1.22 Triangulation Surveys

Triangulation is one of the methods of fixing accurate controls. It is based on the trigonometric proposition that if one side and two angles of a triangle are known, the remaining sides can be computed. A triangulation system consists of a series of triangles in which long side is normally called as *base line*. The base line is measured and remaining sides are calculated from the angles measured at the vertices of the triangles; vertices being the control points are called as triangulation stations (Figure 1.54). Applications of triangulation surveys includes; (i) establishment of accurate control for plane and geodetic surveys as well as photogrammetric surveys covering large areas, (ii) determination of the size and shape of the Earth, and (iii) determination of accurate locations for setting out of civil engineering works, such as piers and abutments of long span bridges, fixing centre line, terminal points and shafts for long tunnels, measurement of the deformation of dams, etc.

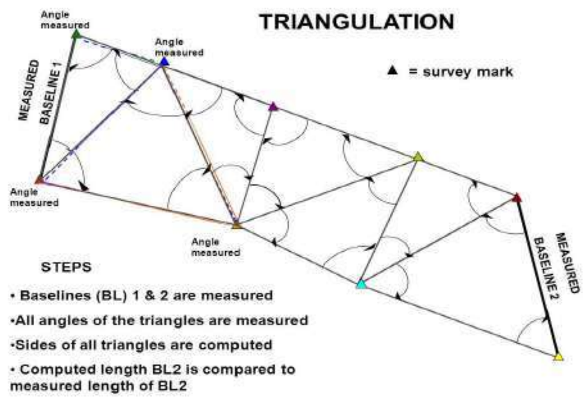


Figure 1.54 Triangulation survey scheme

The area to be covered by triangulation scheme must be carefully studied to select the most suitable positions for the control stations. Existing maps, especially if contoured can be of great value since the size and shape of triangles formed by the stations can be difficult to visualize in the field.

The following factors are considered in selecting the stations-

1. The best shape of a triangle is an isosceles triangle whose base angles are 56° 14' each.
2. Simple triangles should be preferably equilateral triangles which are may treated as well conditioned triangles.
3. The angles of simple triangles should not be less than 30° or more than 120°
4. Braced quadrilaterals are preferred figures but no angle should be less than 30°
5. Centered polygons should be regular, and no angle should be less than 40°
6. No angle of the figure, opposite a known side should be small
7. The sides of the figures should be of comparable length
8. Stations are placed on the highest points of the elevated places such as hilltops, house rooftops, etc., to ensure intervisibility.
9. Easy access to the stations with instruments and equipment.
10. Stations should be useful for providing intersected points and also for a subsequent detailed survey.
11. Very far-off stations should be avoided for plane surveys.
12. Grazing line of sight should be avoided and line of sight should not pass over the industrial area to avoid atmospheric refraction.
13. Cost of clearing and cutting of vegetation along the line of sight should be minimum.

1.22.1 Trilateration

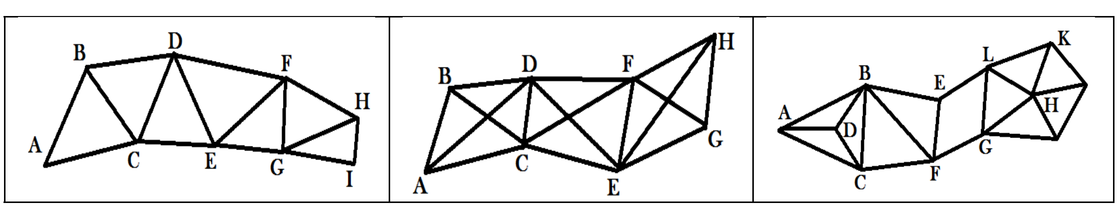
Trilateration is a surveying procedure in which the lengths of the sides of a triangle of triangulation scheme are measured, generally electronically, and included angles are determined trigonometrically. The field techniques for trilateration are similar to those for triangulation, with the exception that only lines are measured while large number of angles are calculated. Trilateration in surveying is used to determine the horizontal positions, in addition to other methods, like triangulation, intersection, resection and GPS positioning.

The trilateration consists of series of joined or overlapped triangles that forms triangles, quadrilaterals and polygons. It employs measurement of sides of the triangle, using the EDMs. With the development of EDMs, trilateration has become highly accurate and precise for expanding and establishing the horizontal controls. The areas that is subjected to seismic activity, trilateration is employed to study the gradual and secular movement in Earth's crust. It is used for determining the position of objects on the surface of Earth, involving the distance and angles. Trilateration is also used in control expansion or densification for future metropolitan growth; coastline control; inland waterways; control extension; densification for land subdivisions and construction; and deformation surveys of dams, geothermal areas, structures, regional/local tectonics, and landslides. It is used to achieve rapid control expansion with utmost accuracy, and is less expensive as compared to triangulation.

1.22.2 Principle of triangulation

In triangulation, entire area to be surveyed is divided into framework of triangles. Vertices of the individual triangles are known as triangulation stations. Triangulation is more accurate than the theodolite transverse, as there is less accumulation of error than that in theodolite traverse. A series of triangulation may consist of (i) a chain of simple triangles, (ii) braced quadrilaterals, and (iii) centered polygons, as shown in Figures 1.55a, b and c. Simple triangles of a triangulation system do not provide many checks on the accuracy of observations as there is only one route through which distance can be computed. In this layout, checking baselines and azimuth at frequent intervals are necessary to avoid the errors.

Although triangle is the smallest figure in triangulations scheme, but braced quadrilateral containing four corner stations is preferred. Braced quadrilaterals system is treated to be the best arrangement, as it provides a means of computing the length of the sides using different combination of the sides, diagonals and angles, and thereby offering more checks on measured/computed data. The braced quadrilateral system is considered as the best arrangement of triangles as it provides several ways to compute the length of sides using various combinations of sides and angles. A series of triangulation scheme may also consist of centered polygons, where one station is within the polygon. Such figure provides many more checks than a simple quadrilateral, so it offers more accurate solution, but takes more computational time. The figures containing centered polygons (such as quadrilaterals, pentagons, or hexagons) and centered triangles is known as *centered figures*. This layout is generally preferred for covering a vast area. This system provides a proper check on the accuracy of work, but the progress of work is slow as more settings of instrument are required.



Figures 1.55 (a) Chain of triangles, (b) quadrilaterals, and (c) centred polygons.

1.22.3 Types of triangulations schemes

The triangulation schemes are classified into three categories: (i) First order or Primary triangulation, (ii) Second order or Secondary triangulation, and (iii) Third order or Tertiary triangulation. The basis of the classification of triangulation schemes is the accuracy with which the length and azimuth of a line of the triangulation are determined. Triangulation systems of different accuracies depend on the extent and the purpose of the survey. The first order triangulation is of the highest order, and is employed either to determine the Earth's figure or to furnish the most precise control points to which secondary triangulation may be connected. The specifications of each category of triangulation are given in Table 1.9.

Table 1.9 Specifications of triangulation schemes

S.No.	Specifications	Primary triangulation	Secondary triangulation	Tertiary triangulation
1	Length of the baseline	5 km to 15 km	1.5 km to 5 km	0.5 km to 3 km
2	Length of the sides of the triangle	30 km to 150 km	8 km to 65 km	1.5 km to 10 km
3	Average triangular error or closure	< 1 sec	3 sec	6 sec
4	Maximum station closure	> 3 sec	8 sec	12 sec
5	Actual error of base	1: 300000	1:150,000	1:75,000
6	Probable error of base	1 in 10,00,000	1 in 5,00,000	1 in 250,000
7	Discrepancy between two measures	10mm/km	20 mm/km	25 mm/km
8	Probable error of computed distance	1 in 60000 to 1 in 250000	1 in 20,000 to 1 in 50,000	1 in 5,000 to 1 in 20,000
9	Probable error in astronomical observation	0.5 sec	2 sec	5 sec

In primary triangulation, the length of base line is between 5 to 15 km, and length of the sides of triangles is between 30 to 150 km. The secondary triangulation consists of a number of points fixed within the framework of primary triangulation. The stations are fixed at close intervals so that the triangles formed are smaller than the primary triangulation. The third-order triangulation consists of a number of points fixed within the framework of secondary triangulation, and forms the immediate control for detailed engineering and other surveys. The sizes of the triangles are small and instrument with moderate precision may be used.

The purpose of triangulation is to establish the accurate control points for plane and geodetic surveys of large areas, and locating engineering works accurately. The triangulation is based on the principle that if the length and bearing of one side and three angles of a triangle are measured precisely, the lengths and directions of other two sides can be computed. So, it is important to measure the base line precisely as it can be used at a starting point for computation. If the surveyed area is large, more than one base line is measured at suitable distances to minimize accumulation of errors in lengths. As a check, the length of one side of last triangle is also measured and compared with the computed length, known as the check line, as shown in Figure 1.56. If the coordinates of any vertex of the triangles and azimuth of any side are also known, the coordinates of the remaining vertices may be computed. This way, coordinates of all triangulation stations are computed.

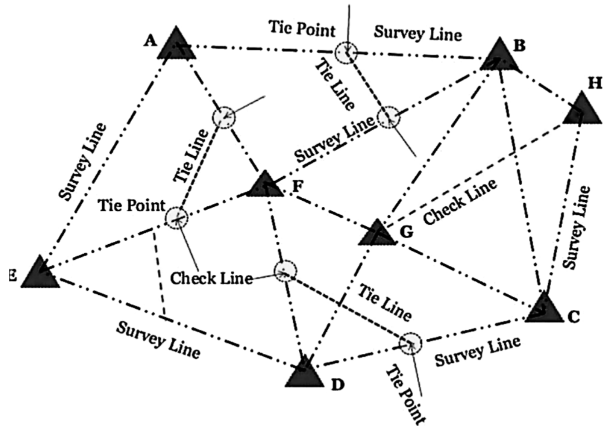


Figure 1.56 Various lines in a triangulations scheme

The following corrections for base line measurement are to be done¬

(i) Correction for the absolute length
(ii) Correction for temperature
(iii) Correction for pull or tension
(iv) Correction for sag
(v) Correction for slope
(vi) Correction for alignment
(vii) Reduction to mean sea level
(viii) Axis signal correction
(ix) Correction for the unequal height
(x) Reduction to chord to arc

1.22.4 Technical terms

(a) Triangulation station marks: Stations are marked with a permanent mark buried below the surface on which a target/signal or instrument is to be centred for taking observations. This mark can be bronze or copper mark cemented into surface.

(b) Laplace station: The triangulation station at which astronomical observation for azimuth is made is called Laplace station.

(c) Satellite stations: In order to secure well condition triangle or have better intervisibility, objects such as church tops, flag poles or towers etc., are sometimes selected as triangulation stations, called satellite stations. Sometimes, it is not possible to set the instrument over the triangulation station, so a subsidiary station known as a satellite station or false station is selected as near as possible to the main station. Observations are made to the other triangulation stations with the same precision from the satellite station.

(d) Signals: Signals are the devices erected on the ground to define the exact position of a triangulation station. It is placed at each station so that the line of sights may be established between the triangulation stations. The signals are classified in two broad classes: (i) luminous signals, and (ii) opaque signals.

(i) Luminous signals: These are further divided into two categories; sun signals, and night signals. Sun signals reflect the rays of the sun towards the station of observation, and are known as *heliotropes*. Such signals can only be used in clear weather. While making observations at night, night signals are used. These includes, such as various forms of oil lamps with a reflector that can be used for sights less than 80 km, and Acetylene lamps which are used for sights more than 80 km.

(ii) Opaque signals: The opaque, or non-luminous signals, are used during day. Most commonly used are the Pole signal, Target signal, Pole and Brush signals, Stone cairn, and Beacons.

(e)Towers: Intervisibility between the stations is the most essential condition in triangulation. When the distance between the stations is too large or the elevation difference is less, both signal and station are to be elevated to overcome the effect of the Earth curvature. A tower is erected at the triangulation station when the station or the signal or both are to be elevated to make them intervisibile between the stations. These towers generally have two independent structures; Outer structure- which is supporting the observer and the signal, and Inner structure- which is supporting the instrument only. This arrangement does not disturb the instrument setting due to the movement of observer. These towers may be made of masonry, timber or steel, but timber scaffolds are most commonly used to heights over 50 m.

The computation of height of signal depends upon the (i) distance between the stations, (ii) relative elevation of the stations, and (iii) profile of the intervening ground. If the intervening ground gives the clear visibility, the distance of horizon from the station of known elevation is given by the following formula-

$$h = 0.06735\ D^2 \qquad (1.34)$$

or $D = \sqrt{(h / 0.06735)}$

Where h is the height of station above datum (in m), and D is the distance of visible horizon (in km).

Let, A and B be two stations having their ground portion elevated to A' and B', respectively (Figure 1.57).

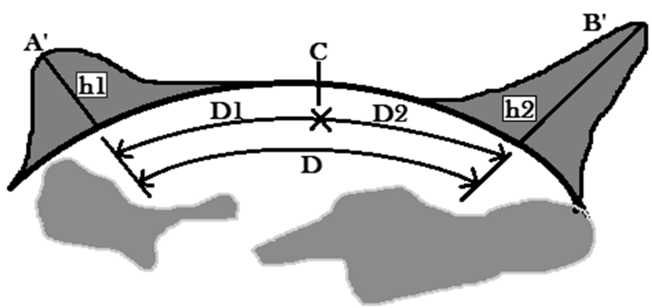


Figure 1.57 Intervisibility between two triangulation stations

h_1 = RL of station A'

h_2 = minimum required elevation of station B so that it is visible from A

D_1 = distance of visible horizon from A

D_2 = distance of visible horizon from B at an elevation h_2

D = known distance between A and B

Now, $D_1 = \sqrt{\frac{h_1}{0.06735}} = 3.858.\sqrt{h_1}$

and, $D_2 = 3.858.\sqrt{h_2}$

But, $D = D_1 + D_2$ *or* $D_2 = D - D_1$

Normally the line of sight is kept at 3 m above the ground to avoid grazing sights as the refraction is maximum near the ground.

(f) Axis signal correction: Signals are installed at each station with different heights to ensure the intervisibility of stations and measuring the vertical angles. The height of instrument axis and the signal at the station sighted are also different, which may result in error in the observed vertical angles. The corrections applied to vertical angles is known as axis signal correction, or eye and object correction, or instrument and signal correction. Let A be the instrument station and B be the station at which signal is kept. The observed vertical angle from A to B will be the true vertical angle, if height of instrument at A (h_1) is equal to the height of signal at B (S_2). But, these two heights, i.e., h_1 and S_2 (Figure 1.58a), respectively, are never same, a correction has to be applied to the observed vertical angle α. Let α_1 be the corrected vertical angle for the axis signal, and D be the horizontal distance AB (Figure 1.58b) equal to the spheroidal distance AA. The axis signal correction is α_1 (∠BAE in Fig. 1.59b). In triangle ABO,

$\angle BAO = \angle A_1AO + \angle BAA_1 = 90° + \alpha$

$\angle AOB = \theta$

$\angle ABO = 180° - [(90° + \alpha) + \theta] = 90° - (\alpha + \theta)$

$\angle EBF = 90° - [90° - (\alpha + \theta)] = \alpha + \theta$

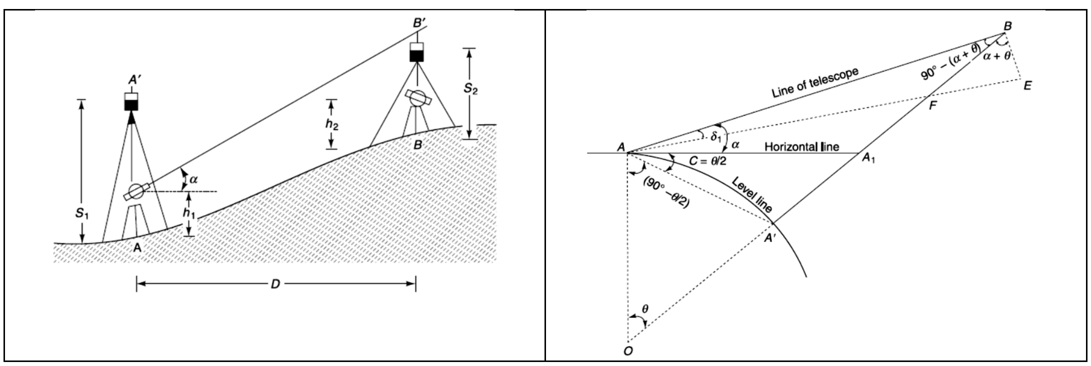


Figure 1.58 (a) Angle observations between triangulation stations, and (b) Axis signal corrections

The following relationship can be derived using Figures 1.59a and b. Usually θ is very small as compared to α, hence the equation can be derived as-

$\tan \delta_1 = (S_2 - h_1) \cos^2\alpha / D$ (1.35)

If the vertical angle is small, the equation reduces to-

$\delta_1 = (S_2 - h_1) / (D \sin 1'')$ (1.36)

Similarly, if observations are made from B towards point A with β as the observed vertical angle and β_1 the corrected vertical angle for axis signal correction, it can be shown that

$\tan \delta_2 = [(S_1 - h_2) \cos^2\beta] / D$ (1.37)

The axis signal correction is negative for an angle of elevation and positive for an angle of depression.

(g) Reduction to center: The angles measured at satellite station are then corrected and reduced to what they would from the actual triangulation station. The operation of applying this correction due to the eccentricity of the station is generally known as reduction to centre. Distance between the actual triangulation station and satellite station is determined by using trigonometric levelling.

1.22.5 Triangulation survey work

It involves two major steps; (i) Field work, and (ii) Computational work. The field work involves (a) Reconnaissance survey, (b) Erection of signals, (c) Measurement of starting baseline, (d) Measurement of horizontal and vertical angles, (e) Observations to determine the azimuth of sides, and (f) Measurement of closing baseline. The computational work includes:

- Checking the observed angles
- Checking the triangulation errors
- Checking the closing horizon error at each station (i.e., sum =360^0)
- Computation of corrected length of baseline
- Computation of sides of triangles
- Computation of the latitude and departure of each side of the triangulation network, and
- Computation of independent coordinates of the triangulation stations.

The reconnaissance of the area involves; (i) proper examination of the terrain, (ii) selection of suitable positions for baselines, (iii) selection of suitable positions of triangulation stations, and (iv) determination of intervisibility of triangulation stations.

1.22.6 Accuracy of triangulation

It can be computed using the relationship given below:

$$m = \sqrt{\frac{\Sigma E^2}{3n}} \qquad (1.38)$$

Where m is the root mean square error of unadjusted horizontal angle (in secs.) as obtained from the triangular error, ΣE^2 is the sum of the square of all the triangular errors in the triangulation series, and n is the total number of triangles in the series.

Unit Summary

This unit discusses 22 different topics of surveying, starting from setting–up the instruments, data collection, error minimisation to map preparation. Components of various surveying equipment, such as levels, theodolites, compass, etc., are explained. Distance, elevations, berainngs and angles measurements play an important role for providing horizontal and vertical controls as well as map preparation in engineering projects. Observation methods have been explained and various parameters computed. These observations need to be accurate enough to create maps useful for project planning. The utility of each equipment is highlighted.

Solved Examples

Example 1.1:
Compute the plotting accuracy of a map at 1:50,000 scale.

Solution:
Plotting accuracy = 0.25 mm x scale
= 0.25 x 50,000 = 12,500 mm
= 1250 cm = 12.5 m

So, any detail smaller than 12.5 m on the ground can't be plotted at 1: 50,000 scale.

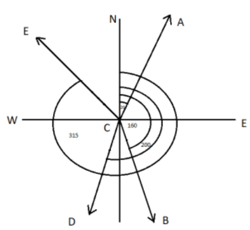
Example 1.2:
How many Survey of India toposheets are there at 1:50,000 scale to cover equivalent area of 1: 250,000 scale?

Solution:
One Survey of India toposheet at 1:250,000 scale covers 1^0 latitude and 1^0 longitude, while on toposheet at 1:50,000 scale covers 15′ x 15′ area. So, 16 toposheets at 1:50,000 would be required to cover equivalent area at 1:250,000 scale.

Example 1.3:
How much length is covered by 1^0 latitude and 1^0 longitude at the equator?

Solution:
Each degree of latitude is approximately 69 miles (111 kilometers) apart. At the equator, the distance is 68.703 miles (110.567 kilometers). A degree of longitude is widest at the equator with a distance of 69.172 miles (111.321 kilometers), and it gradually reduces to zero as all longitude lines meet at the poles.

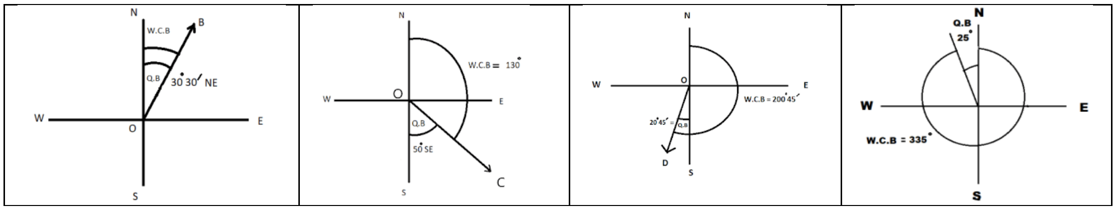
Example 1.4:
Convert the following whole circle bearing to reduced bearing (with quadrant)
a. 30°
b. 160°
c. 200°
d. 315°
Solution:



For 30°, Reduced Bearing (RB) = N 30° E
For 160°, RB = (180° – 160°) = S 20° E
For 200°, RB = (200° – 180°) = S 20° W
For 315°, RB = (360° – 315°) = N 45°W

Example 1.5:
Convert the following quadrantal bearing into whole circle bearing:
1. N 30°30′ E
2. S 50° E
3. S 20°45′ W
4. N 25° W

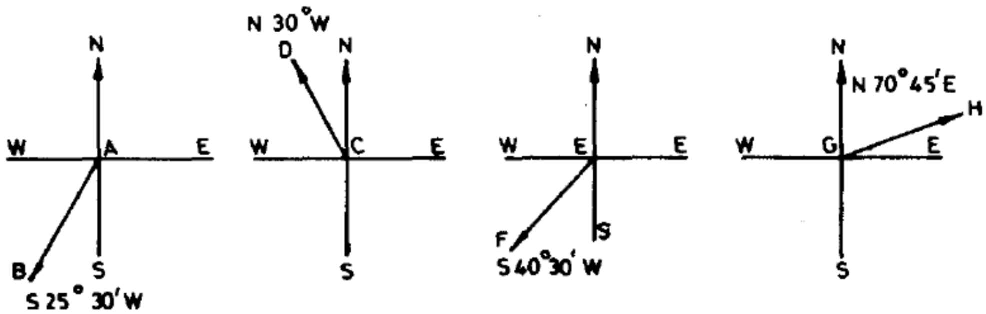
Solution:
Now, the whole circle bearing can be written directly as below:



1. WCB = 30° 30′
2. WCB =180° – 50° = 130°
3. WCB = 180° + 20° 45′ = 200° 45′
4. WCB = 360° – 25° = 335°

Example 1.6:
The fore bearings of the four lines AB, CD, EF and GH are, respectively, as under: (i) S 25^0 30′ W; (ii) N 30^0 W; (iii) S 40^0 30′ W; (iv) N 70^0 45′E. Determine their back bearings.

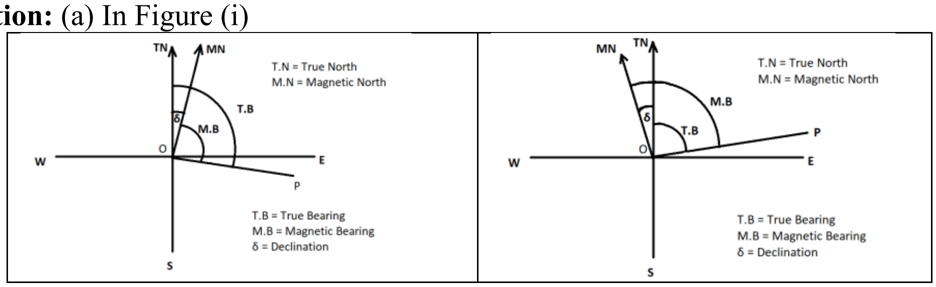
Solution:



Line AB N 25^0 30′ E
Line CD N 30^0 00′ E
Line EF N 40^0 30′ E
Line GH N70^0 45′W

Example 1.7:
The magnetic bearing of a line OP is 89° 45′. Determine the true bearing if the magnetic declination is (a) 5° 30′ E, and (b) 4°15′ W.

Solution: (a) In Figure (i)



True Bearing = Magnetic Bearing + Declination
= (89°45′ + 5°30′) = 95°15′
(b) In Figure (ii)
True Bearing = (Magnetic Bearing – Declination)
= (89°45′ – 4°15′) = 85°30′

Example 1.8:
The compass observations as follows were taken of a closed traverse ABCDEA, find the station affected by the local attraction and correct the observed bearing of the lines.

Line	Bearing
AE	$319^0\,00'$
AB	$72^0\,45'$
BA	$252^0\,00'$
BC	$349^0\,00'$
CB	$167^015'$
CD	$298^0\,30'$
DC	$118^0\,30'$
DE	$229^0\,00'$
ED	$48^0\,00'$
EA	$135^0\,30'$

Solution:
By inspecting the fore and back bearings of the lines, it is clear that the observed fore and back bearings of the line CD (i.e., bearings of CD and DC) exactly differ by 180^0. Therefore, both the stations C and D are considered free from local attractions. Therefore, all the bearings observed from these stations should be correct bearings. The bearings of the line DE should be correct bearing.

Therefore, bearing of ED (back bearing of DE) = 229^0 - 180^0 = 49^0. But the observed bearing of the line ED = 48^0, so station E is affected by local attraction by 1^0. To correct the bearing of the lines at the station E, a value of 1^0 should be added.

The error at the station E is said to be $+1^0$. To correct the values of bearings observed at the station E, 1^0 should be added to the observed bearings, therefore, the correct bearings of lines ED and EA are 49^0 and $136^0\,30'$, respectively.

From the correct bearing of the line EA, the station A may be corrected: Bearing of AE (back bearing of EA) =$136^0\,30'$ (bearing of EA) + 180^0 = $316^0\,30'$. But the observed bearing of the line AE is 319^0. Therefore, the station A is affected by local attraction. To correct the bearing of AE, observed bearing from station A, $2^0\,30'$ should be deducted. Therefore, the correction for local attraction - $2^0\,30'$.

The correct bearings of AE and AB are, therefore, $316^0\,30'$ and $70^0\,15'$, respectively. Therefore, the correct bearing of BA (back bearing of AB) = $70^015'$ (bearing of AB) + 180^0 = $250^0\,15'$. But the observed bearing of BA is 252^0. Therefore, the station B is affected by local attraction with the correction for local attraction as $1^0\,45'$. Therefore, the correct bearings of BA and BC are $250^0\,15'$ and $347^0\,15'$ respectively.

Line	Bearing	Correction due to local attraction	Corrected bearings
AE	$319^0\,00'$	$-2^0\,30'$	$316^0\,30'$
AB	$72^0\,45'$	$-2^0\,30'$	$70^0\,15'$
BA	$252^0\,00'$	$-1^0\,45'$	$250^015'$
BC	$349^0\,00'$	$-1^0\,45'$	$347^015'$
CB	$167^01\,5'$	$0^0\,00'$	$167^015'$
CD	$298^0\,30'$	$0^0\,00'$	$298^0\,30'$
DC	$118^0\,30'$	$0^0\,00'$	$118^0\,30'$

DE	$229^0\,00'$	$0^0\,00'$	$229^0\,00'$
ED	$48^0\,00'$	$1^0\,00'$	$49^0\,00'$
EA	$135^0\,30'$	$1^0\,00'$	$136^0\,30'$

Example 1.9:
Following data of a closed compass traverse PQRSP was taken in a clockwise direction:
(i) Fore bearing and back bearing at station P= 55^0 and 135^0, respectively
(ii) Fore bearing and back bearing of line RS = 211^0 and 31^0, respectively
(iii) Included angles at Q = 100^0 and at R = 105^0
(iv) Local attraction at station R = 2^0 W
From the above data, calculate the local attraction at stations P and S, and calculate the corrected bearings of all the lines.

Solution:

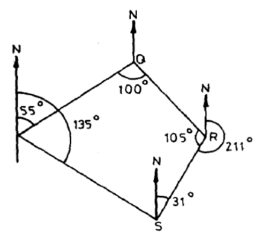


Figure shows the traverse, as FB and BB of the line RS differ exactly by180^0, stations R and S are either free from local attraction or affected by it equally. As the station R is affected, the station S is also affected. The local attraction at S is given as 2^0 W. In other words, the bearing measured at R and S stations are added with 2^0 due to local attraction.

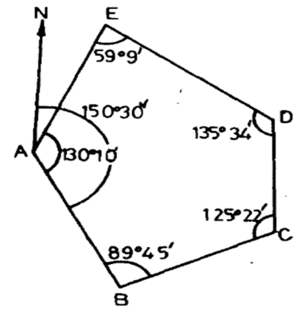
Therefore, corrected FB of RS = $211^0 – 2^0 = 209^0$
Interior $\angle$QPS = $135^0 – 55^0 = 80^0$

Interior $\angle$ PSR = $360^0 – (80^0 + 100^0 + 105^0) = 75^0$
The bearings of all the lines can be determined from the included angles and the corrected bearing of the line RS is 209^0.

BB of RS = $209^0 – 180^0 = 29^0$
FB of SP = $29^0 + (360^0 – 75^0) = 314^0$
BB of SP = $314^0 – 180^0 = 134^0$
FB of PQ = $134^0 – 80^0 = 54^0$
BB of PQ = $54^0 + 180^0 = 234^0$
FB of QR = $234^0 – 100^0 = 134^0$
BB of QR $134^0 + 180^0 = 314^0$
FB of RS $314^0 – 105^0 = 209^0$ (*OK, checked*)

Example 1.10:
In a closed traverse ABCDE, the bearings of the line AB was measured as 150^0 30′. The included angles were measured as under: $\angle$A = 130^0 10′, $\angle$B = 89^0 45′, $\angle$C = 125^0 22′, $\angle$ D = 135^0 34′, and $\angle$E = 59^0 09′. Calculate the bearings of all other lines.

Solution:



Bearing of BC = Bearing of A + ∠ABC

$= (150^0\ 30' + 180^0) + 89^0\ 45' = 420^0\ 15' = 60^0\ 15'$

Bearing of CD = Bearing of CB + ∠BCD

$= (60^0\ 15' + 180^0) + 125^0\ 22' = 365^0\ 37' = 5^0\ 37'$

Bearing of DE = Bearing of DC + ∠CDE

$= (5^0\ 37' + 180^0) + 135^0\ 34' = 321^0\ 11'$

Bearing of EA = Bearing of ED + ∠DEA = $(321^0\ 11' - 180^0) + 59^0\ 9' = 200^0\ 20'$

Check: For checking the calculations, it is advisable to calculate the bearing of the first line from bearings of the last line.

Bearing of AB = Bearing of AE + ∠EAB

$= (200^0\ 20' - 180^0) + 130^0\ 10' = 150^0\ 30'$

Hence, checked.

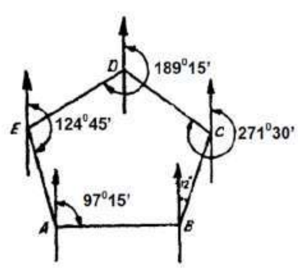
Example 1.11:

The bearing of the sides of a traverse ABCDE are as follows:

Side	**FB**	**BB**
AB	$97^0\ 15'$	$277^0\ 15'$
BC	$12^0\ 00'$	$192^0\ 00'$
CD	$271^0\ 30'$	$91^0\ 30'$
DE	$189^0\ 15'$	$9^0\ 15'$
EA	$124^0\ 45'$	$304^0\ 45'$

Calculate the interior angles of the traverse.

Solution:



Bearing of AE = BB of EA – FB of AB

$= 304^0 45' - 97^0 15' = 207^0 30'$ (exterior angle)

Interior angle at A = $360^0 - 207^0 30' = 152^0 30'$

Bearing of BA = BB of AB – FB of BC

$= 277^0 15' - 12^0 00' = 265^0 15'$ (exterior angle)

Interior angle at B = $360^0 - 265^015' = 94^045'$
Bearing of CB = BB of BC – FB of CD
$= 192^000' - 271^030' = -79^030'$
Interior angle at C = $79^030'$
Bearing of DC = BB of CD – FB of DE
$= 91^030' - 189^015' = -97^045'$
Interior angle at D = $97^045'$
Bearing of ED = BB of DE – FB of EA
$= 09^015' - 124^045' = 115^030'$
Interior angle at E = $115^030'$

Check = $(2*5 - 4) * 90^0 = 540^0$
$= 152^030' + 94^045' + 79^030' + 97^045' + 115^030'$
$= 540^0$

Example 1.12:
First four columns show the readings that are taken in the field. Compute the reduced levels of various points using Rise and Fall method.

Solution:

BS (m)	FS (m)	Rise (m)	Fall (m)	RL (m)	Station
1.575				100	10
0.355	1.355	0.22		100.22	1
0.605	2.685		2.33	97.89	TP1
0.485	2.355		1.75	96.14	TP2
1.37	1.68		1.195	94.945	2
1.025	1.57		0.2	94.745	3
1.15	1.99		0.965	93.78	TP1
1.71	1.84		0.69	93.09	TP2
1.38	1.955		0.245	92.845	4
1.31	1.55		0.17	92.675	5
1.31	1.61		0.3	92.375	6
1.635	1.13	0.18		92.555	7
2.865	0.66	0.975		93.53	8
2.95	0.055	2.81		96.34	TP
2.43	0.43	2.52		98.86	9
0.435	0.245	2.185		101.045	TP
	1.4		0.965	100.08	10

Checks:
Σ FS - Σ BS = Σ Rise – Σ Fall = Change in RL
Σ FS - Σ BS =0.08
Σ Rise – Σ fall =0.08
Change in RL =100.08-100= 0.08
Hence, checked.

Example 1.13:
The following readings were observed with a levelling instrument, the instrument was shifted after 5th and 11th reading, 0.585, 1.010, 1.735, 3.295, 3.775, 0.350, 1.300, 1.795, 2.575, 3.375 3.895, 1.735, 0.635, and 1.605 m. Determine the RLs of various points using height of collimation method if the RL of a point on which the first reading was taken is 136.440 m. Apply necessary checks.

Solution:

BS (m)	IS (m)	FS (m)	HI (m)	RLs (m)	Remarks
0.585			137.025	136.440	1, 2, 3, 4
	1.010			136.015	
	1.735			135.290	
	3.295			133.730	
0.350		3.775	133.600	133.250	5
	1.300			132.300	6, 7, 8, 9, 10
	1.795			131.805	
	2.575			131.025	
	3.375			130.225	
1.735		3.895	131.440	129.705	
	0.635			130.805	11, 12
		1.605		129.835	
ΣBS=2.670		ΣFS=9.275			

Check

ΣBS - ΣFS = Last RL – First RL

2.670 – 9.275 = 129.835 – 136.440

-6.605 = -6.605 m

Example 1.14:

The following consecutive reading were taken with a level and 4 m staff at a common interval of 30 m, as 0.725 on A, 0.935, 2.845, 3.745, 3.935, 0.965, 1.135, 1.785, 2.625, 3.845, 0.965, 1.575 and 2.015 on B. The elevation of point A is 320.50 m. Enter the levels in a table, calculate the reduced levels of points, and apply the checks. Also calculate the gradient of line AB.

Solution:

Interval	BS (m)	IS (m)	FS (m)	Rise (m)	Fall (m)	RL (m)	Remark
0	0.725					320.500	A
30		0.935			0.210	320.290	
60		2.845			1.910	318.380	
90		3.745			0.900	317.480	
120	0.965		3.935		0.190	317.290	CP-1
150		1.135			0.170	317.120	
180		1.785			0.650	316.470	
210		2.625			0.840	315.630	
240	0.965		3.845		1.220	314.410	CP-2
270		1.575			0.610	313.800	
300			2.015		0.440	313.360	B
Sum	**2.655**		**9.795**		**7.140**		

Checks:

Σ B.S. –Σ F.S. = 2.655 – 9.795 = -7.140

Σ Rise – Σ fall = 0.00 – 7.140 = -7.140

Last RL – First RL = 313.360 – 320.500 = -7.140

Distance AB = 30 * 10 = 300 m

Gradient of line AB = (First R.L. – Last RL) / Distance AB = (-7.140) / 300 = 0.0238

Example 1.15:

The following consecutive readings were taken with a level and 4 m levelling staff on a continuously sloping ground at 30 m intervals. 0.680, 1.455, 1.855, 2.330, 2.885, 3.380, 1.055, 1.860, 2.265, 3.540, 0.835, 0.945, 1.530 and 2.250 m. The RL of the starting point was 80.750 m. (i) Enter the above readings in a proper table, (ii) Determine the RL of various staff stations, and (iii) Compute the average gradient of measured ground.

Solution:

Interval	BS (m)	IS (m)	FS (m)	Rise (m)	Fall (m)	RL (m)	Remark
0	0.680					80.750	B.M.
30		1.455			0.775	79.975	
60		1.855			0.400	79.575	
90		2.330			0.475	79.100	
120		2.885			0.555	78.545	
150	1.055		3.380		0.495	78.050	C.P.1
180		1.860			0.805	77.245	
210		2.265			0.405	76.840	
240	0.835		3.540		1.275	75.565	C.P.2
270		0.945			0.110	75.455	
300		1.530			0.585	74.870	
330			2.250		0.720	74.150	
Sum	2.570		9.170		6.600		

Checks:
Σ BS –Σ FS = 2.570 – 9.170 = -6.600
Σ Rise – Σ fall = 0.000 – 6.600 = -6.600
Last RL – First RL = 74.150 – 80.750 = -6.600
Distance = 30 x 11 = 330
Gradient = (First RL – Last RL) / Distance = (80.750 – 74.150) / 330 = 0.020

Examples 1.16:
The following level readings are taken from a page of a level book. Some of the readings are missing. Fill up the missing readings and apply the arithmetic checks.

Station	BS	IS	FS	Rise	Fall	RL
A	3.125					
B	×		×	1.325		125.505
C		2.320			0.055	
D		×				123.850
E	×		2.655			
F	1.620		3.205		2.165	
G		3.625				
H			×			123.090

Solution:
The steps in the solution are as follows:
FS of station B= 3.125-1.325=1.800 m
BS of station B= 2.320-0.055=2.265 m
RL of BM= 125.505-1.325=124.180 m
Fall of station E= 125.850-125.115= 0.735 m
IS of station D = 2.655-0.735=1.920 m
BS of station E= 3.205-2.165=1.040 m
Rise of station H=123.090-120.945=2.145 m

FS of station H= 3.625-2.145=1.480 m

The missing entries are filled and presented in the following table:

Station	BS (m)	IS (m)	FS (m)	Rise (m)	Fall (m)	RL (m)	Remarks
A	3.125					124.180	BM
B	2.265		1.800	1.325		125.505	TP
C		2.320			0.055	125.450	
D		1.920		0.400		123.850	
E	1.040		2.655		0.735	125.115	TP
F	1.620		3.205		2.165	122.950	TP
G		3.625			2.005	120.945	
H			1.480	2.145		123.090	BM
Sum	8.050		9.140	3.870	4.960		

Checks:

ΣBS - ΣFS= 8.050 - 9.140= -1.09 m

ΣRise – Σfall =3.87 - 4.96= -1.09 m

LRL - FRL=123.09-124.180=-1.09 m

Since, ΣBS – ΣFS = Last RL – First RL= Σ Rise – Σ Fall

Therefore, the calculations are correct.

Example 1.17:

A level was set up at the mid-point between two pegs A and B, 50 m apart and the staff readings at A and B were 1.22 and 1.06 m, respectively. With the level set up near A, the readings at A and B were 1.55 and 1.37 m, respectively. Find, the collimation error per 100 m length of sight.

Solution:

Distance between A and B = 50 m.

When the level was set up at center, the difference in staff readings =1.22 - 1.06 = 0.160 m

When the level was set up near, the difference in staff readings A = 1.55 -1.37 = 0.180 m

Collimation error is the difference in above two values = 0.160 - 0.180 = -0.020 (- sign indicates downwards).

So, collimation error per 100 m length = (0.020 / 50)*100 = 0.04 m inclined downward.

Example 1.18:

Following observations are taken during a reciprocal levelling.

Instrument near	P	Q
Staff reading at P (in m)	1.824	0.928
Staff reading at Q (in m)	2.748	1.606

If the reduced level of P is 140.815 m, then compute the reduced level of Q.

Solution:

h_P =1.824 m, h_Q = 2.748 m

h'_P = 0.928 m, h'_Q = 1.606 m

Correct difference in elevation between P and Q is

$h = [(h'_Q - h'_P) + (h_Q - h_p)] / 2$

$= (1.606 - 0.928) + (2.748 - 1.824) / 2$

= 0.801 m > 0, and therefore point Q has lower elevation than point P.

The RL of Q = (140.815 - 0.801)

= 140.014 m