Chapter 11

APPROXIMATE FRAME ANALYSIS

1 Despite the widespread availability of computers, approximate methods of analysis are justified by

1. Inherent assumption made regarding the validity of a linear elastic analysis *vis a vis* of an ultimate failure design.
2. Ability of structures to redistribute internal forces.
3. Uncertainties in load and material properties

2 Vertical loads are treated separately from the horizontal ones.

3 We use the design sign convention for moments (+ve tension below), and for shear (ccw +ve).

4 Assume girders to be numbered from left to right.

5 In all free body diagrams assume positivee forces/moments, and take algeebraic sums.

6 The key to the approximate analysis method is our ability to sketch the deflected shape of a structure and identify inflection points.

7 We begin by considering a uniformly loaded beam and frame. In each case we consider an extreme end of the restraint: a) free or b) restrained. For the frame a relativly flexible or stiff column would be analogous to a free or fixed restrain on the beam, Fig. 11.1.

11.1 Vertical Loads

8 With reference to Fig. 11.1, we now consider an *intermediary* case as shown in Fig. 11.2.

9 With the location of the inflection points identified, we may now determine all the reactions and internal forces from statics.

10 If we now consider a multi-bay/multi-storey frame, the girders at each floor are assumed to be continuous beams, and columns are assumed to resist the resulting unbalanced moments from the girders, we may make the following assumptions

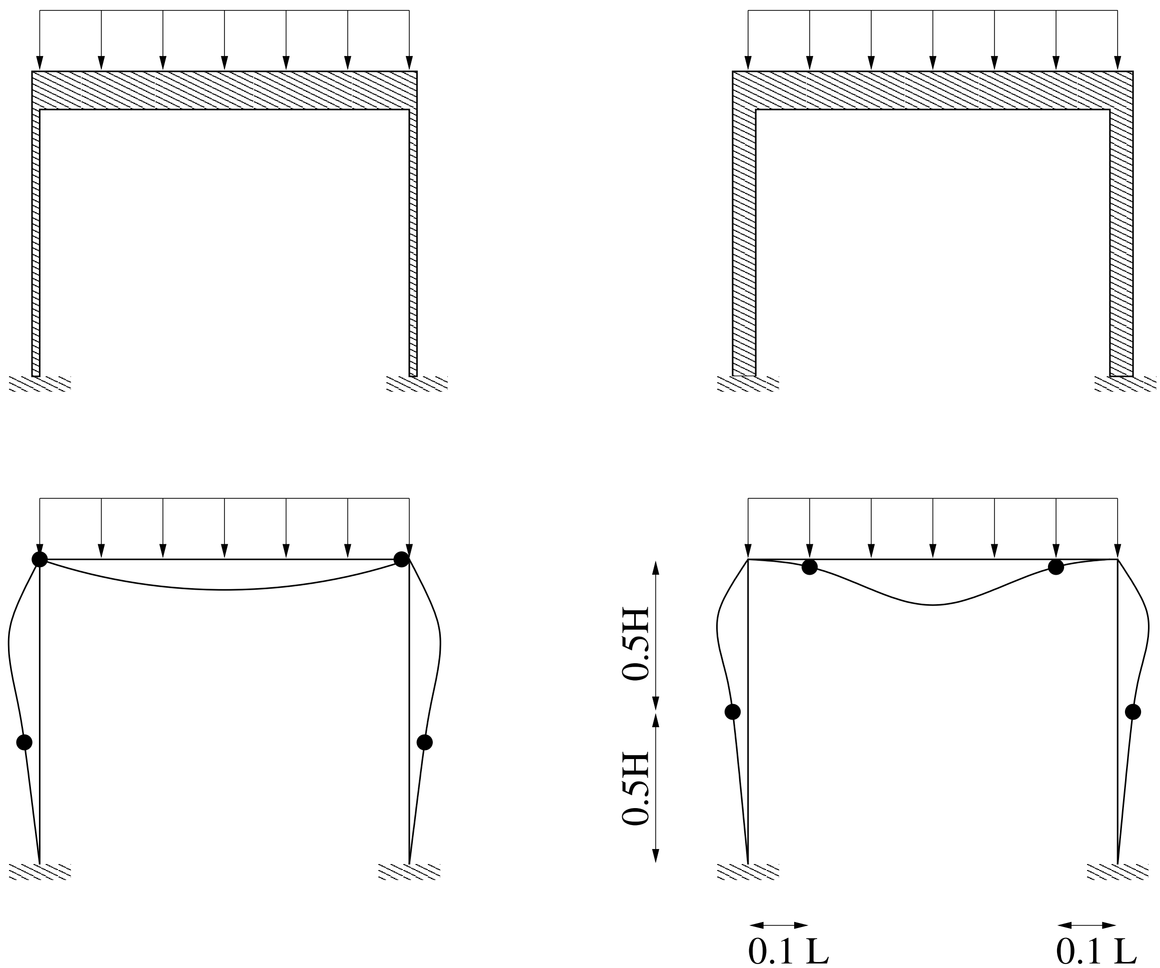

Figure 11.2: Uniformly Loaded Frame, Approximate Location of Inflection Points

$$\boxed{M^+ = \frac{1}{8}wL_s^2 = w\frac{1}{8}(0.8)^2L^2 = 0.08wL^2} \quad (11.1)$$

Maximum negative moment at each end of the girder is given by, Fig. 33.9

$$\boxed{M^{left} = M^{rgt} = -\frac{w}{2}(0.1L)^2 - \frac{w}{2}(0.8L)(0.1L) = -0.045wL^2} \quad (11.2)$$

Girder Shear are obtained from the free body diagram, Fig. 33.10

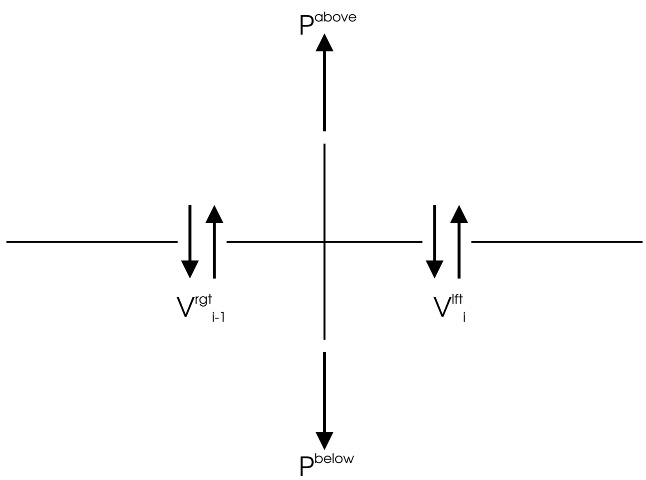

Figure 11.4: Approximate Analysis of Frames Subjected to Vertical Loads; Column Axial Forces

$$\boxed{V^{lft} = \frac{wL}{2} \qquad V^{rgt} = -\frac{wL}{2}} \quad (11.3)$$

Column axial force is obtained by summing all the girder shears to the axial force transmitted by the column above it. Fig. 33.10

$$\boxed{P^{dwn} = P^{up} + V_{i-1}^{rgt} - V_i^{lft}} \quad (11.4)$$

Column Moment are obtained by considering the free body diagram of columns Fig. 33.11

$$\boxed{M^{top} = M_{above}^{bot} - M_{i-1}^{rgt} + M_i^{lft} \qquad M^{bot} = -M^{top}} \quad (11.5)$$

Horizontal Loads, Portal Method

1. Column Shears

$$\begin{array}{lclcl}
V_5 & = & \frac{15}{(2)(3)} & = & 2.5 \text{ k} \\
V_6 & = & 2(V_5) = (2)(2.5) & = & 5 \text{ k} \\
V_7 & = & 2(V_5) = (2)(2.5) & = & 5 \text{ k} \\
V_8 & = & V_5 & = & 2.5 \text{ k} \\
V_1 & = & \frac{15+30}{(2)(3)} & = & 7.5 \text{ k} \\
V_2 & = & 2(V_1) = (2)(7.5) & = & 15 \text{ k} \\
V_3 & = & 2(V_1) = (2)(2.5) & = & 15 \text{ k} \\
V_4 & = & V_1 & = & 7.5 \text{ k}
\end{array}$$

2. Top Column Moments

$$\begin{array}{lclcl}
M_5^{top} & = & \frac{V_1 H_5}{2} = \frac{(2.5)(14)}{2} & = & 17.5 \text{ k.ft} \\
M_5^{bot} & = & -M_5^{top} & = - & 17.5 \text{ k.ft} \\
M_6^{top} & = & \frac{V_6 H_6}{2} = \frac{(5)(14)}{2} & = & 35.0 \text{ k.ft} \\
M_6^{bot} & = & -M_6^{top} & = - & 35.0 \text{ k.ft} \\
M_7^{top} & = & \frac{V_7^{up} H_7}{2} = \frac{(5)(14)}{2} & = & 35.0 \text{ k.ft} \\
M_7^{bot} & = & -M_7^{top} & = - & 35.0 \text{ k.ft} \\
M_8^{top} & = & \frac{V_8^{up} H_8}{2} = \frac{(2.5)(14)}{2} & = & 17.5 \text{ k.ft} \\
M_8^{bot} & = & -M_8^{top} & = - & 17.5 \text{ k.ft}
\end{array}$$

3. Bottom Column Moments

$$\begin{array}{lclcl}
M_1^{top} & = & \frac{V_1^{dwn} H_1}{2} = \frac{(7.5)(16)}{2} & = & 60 \text{ k.ft} \\
M_1^{bot} & = & -M_1^{top} & = - & 60 \text{ k.ft} \\
M_2^{top} & = & \frac{V_2^{dwn} H_2}{2} = \frac{(15)(16)}{2} & = & 120 \text{ k.ft} \\
M_2^{bot} & = & -M_2^{top} & = - & 120 \text{ k.ft} \\
M_3^{top} & = & \frac{V_3^{dwn} H_3}{2} = \frac{(15)(16)}{2} & = & 120 \text{ k.ft} \\
M_3^{bot} & = & -M_3^{top} & = - & 120 \text{ k.ft} \\
M_4^{top} & = & \frac{V_4^{dwn} H_4}{2} = \frac{(7.5)(16)}{2} & = & 60 \text{ k.ft} \\
M_4^{bot} & = & -M_4^{top} & = - & 60 \text{ k.ft}
\end{array}$$

4. Top Girder Moments

$$\begin{array}{lclcl}
M_{12}^{lft} & = & M_5^{top} & = & 17.5 \text{ k.ft} \\
M_{12}^{rgt} & = & -M_{12}^{lft} & = - & 17.5 \text{ k.ft} \\
M_{13}^{lft} & = & M_{12}^{rgt} + M_6^{top} = -17.5 + 35 & = & 17.5 \text{ k.ft} \\
M_{13}^{rgt} & = & -M_{13}^{lft} & = - & 17.5 \text{ k.ft} \\
M_{14}^{lft} & = & M_{13}^{rgt} + M_7^{top} = -17.5 + 35 & = & 17.5 \text{ k.ft} \\
M_{14}^{rgt} & = & -M_{14}^{lft} & = - & 17.5 \text{ k.ft}
\end{array}$$

6. Top Girder Shear

$$\begin{aligned}
V_{12}^{lft} &= -\frac{2M_{12}^{lft}}{L_{12}} = -\frac{(2)(17.5)}{20} &= -1.75 \text{ k}\\
V_{12}^{rgt} &= +V_{12}^{lft} &= -1.75 \text{ k}\\
V_{13}^{lft} &= -\frac{2M_{13}^{lft}}{L_{13}} = -\frac{(2)(17.5)}{30} &= -1.17 \text{ k}\\
V_{13}^{rgt} &= +V_{13}^{lft} &= -1.17 \text{ k}\\
V_{14}^{lft} &= -\frac{2M_{14}^{lft}}{L_{14}} = -\frac{(2)(17.5)}{24} &= -1.46 \text{ k}\\
V_{14}^{rgt} &= +V_{14}^{lft} &= -1.46 \text{ k}
\end{aligned}$$

7. Bottom Girder Shear

$$\begin{aligned}
V_{9}^{lft} &= -\frac{2M_{12}^{lft}}{L_{9}} = -\frac{(2)(77.5)}{20} &= -7.75 \text{ k}\\
V_{9}^{rgt} &= +V_{9}^{lft} &= -7.75 \text{ k}\\
V_{10}^{lft} &= -\frac{2M_{10}^{lft}}{L_{10}} = -\frac{(2)(77.5)}{30} &= -5.17 \text{ k}\\
V_{10}^{rgt} &= +V_{10}^{lft} &= -5.17 \text{ k}\\
V_{11}^{lft} &= -\frac{2M_{11}^{lft}}{L_{11}} = -\frac{(2)(77.5)}{24} &= -6.46 \text{ k}\\
V_{11}^{rgt} &= +V_{11}^{lft} &= -6.46 \text{ k}
\end{aligned}$$

8. Top Column Axial Forces (+ve tension, -ve compression)

$$\begin{aligned}
P_5 &= -V_{12}^{lft} &= -(-1.75) \text{ k}\\
P_6 &= +V_{12}^{rgt} - V_{13}^{lft} = -1.75 - (-1.17) &= -0.58 \text{ k}\\
P_7 &= +V_{13}^{rgt} - V_{14}^{lft} = -1.17 - (-1.46) &= 0.29 \text{ k}\\
P_8 &= V_{14}^{rgt} = -1.46 \text{ k}
\end{aligned}$$

9. Bottom Column Axial Forces (+ve tension, -ve compression)

$$\begin{aligned}
P_1 &= P_5 + V_9^{lft} = 1.75 - (-7.75) &= 9.5 \text{ k}\\
P_2 &= P_6 + V_{10}^{rgt} + V_9^{lft} = -0.58 - 7.75 - (-5.17) &= -3.16 \text{ k}\\
P_3 &= P_7 + V_{11}^{rgt} + V_{10}^{lft} = 0.29 - 5.17 - (-6.46) &= 1.58 \text{ k}\\
P_4 &= P_8 + V_{11}^{rgt} = -1.46 - 6.46 &= -7.66 \text{ k}
\end{aligned}$$

Design Parameters On the basis of the two approximate analyses, vertical and lateral load, we now seek the design parameters for the frame, Table 33.2.

■

Portal Method PORTAL.XLS Victor E. Saouma

	A	B	C	D	E	F	G	H	I	J	K	L	M	N	O	P	Q	R	S
1	**PORTAL METHOD**																		
2	**# of Bays**					**3**			**L1**				**L2**				**L3**		
3									**20**				**30**				**24**		
4								**MOMENTS**											
5	**# of Storeys**					**2**		**Bay 1**				**Bay 2**				**Bay 3**			
6			Force		Shear			Col	Beam		Column		Beam		Column		Beam		Col
7		**H**	**Lat.**	Tot	Ext	Int			Lft	Rgt			Lft	Rgt			Lft	Rgt	
8									=+H9	=-I8			=+J8+K9	=-M8			=+N8+O9	=-Q8	
9	**H1**	**14**	**15**	=+C9	=+D9/(2*F2)	=2*E9		=+E9*B9/2			=+F9*B9/2				=+K9				=+H9
10								=-H9			=-K9				=+K10				=+H10
11									=+H12-H10	=-I11			=+K12-K10+J11	=-M11			=+O12-O10+N11	=-Q11	
12	**H2**	**16**	**30**	=SUM(C9:C12)	=+D12/(2*F2)	=2*E12		=+E12*B12/2			=+F12*B12/2				=+K12				=+H12
13								=-H12			=-K12				=+K13				=+H13
14								**SHEAR**											
15								**Bay 1**				**Bay 2**				**Bay 3**			
16								Col	Beam		Column		Beam		Column		Beam		Col
17									Lft	Rgt			Lft	Rgt			Lft	Rgt	
18									=-2*I8/I$3	=+I18			=-2*M8/M$3	=+M18			=-2*Q8/Q$3	=+Q18	
19								=+E9			=+F9				=+F9				=+E9
20								=+H19			=+K19				=+O19				=+S19
21									=-2*I11/I$3	=+I21			=-2*M11/M$3	=+M21			=-2*Q11/Q$3	=+Q21	
22								=+E12			=+F12				=+F12				=+E12
23								=+H22			=+K22				=+O22				=+S22
24								**AXIAL FORCE**											
25								**Bay 1**				**Bay 2**				**Bay 3**			
26								Col	Beam		Column		Beam		Column		Beam		Col
27									0				0				0		
28								=-I18			=+J18-M18				=+N18-Q18				=+R18
29									0				0				0		
30								=+H28-I21			=+K28+J21-M21				=+O28+N21-Q21				=+S28+R21

Figure 11.19: Portal Method; Equations in Spread-Sheet

Mem.		Vert.	Hor.	**Design Values**
9	-ve Moment	9.00	77.50	**86.50**
	+ve Moment	16.00	0.00	**16.00**
	Shear	5.00	7.75	**12.75**
10	-ve Moment	20.20	77.50	**97.70**
	+ve Moment	36.00	0.00	**36.00**
	Shear	7.50	5.17	**12.67**
11	-ve Moment	13.0	77.50	**90.50**
	+ve Moment	23.00	0.00	**23.00**
	Shear	6.00	6.46	**12.46**
12	-ve Moment	4.50	17.50	**22.00**
	+ve Moment	8.00	0.00	**8.00**
	Shear	2.50	1.75	**4.25**
13	-ve Moment	10.10	17.50	**27.60**
	+ve Moment	18.00	0.00	**18.00**
	Shear	3.75	1.17	**4.92**
14	-ve Moment	6.50	17.50	**24.00**
	+ve Moment	11.50	0.00	**11.50**
	Shear	3.00	1.46	**4.46**

Table 11.2: Girders Combined Approximate Vertical and Horizontal Loads