Chapter 21

UNBRACED ROLLED STEEL BEAMS

21.1 Introduction

1 In a previous chapter we have examined the behavior of laterally supported beams. Under those conditions, the potential modes of failures were either the formation of a plastic hinge (if the section is compact), or local buckling of the flange or the web (partially compact section).

2 Rarely are the compression flange of beams entirely free of all restraint, and in general there are two types of lateral supports:

1. Continuous lateral support by embedment of the compression flange in a concrete slab.

2. Lateral support at intervals through cross beams, cross frames, ties, or struts.

3 Now that the beam is not laterally supported, we ought to consider a third potential mode of failure, lateral torsional buckling.

21.2 Background

4 Whereas it is beyond the scope of this course to derive the governing differential equation for flexural torsional buckling (which is covered in either *Mechanics of Materials II* or in *Steel Structures*), we shall review some related topics in order to understand the AISC equations later on

5 There are two types of torsion:

Saint-Venant's torsion: or pure torsion (torsion is constant throughout the length) where it is assumed that the cross-sectional plane prior to the application of torsion remains plane, and only rotation occurs.

Warping torsion: out of plane effects arise when the flanges are laterally displaced during twisting. Compression flange will bend in one direction laterally while its tension flange will bend in another. In this case part of the torque is resisted by bending and the rest by Saint-Venant's torsion.

$$X_1 = \frac{\pi}{S_x}\sqrt{\frac{EGJA}{2}} \quad \text{(21.4-a)}$$

$$X_2 = 4\frac{C_w}{I_y}\left(\frac{S_x}{GJ}\right)^2 \quad \text{(21.4-b)}$$

are not really physical properties but a combination of cross-sectional ones which simplifies the writing of Eq. 21.3. Those values are tabulated in the AISC manual.

9 The flexural efficiency of the member increases when X_1 decreases and/or X_2 increases.

21.3.2 Governing Moments

1. $L_b < L_p$: "very short" Plastic hinge

$$\boxed{M_n = M_p = Z_x F_y} \quad (21.5)$$

2. $L_p < L_b < L_r$: "short" inelastic lateral torsional buckling

$$\boxed{M_n = C_b\left[M_p - (M_p - M_r)\left(\frac{L_b - L_p}{L_r - L_p}\right)\right] \leq M_p} \quad (21.6)$$

and

$$\boxed{C_b = 1.75 + 1.05\left(\frac{M_1}{M_2}\right) + 0.3\left(\frac{M_1}{M_2}\right)^2 \leq 2.3} \quad (21.7)$$

where M_1 is the smaller and M_2 is the larger end moment *in the unbraced segment* $\frac{M_1}{M_2}$ is negative when the moments cause single curvature (i.e. one of them is clockwise, the other counterclockwise), hence the most severe loading case with constant M gives $C_b = 1.75 - 1.05 + 0.3 = 1.0$.

M_r is the moment strength available for service load when extreme fiber reach the yield stress F_y;

$$M_r = (F_y - F_r)S_x \quad (21.8)$$

3. $L_r < L_b$ "long" elastic lateral torsional buckling, and the critical moment is the same as in Eq.21.2

$$\boxed{M_{cr} = C_b\frac{\pi}{L_b}\sqrt{\left(\frac{\pi E}{L_b}\right)^2 C_w I_y + EI_y GJ} \leq C_b M_r \text{ and } M_p} \quad (21.9)$$

or if expressed in terms of X_1 and X_2

$$\boxed{M_{cr} = \frac{C_b S_x X_1 \sqrt{2}}{\frac{L_b}{r_y}}\sqrt{1 + \frac{X_1^2 X_2}{2\left(\frac{L_b}{r_y}\right)^2}} \leq C_b M_r} \quad (21.10)$$

10 Fig. 21.2 summarizes the governing equations.