dimensions of this variable.

So, we have length L also is there, T is also there, F is there. Is there any other term? Every all this 5 of these terms has basic dimensions length T and F, F, L, T. So, that means, suppose if there was no F in any of the 5 variables, then reference dimensions could have been 2. Here, L is also there, T is also their, F is also there, so, r will be 3. So, number of Pi terms is going to be k – r, according to the Buckingham Pi theorem. So, 2 Pi terms, this is just to explain you .

So, step 3 says, determine the required number of pi terms. So, in our, in this particular case, the basic dimensions are F, L, T. So, that means basic dimensions are 3 total, same thing what I have written before. Then the number of Pi terms are the number of variables, 5 minus the number of basic dimensions, 3. So, there should be 2 Pi terms for this case. We have already written it and seen it. So, 3 steps we have done. First step was, determining the, writing the all the variables, second writing their dimensions and third number determining the number of Pi terms. This forms very crucial first 3steps.

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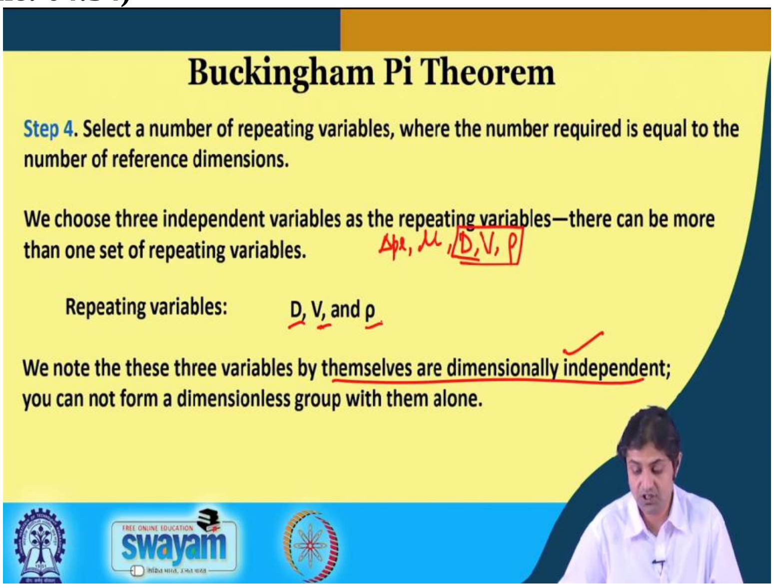

Now, step 4, we have to select a number of repeating variables, where the number required is equal to the number of reference dimension. So, repeating variables will be equal to the number of reference dimension. In this case, we have how many references dimension? 3. So, number of repeating variables that will be there in all the dimensionless Pi terms will be 3, I mean, repeating variables. So, 3 repeating variables and this will depend upon, 3 repeating variables for this case, not always, for this case of pipe flow.

And why? Because the repeating variable should be equal to the number of reference dimensions, in our case it is 3. So, we choose 3 independent variables as repeating variables, so, this is also important. So, those 3 repeating variables should be independent, one should not, one cannot be obtained from the other. There can be more than one set of repeating variables. So, one set of repeating variables μst be 3 independent variables. So, you cannot have both, for example, μ and nu as repeating variable, for example, or gamma and ρ. You should try to avoid this type of repeating value variables, try to avoid.

Now, in our this pipe flow case, we have chosen repeating variables as, we had 5 variables, delta pl, we had μ, we have diameter, we have velocity and we have ρ. So, this 3 we have chosen as repeating variables. We note that these 3 variables by themselves are dimensionally independent. So, this is an important criteria. Which, I mean, what does this dimensionally independent means? You are not able to form a dimensionless group with them alone.

So, you cannot, no way if you choose D, V and ρ, you will be able to form a dimensionless group. So, they are dimensionally independent. That is one of the important criterias of choosing a repeating variable.

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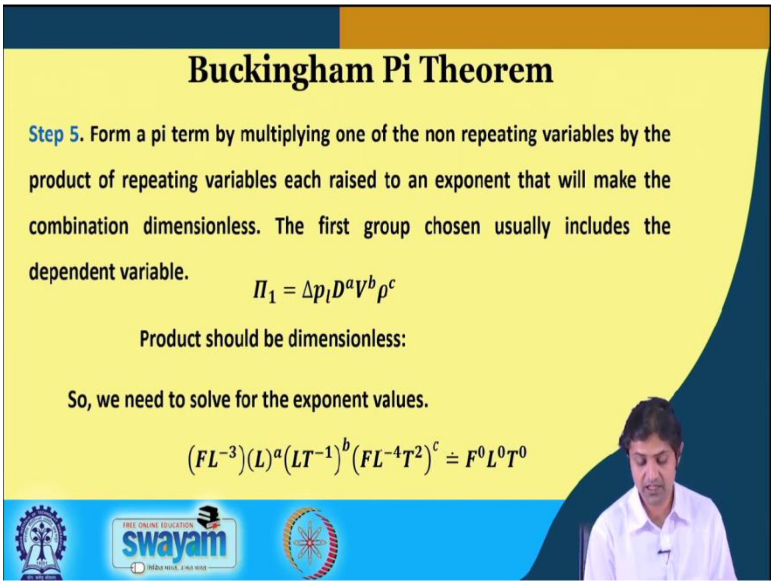

So, now, the step 5. So, step 4 was, we have to select the number of repeating variables, we have chosen that. Now, step 5 is, we have to form a Pi term. Now, how is that is formed? We have to form a Pi term by µltiplying one of the non repeating variables by the product of the repeating variables each raised to an exponent that will make the combination dimensionless. So, this means now, for in our particular case, where we had 5 variables, pressure drop per unit length, µ, D, ρ and V. We chose 3 as repeating variables, which means we were left with 2.

So, first pi turn will be found, will be formed by µltiplying one of these. So, out of left two, we will µltiply one. So, we will see, which we, so, we have D,V and ρ. So, we have delta pl left. So, this is the first Pi term, Pi 1 is going to be, µltiplying one of the non repeating, so I am µltiplying delta pl µltiplied by product of the repeating variables. So, D to the power a, V to the power b and ρ to the power c, something like that.

I mean, we will see exactly how I have done in this slide, but yeah. So, but the main message is, µltiplying one of the repeating variables, one of the non repeating variables. So, which is this pressure drop per unit length by product of repeating variables and each raise to power of an exponent, a, b, c, which are still unknown. So, this is how we are forming one pi term. Similarly, we can form the second pi term, as well. So, out of the in this case, we µltiplied only one, for the second Pi term you can µltiply the other left one.

So, as I did, so, Pi 1 is going to be, delta pl D to the power a V to the power b ρ to the power c. Now, this product should be dimensionless, that is the purpose. So, this should be dimensionless. So, we need to solve for these exponent values. How? So, simply, what we have done here, so, we have written the dimensions of pressure per unit length, because we wrote the dimensions of each of these, you know, variables, D to the power a, that means, L to the power a. V is LT – 1., so, V to the power b and ρ is FL - 4T square and this to the power c should be having no dimensions.

So, no dimension means, F to the power 0, L to the power 0, T to the power 0. Normally, how would you solve that? You will collect the powers of F and equate it to 0, you will collect the powers of L and equate it to 0 and you will collect the powers of T and equate it to 0, we will see.

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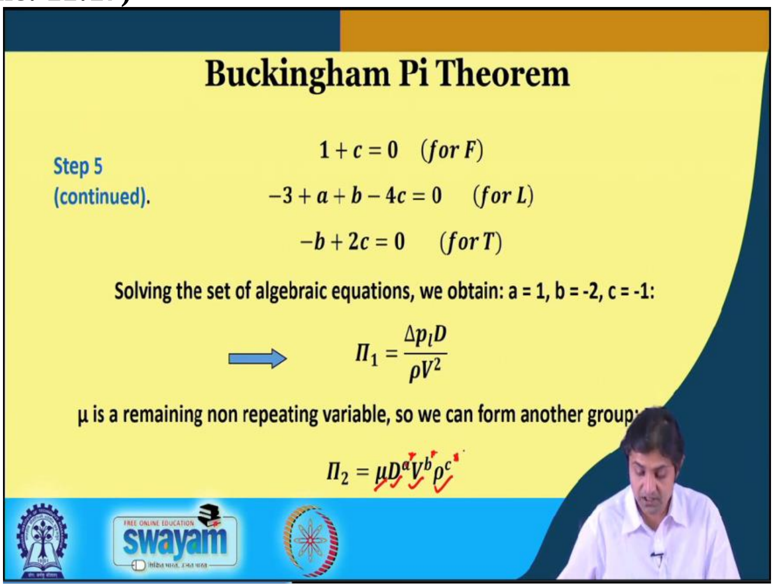

So, step 5 still continuing because we are forming the, so, you see, I will just take. So, F appears here, so, F to the power 1 and F to the power c. So, this will be, there is only 2 Fs, so it will become 1 + C is equal to 0. For L, it is going to be minus 3 + a + b – 4c is equal to 0. And similarly, for T, minus b + 2c is equal to 0. But we have written it here, in a μch, so, see, 1 + c = 0, this will give C as, minus 1. So, b is equal to 2c, this implies, b is equal to minus 2 and if we know c and b we can get a here.

So, solving these things, we get, a is equal to 1, b is equal to minus 2, c is equal to 1, as we have already found out here. So, this will give us, so, if we put back a, b and c in that dimensionless group pi one will become delta pl into D divided by ρ V Square . So, now so, first dimensionless we have found out. So, now there is only μ that is remaining, that is, non repeating variable. So, in similar way we can form another group.

So, we form another group, where we have, μ D to the power a V to the power b ρ to the power c. One important thing to note that, this a and this a are not equal. So, actually we can write, a star, b star or c star, so that, you do not get confused. But we have to solve in a similar way, this a b c will be different from what we have obtained for Pi 1. Do not simply substitute a b c that is obtained from the first dimensionless group. It is a new a b c, small a, small b, small c, very important to know.

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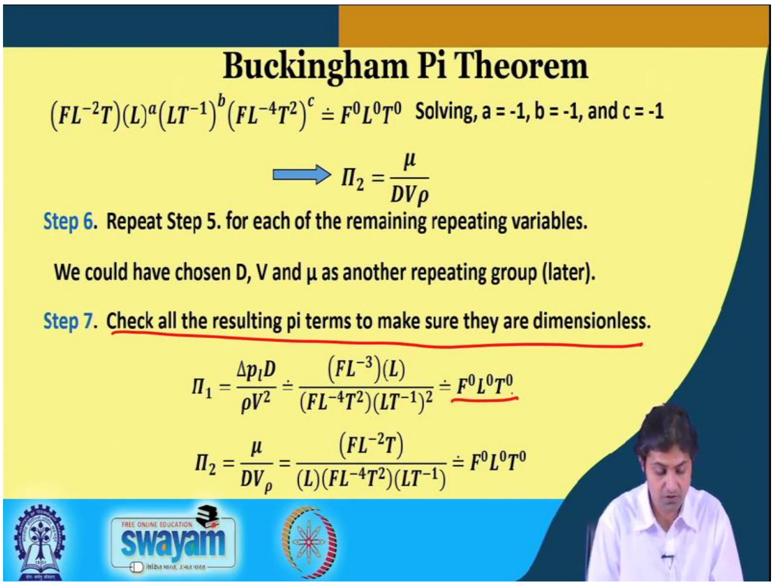

So, we follow the procedure again. So, this was μ, L to the power a V to the power b ρ to the power c is equal to F0 L0 T0, F to the power 0 L to the power 0 and T to the power 0. So, if you solve this, we are going to get, a is equal to minus 1, b is equal to minus 1, c is equal to minus 1. So, 3 equations you can obtain. So, if you see, so it will be same, 1 + c is equal to 0. So, F is from here, and F is here. So if you start collecting b, so it will b, 1 – b, 1 – b +2c is equal to 0, see, we have already obtained. So, 1- 2b - 2 is equal to 0. So, if you take 2b, 1 - 2 is - 1, and this is what we get

And similarly, for b, so - 2 + a + b – 4c is equal to 0 and this is going to give us, a also as, minus 1. But anyways, I will rub this and you can see if we solve this equation, we are going to get a is equal to – 1, b is equal to -1 and c is equal to – 1. And our new dimensionless group is, μ / DV ρ. And is this term look, does this term look similar to you? One by Reynolds number. We will come to that.

Now, the step 6 of a general procedure is, we have to repeat this step 5, if there are more number of repeating variables that are left. So, we have to repeat step 5 for each of the remaining repeating variables. There will be cases where you will not get only 2 dimensionless group, but let us say, 3, 4, 5 depends how many. In another way, we could also have chosen DV and μ, as another repeating group.

I mean, it is not only that we had to choose the one that we chose before. Instead of ρ, you could have chosen μ as repeating variable and ρ could have been chosen as non repeating variable. The one, I mean, delta pl here, which was to be found, should always be in a non-repeating variable, otherwise your equation will become implicit and that is what we do not want. If there is something on both on the left hand side and the right hand side it does not help. So, the best thing to do is, to keep the dependent variable as one of the non repeating variables.

Now, the step 7 is, you have to check all the resulting pi terms to make sure that they are dimensionless. That is an important step. So, here we check again, we actually have done the same steps, I mean, and same checking of this term in this, I mean, last lecture. So, we can see, Pi 1 is F to the power 0 L to the power 0 T to the power 0 and similarly, this is also 0. This is step 7. So, both Pi 1 and Pi 2 are dimensionless.

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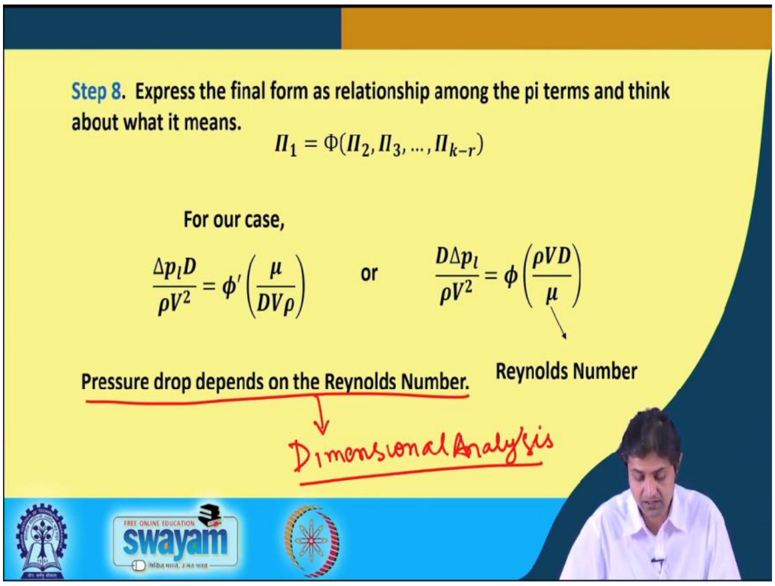

Now, in the end, express the final form as a relationship among Pi terms and think about what it means. So, the final step is Pi 1 is a function of Pi 2, Pi 3. Here, we have only 2 so we can simply write; Pi 1 is a function of Pi 2. So, Pi 1 is a function of Pi 2, or we can simply write in a reverse format as well, D delta L ρ V square is equal to, if this is dimension, so 1 by this is also dimensions so we have written this and this is as I told you, is the Reynolds number.

So, important message here is pressure drop depends on Reynolds number and as you see, we have obtained this result, an important result that the pressure drop depends on the Reynolds number using dimensional analysis. So, I hope at this by this point in time, you would be appreciating how powerful dimensional analysis can be for a future, you know, aspiring experimentalists this is one topic which you µst understand, otherwise it is impossible to carry on the experiments.

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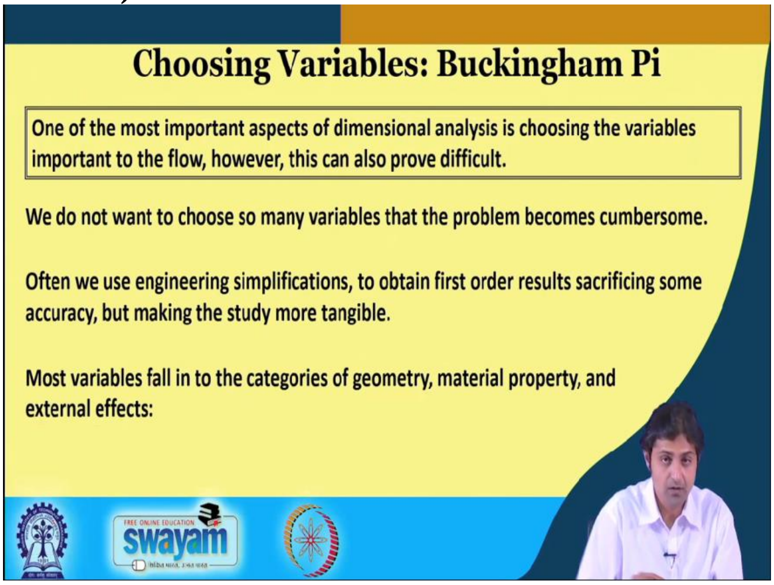

Now, there are some rules or guidelines for choosing variables for Buckingham Pi theorem. So, one of the as I said one of the most important aspect of dimensional analysis is choosing the variables important to the flow, however, this can be very difficult. So, in a very, you know, big problem, large problem, there could be many, many variables. Now, the challenge is, how to choose the variables which are more important to the flow than the other.

The guideline is, we do not want to choose, we should not choose so many variables that the problem becomes cumbersome. If we choose hundred variables then the problem will be large, you know, I mean, it is very difficult to even solve it using dimensional analysis. So, to be able to do that we use engineering simplifications. . So, what does this do? We obtain in this first order results sacrificing some accuracy, but making study more tangible.

So, while choosing these variables, we have found out that most of these variables will fall into one of these 3 categories, most of them, geometry, material property and external effects. So, best is to, you know, choose carefully. So, all the variables should be falling, but this is only guideline, every problem is different, but most, more or less majority of the problem will satisfy these.

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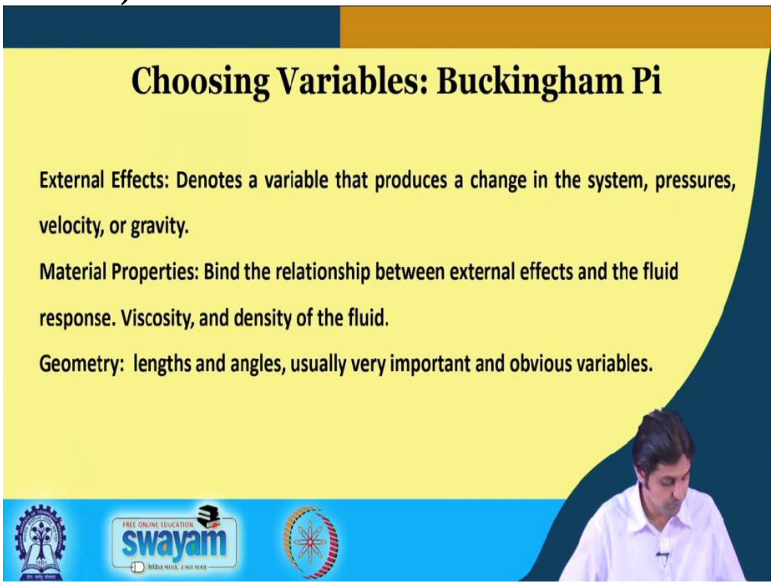

External effects. What are external effects? External effects denotes a variable that produces change in the system like pressures, velocity or gravity. These are external effect. So, by definition, it denotes variable that produces a change in the system. What are material properties? Material properties; it binds the relationship between the external effects and fluid response. For example, the viscosity and density of the fluid can be listed under material properties.

Geometry; it is the lengths and angles usually very important and obvious variables, these most common one. You cannot neglect, that for example, the diameter of the pipe in our case.

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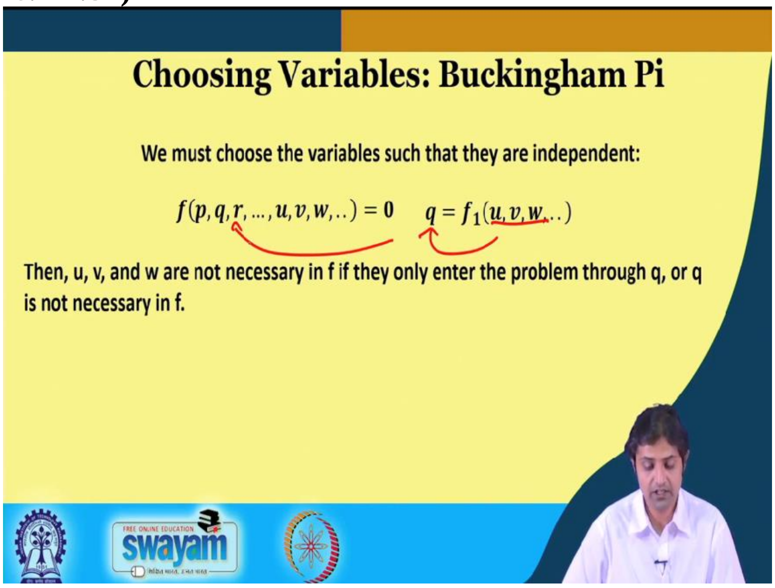

As I said, we µst choose the variables such that they are independent. There is no point in choosing all these 3, µ, nu and ρ, we should not be choosing all 3 of them. We can either choose nu or ρ, or µ and ρ, or even in some case, µ and nu, but preferably µ and nu and ρ. Because why I am saying this, because these are not independent, µ is equal to ρ into nu, this is the forµla. So, because of this relationship, they are not independent, all 3 of them, for instance.

So, f p, q, r…u, v, w is equal to 0, this is one equation. And q can be written as, f 1 of say u, v, w, for example. Then u, v and w are not necessarily in f, if they only enter the problem through q, or q is not necessary in f. So, we µst not, you know, put it in both places, both in f or q. So, that is one of the other way of saying that it should be independent. So, either you enter the variable u, v and w through q, or enter it through f here, in this one.

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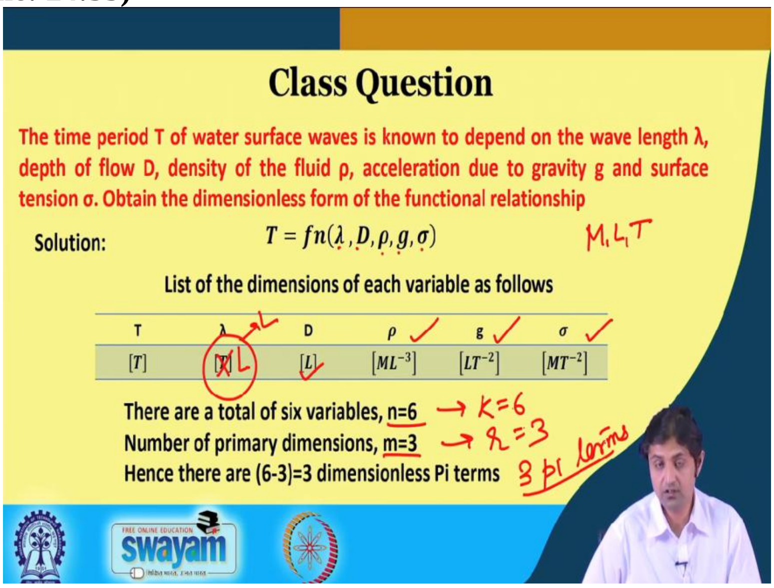

So, we will start with a question, another question. So, the time period T of the water surface waves is known to depend on the wave length lambda, depth of the flow D, density of the fluid ρ and acceleration due to gravity g and surface tension sigma. So, using the Buckingham Pi theorem, obtained the dimensionless form of the functional relationship. This is a classical problem of, you know, Buckingham Pi theorem, which we are going to solve in the class today.

So, first, we write down all the variables which already have been given. It says, time period T, that is how we have T. We have to find out T and it is known to depend on the wavelength lambda, so, lambda is here, depth of the flow D. It is our good luck that we already have been told what variable this time period depends upon and so, T is not only non repeating variable, we will see later, but it is also something that needs to be found out, that is why we have written T as a function of.

So, going back again, the time period T depends upon wavelength lambda, we have already ticked it, depth of the flow D, we have already ticked it, density of the fluid ρ, acceleration due to gravity g and the surface tension sigma. So, if you see, how many terms are there, 1, 2, 3, 4, 5, 6. So, total 6 variables are there. Second step is listing all the dimensions and that is going to be, so, T is time period is time period T. Lambda is, so, this is actually lambda is wavelength L and the depth of the flow is also L, ρ is the density of the fluid.

So, it can be, what we have chosen, we have chosen the basic dimension as M, L and T. So, sorry for this typo, this should be L. P is M L – 3, G is LT – 2 and surface tension sigma is MT - 2. So, as we have seen, 1, 2, 3, 4, 5, 6, the total number of variables is 6. So, n is equal to 6 or k is equal to 6 and number of primary dimension m is equal to, we had been calling in Buckingham Pi theorem k= 6 and r as, 3. So, number of Pi terms will be k – r, so total 3 Pi terms.

So, some of our steps are complete, we have determined k = 6 r as 3 and therefore number of Pi term is going to be 6 – 3 = 3. These are some very, very important steps, this µst be followed.

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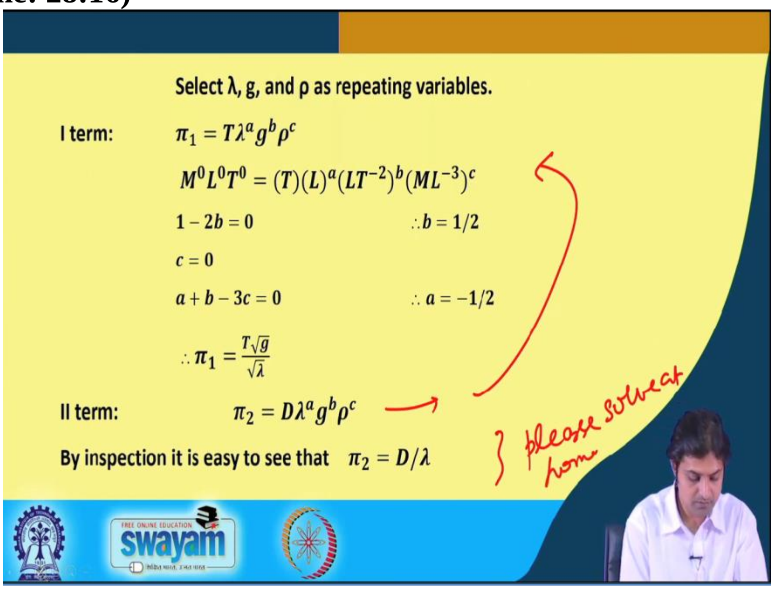

Now, we have selected, we have to select the repeating variables. So, we have selected lambda, G and ρ as repeating variable. So, if we have, so that is why first we because if there are 3 repeating variables. So, Pi 1 will be µltiplied by say first non-repeating. So, T µltiplied by lambda to the power a, g to the power b, ρ to the power c, so this should be dimensionless, Pi 1. So, we have written down the dimensions, M to the power 0, L to the power 0, T to the power 0. T is T, this T is T, lambda has a dimension of L, so, L to the power a.

g has dimension of LT to the power - 2. So, therefore it is LT minus 2 to the whole power b and ρ is M L – 3, so M L - 3 to the power c. So, if we solve this, we get 1 - 2 b. Why? See, we have done M. No, we have done T. So, T so we have equated the coefficient of T, 1 – 2b is equal to 0, this gives b is equal to half. Secondly, we have c = 0 is what we get, if we compute the, so, b equal to and we have to calculate a + b - 3 c.

So, actually we should be, so the other term is going to be so, a, so we will calculate b so it will be B minus. So, how are we going to get the C is equal to 0 we equate M to the power 0. So, M is only in one term. So, M to the power C therefore C is equal to 0 and to equate you know we have to do for L, L to the power so, it will be A - A + B - 3 C is equal to 0 correct. So, this has been gotten for equating exponent of T and this has been by equating the exponent of M and the third equation by equating the exponent of L.

So, these are we get these equal to half c equal to 0 and as equal to - half. So, our first term is going to be T under root g / under root lambda. Now, the second pi term is D lambda to the power a, g to the power b and ρ to the power c, so, these repeating terms are the same, of course, exponents we will recalculate. So, we are going to recalculate now because this is not the same and instead of T we have put D. So, by inspection, it is easy to see that I mean if you do the same analysis as this, you will find out another Pi 2 terms as d / lambda. Please solve at home.

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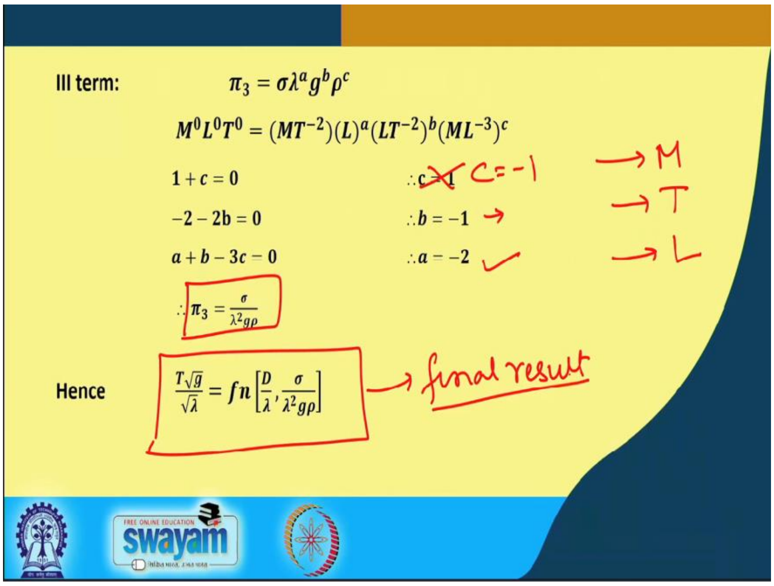

The third is going to be the third repeating variable non repeating variable sigma that is the surface tension and lambda to the power a, g to the power b, ρ to the power c and this will same procedure lambda, sorry, sigma dimension this is L is lambda, this is g and this is P and we calculate it we can get c is equal to. There seems to be a problem 1 +. So, we have done M, 1 + C. So, C will be - 1 for you know and then this is equating M , exponent of M , L will be. So, we are going to equate T, so - 2 - 2 b and this gives us b equal to - 1.

This is equating T and now if equate L, so L to the power a + b – 3c and this will give us a is equal to 3 c - b. So, 3 into - 1 - so that will become a is equal to - 2. So, this is also correct. So I will just you know, sorry for the error here. But the calculation c should be - 1 and this is computed by equating the exponents of L. So Pi 3 is sigma / lambda square P g, so this is another pi term. So we have found out Pi 1, Pi 2 and Pi 3 using this Buckingham Pi theorem some steps.

Now, the last step is expressing Pi 1 or any Pi’s as a function of the other 2. So, this is our final result. So, I will like to close this lecture at this point and we will proceed in our next lecture. Thank you so µch.