4.10 Relief Displacement of a Vertical Photograph

The relief displacement is present on the vertical photograph due to height of various objects. It is the displacement of the objects on a vertical aerial photograph from their true plan positions. Aerial photograph is based on the central or perspective projection, and all the objects on aerial photographs appear to be displaced from their true positions. However, the objects near the principal point will be free from any relief displacement. Due to varying heights of different ground points, these points appear to be displaced on the photograph from their true geographic location. The higher objects, such as tall buildings, trees, towers, hills etc., are displaced away (outward) from the principal point, while the valleys, depressions etc., are displaced (inward) towards the principal point. The relief displacement is measured from the principal point of the photograph. It is always radial from the principal point, and therefore it is also known as the *radial displacement*.

Let us establish a relationship for relief displacement. In Figure 4.13, let AA_0 be a tower of height h above the ground. A photograph of the tower is taken from exposure station O. The a and a_0, respectively are the corresponding images of top and bottom of the tower on the photograph. The aa_0 distance (d) on the photograph is called the relief displacement as it is the horizontal displacement of tower on the photograph due to its height. The distance (d) can be computed as-

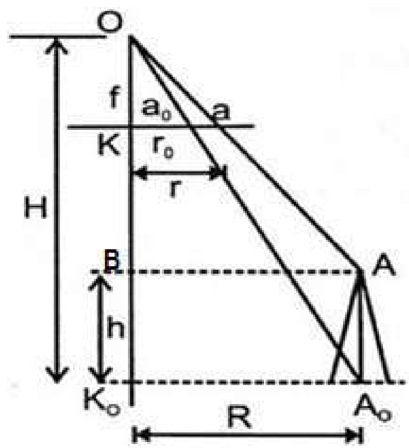


Figure 4.13 Relief displacement of a tower on a vertical photograph (Garg, 2019)

Let f = focal length of camera lens

$R = K_0A_0$

$r = Ka$

$r_0 = Ka_0$

$$d = aa_0 = r - r_0 \qquad (4.12)$$

From similar triangles OKa_0 and OK_0A_0, we can write as-

$$\frac{r_0}{R} = \frac{f}{H}$$

$$r_0 = \frac{fR}{H} \qquad (4.13)$$

Similarly, from similar triangles OKa and OBA, we can write-

$$\frac{r}{R} = \frac{f}{H-h}$$

$$r = \frac{fR}{H - h} \tag{4.14}$$

So relief displacement from equations 4.12. 4.13 and 4.14 would be-

$$d = r - r_0 = \frac{fR}{H-h} - \frac{fR}{H}$$

$$d = \frac{fR}{H(H-h)}(H - H + h)$$

$$d = \frac{fRh}{H(H-h)}$$

As per equation 4.13, $R = \left(\frac{H-h}{f}\right) r$

So $$d = \frac{fh}{H(H-h)} \times \frac{(H-h)}{f} r$$

$$d = \frac{rh}{H} \tag{4.15}$$

It is seen from equation 4.15 that the relief displacement (*d*) is directly proportional to radial distance of displaced image point from principal point (*r*), and as (*r*) increases (*d*) also increases. It is therefore advisable that the objects situated at the edges of a photograph may not be selected for any measurement as it may have larger error which is required to be eliminated before use. Relief displacement is directly proportional to the height of the object, as (*h*) increases (*d*) also increases. Whereas, (*d*) is inversely proportional to the flying height, as (*H*) increases (*d*) decreases. The relief displacement is maximum at the edges of photograph and is zero at the principal point. Relief displacement in the photograph is also one of the main reasons, a photograph is not directly used as a map for measurement purpose. From equations 4.13 and 4.14, we can write as-

$$\frac{r}{r_0} = \frac{H}{H-h}$$

$$\frac{r}{H} = \frac{r_0}{H-h}$$

So relief displacement can be written in terms of r_0

$$d = \frac{rh}{H} = \frac{r_0}{H-h} h \tag{4.16}$$

If the relief displacement is known, the height of an object can be computed as:

$$h = \frac{dH}{r} \tag{4.17}$$

Equation 4.17 has been used to approximately compute the height of the object from a single vertical aerial photograph.

4.11 Stereoscopy

Stereoscopy is a technique for creating a 3D model of depth perception by means of a stereo-pair. Human eyes are the best example of stereoscopy that will see two images of any object, which are only different in view angle, and orientation. The human-being has a good stereoscopic vision to view 3D with both eyes using the basic principle of stereoscopy that left eye will see left image and right eye will focus on the right image. It will allow interpreter to judge the distance and depth in the overlap region (Figure 4.14a). Human eyes fixed on same object provide two points of observations which are essentially required for creating parallax.

The perception of depth starts with the acquisition of visual information through the human eyes, and merging these images together in the human brain. When two images of the same object reach the brain, it tries to fuse both the images into a single 3D image, and starts interpreting the information. For example, the brain makes use of a number of clues (e.g., colour, shape, size, pattern, texture, orientation, shadow, location of objects etc.), to determine the relative distance and depth between various objects present in the scenes, as well as carry out the interpretation.

In the similar manner, a 3D model giving depth perception may be created from stereo-pair of photographs when viewed through a stereoscope. In 3D model, the topographic features appear much taller than in reality and slope appears much steeper due to the exaggeration of vertical heights with respect to the horizontal distances. The vertical exaggeration may be useful in photo-interpretation for identifying many features, such as slopes, low ridge and valley, topography, small depressions and elevations. Factors affecting the vertical exaggeration are photographic and stereoscopic factors.

The vertical exaggeration is fundamentally related to the ratio of air base (B) with the flying height of aircraft (*B/H*). It is directly proportional to air base, and can be correlated with focal length (*f*), distance (D) from the eye at which the stereo-model is perceived (approximately 45 cm), and eyebase (*b*). The vertical exaggeration is inversely proportional to flying height (*H*), and eye base (*b*) (approximately 6 cm). Vertical exaggeration ratio (*R*) is determined by using the formula:

$$R = \frac{BD}{bH} \tag{4.18}$$

In stereoscopy, when a human being's two eyes (binocular vision) are focused on a certain object, the optical axes of the eyes' converge at that point forms a parallactic angle (α). The near the object, the greater the parallactic angle, and vice versa (Figure 4.14b). The brain has learned to associate distance with the corresponding parallactic angles, and provides the visual and mental impression which object is closer and which one is farther. This is the basis of depth perception. If the two objects are exactly the same distance from the viewer, then the parallactic angles will be the same, and the viewer would perceive them at as being the same distance away. The maximum distance at which distinct stereoscopic depth perception is possible is approximately 1000 m for the average adult.

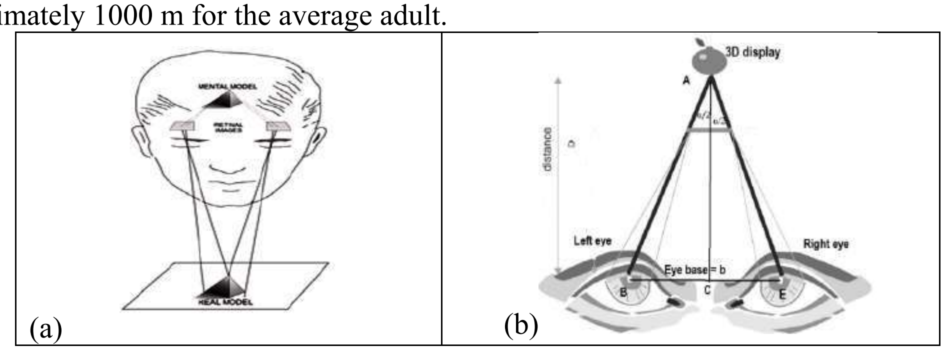


Figure 4.14 Stereoscopy (a) Human vision creating 3D (Quirós, 2018), and (b) parallactic angle (Nam et al., 2012)

4.11.1 Stereoscopic model

A stereoscopic model can be created with a stereo-pair or stereoscopic images or a stereogram. When two successive photographs taken from two different exposure stations cover part of the common scene which is called a stereo-pair. The 3D model created for the common area to both photographs is called a *stereoscopic model* or *stereo-model*. Stereoscopic model thus provides the ability to resolve parallax differences between far and near objects. The 3D view can be created from a stereo-pair through stereoscopic process, using simple equipment, such as stereoscopes or sophisticated equipment, such as stereo-comparator, or digital photogrammetric systems.

Some people with their normal vision in both eyes and having experience in photogrammetry can develop a stereoscopic vision without the use of a stereoscope. Seeing stereoscopically with the eyes without a stereoscope can be practiced with a famous “sausage-link” exercise, as shown in Figure 4.15a. Hold your hands at eye level about 10 inches in front of you, and select the homogeneous background with a different color than your fingers. Point your index fingers against each other, leaving about 1 inch distance between them. Now look through your fingers, into the distance behind them. An extra piece of finger “like a sausage” should appear (Figure 4.15b). If you try to look at the sausage, it will disappear, it is only present if you look at something more distant than your fingers. Move your fingers closer or farther apart to adjust the length of the "sausage." Close one eye or the other and the illusion will disappear. If you focus your attention back to your finger tips the illusion will also disappear.

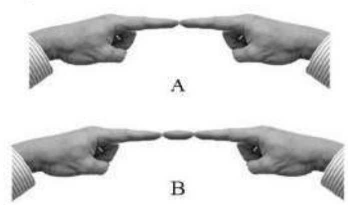


Figure 4.15 Stereoscopic exercise with index fingers (Handerson, 2010)

4.11.2 Requirements of a stereoscopic vision

To obtain a 3D model from the two dimensional photos, stereo-pair of an area must fulfil the following essential conditions:

1. Both the photographs must cover the same common area with minimum 60% overlap.
2. Time of exposure and contrast of both the photographs must be nearly same.
3. Scale of two photographs should be the same.
4. The brightness of both the photographs should be similar.
5. Parallax must be present in the stereo-pair (i.e., the photographs must have been taken from two different exposure stations).
6. Base to height ratio must have an appropriate value; the normal value is up to 2.0 but not less than 0.25. If this ratio is too small, say 0.02, the stereoscopic view will not provide a better depth impression. Base to height ratio increases when the overlap decreases. Short focal length wide angle lens cameras give better base height ratio which is important in natural resource inventory and mapping.

4.11.3 Stereoscopes

Stereoscopic vision is the basic pre-requisite for photogrammetry and photo-interpretation in 3D environment. Many applications need information extraction with stereo-images rather than mono-images. This requires two views of a single object from two slightly different positions of camera. The photographs are taken from two different positions with overlap in order to reproduce the objects in a manner as they are individually seen by the eyes.

Stereoscope is an optical device for 3D viewing of landscapes or objects. A stereoscope helps viewing a stereo-pair; left image with the left-eye and right image with the right-eye, to create a 3D model. To do so, the photographs are to be properly oriented below the stereoscope in the same manner as they were taken at the time of photography. When each eye views the respective image, these two images help creating a 3D view in overlap region. The fusion of common area of these two images in the brain will allow the judgement of depth or distance.

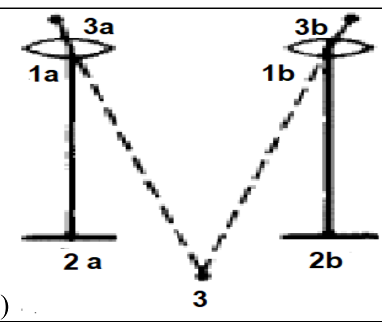
There are two type of stereoscopes used for three dimensional studies of aerial photographs; (i) Lens stereoscopes, and (ii) Lens and mirror stereoscopes.

(i) Lens stereoscopes

Lens stereoscope is also called as *pocket stereoscope,* as it can be kept in pocket owing to its small size. Being light-weight, it is easy to transport in the field, if required. It consists of two plano-convex lenses with magnifying capability, which are mounted on a metallic frame (Figure 4.16a). The distance between these two lenses is adjustable as per the comfort of users' eyes. The eye base average distance is approximately 65 mm for a human-being. The height of pocket stereoscope is normally 10 cm, but its legs can be folded when not in use. Distance between legs of the stereoscope and the focal length of lenses are normally so designed that the stereo-model can be created.

Figure 4.16b shows the line diagram of rays from stereo-photographs to human eyes through lens of a pocket stereoscope. Lens stereoscopes are handy, economical and light-weight, and thus convenient for studying the small format aerial photographs. They have disadvantages, such as limited magnification (2x–4x), and limited field of view to view the larger size photos in 3D at one single glance.


(a)



(b)

Figure 4.16 (a) Lens stereoscope, and (b) line diagram of rays from lens stereoscope (Garg, 2019)

(ii) Lens and mirror stereoscopes

Mirror stereoscope is also called as *Reflecting stereoscope* for viewing of stereo-photographs in 3D. It consists of an arrangement of prisms and mirrors allowing almost entire overlap area to be viewed at a glance in the field of view (Figure 4.17a). Mirrors are fixed to both the legs at 45˚ angle, along with two right angled prisms near the eyepiece lens. The function of mirrors is to reflect the rays 90^0 towards the prisms. The rays will strike to hypotenuse of the prisms and deflect 90^0 towards the eyepiece lenses. The binoculars provide 3x to 8x magnification. One custom made screw situated at one foot allows to level the remaining three legs for providing a stable base to the stereoscope. Figure 4.17b shows the line diagram of rays from photographs to eyes.

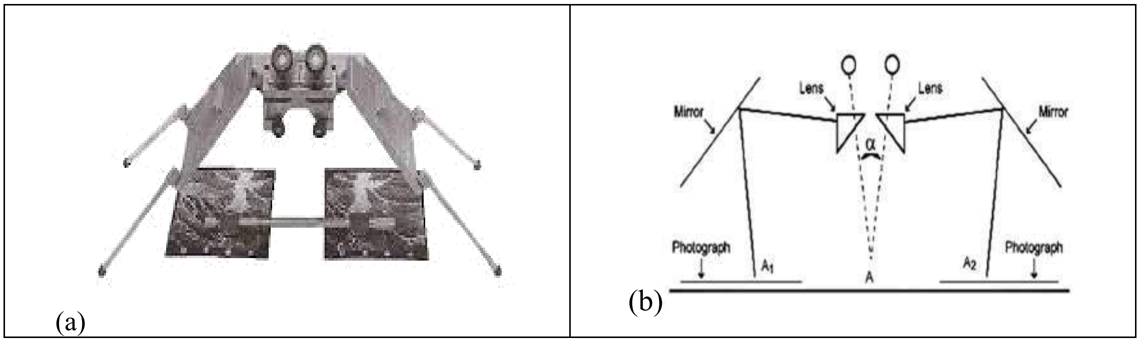


Figure 4.17 (a) Mirror stereoscope, and (b) line diagram rays of mirror stereoscope (Garg, 2019)

The main advantage of a mirror stereoscope is that the observer can see the entire overlap area from both the images in a magnified form, utilising the property of optics, even-though these photographs may not necessarily be located just below the lens. They are however heavier to carry in the field. The mirror stereoscopes are most widely used in stereo-photogrammetry and 3D measurements, in combination with an instrument called, *parallax bar*. The measurements of points in the stereo-pair are done using parallax bar to determine the elevations of those points.

4.12 Determination of Height from Vertical Aerial Photographs

One of the important application of stereo-photographs is the determination of elevations of various points in the overlap region. For this purpose, a parallax bar, also known as a *stereo-meter*, is used for taking the measurements. The pre-requisite is that the elevation of at least one control point is known in the common area (like ground levelling method) before the computation of elevations of other unknown points. The details of the control points are sometimes supplied along with the aerial photographs, else some permanent point is selected and its elevation is determined either from the contour map or levelling or GPS observations. The procedure for height determination from stereo-pair is described below.

4.12.1 Orienting a stereo-pair of photographs

The first step in creating a stereo-model is that the stereo-pair must be properly oriented under the stereoscope. The process of orientation is called *base lining*, which is performed as below:

1. On both the photographs, their respective principal point and conjugate principal point are marked, as shown in Figure 4.18a. Principal point and conjugate principal point are joined by a straight line and the line extended on each photo. This line represents the flight line.
2. Under a mirror stereoscope, two photographs are to be kept apart in the direction of flight line on a flat surface with overlap region inwards.
3. The stereo-pair is aligned in such a way that the line drawn on both the photos lie in a straight line, as shown in Figure 4.18a.
4. Stereo-pair is seen through the stereoscope so that left lens is over the left photograph and the right lens is over the right photograph. The line joining the centre of the lens should almost be matching with the direction of flight line (Figure 4.18b).
5. The distance between the photographs may be adjusted inward or outward till the two images are fused in the brain and a 3D model of the overlap region is created.
6. In the beginning, it might appear a bit difficult to see the two image fusing together to create a stereo-vision, but with little more practice and concentration, it will appear to be easy.
7. Once the perfect 3D model is created and the lines drawn on the photographs fall in a line, the photographs are said to be properly oriented (or base lining is completed).

8. Select the features/points on both the photographs in the overlap region whose heights are to be determined. The visual interpretation elements (size, shape, shadow, tone, texture and surrounding objects) will help identifying the objects; but now with the addition of relief, a more natural view of terrain may be seen.
9. Use parallax bar for taking the measurements of these points. The difference between two parallax bar readings will provide parallax differences between the two points.

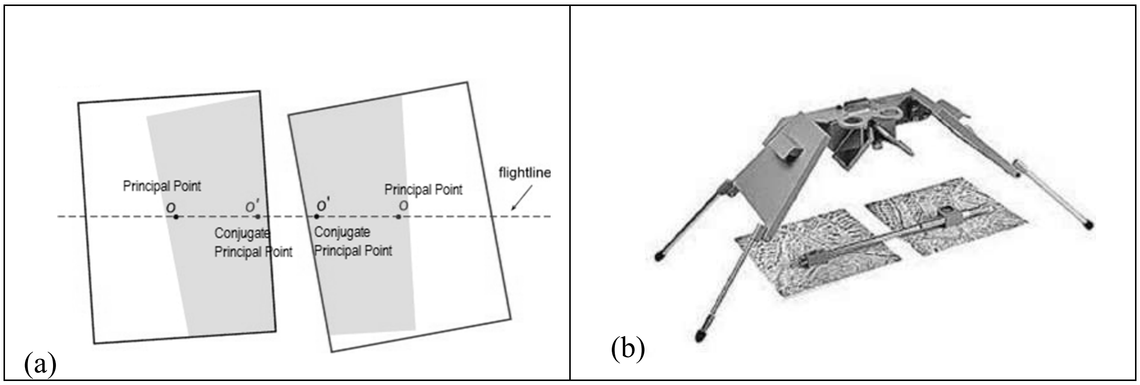


Figure 4.18 (a) Base lining of a stereo-pair, (Aber et al., 2010) and (b) Creating a stereovision (JARS, 1996)

4.12.2 Measurements by Parallax Bar

The change in position of an image from one photo to the next due to aircraft's motion is called *stereoscopic parallax, x-parallax,* or *simply parallax*. The parallax is directly related to the elevation of the point, and is greater for high points than for low points. The parallax bar is a device used to measure the difference of parallax between any two points on the stereo-photographs, more precisely.

The parallax bar consists of a graduated metallic rod (in mm) attached with two bars, as shown in Figure 4.19. Left bar is fixed with the rod and right bar is movable with micrometer drum attached on right end of the graduated rod. A pair of glass graticules, one at each bar can be inserted into the grooves provided in the bar. Each graticule is etched with three small identical index marks (cross, dot or circle), called *floating marks*. The left hand bar is generally clamped at any required distance so that the entire overlap area is covered by the separation of left and right floating marks.

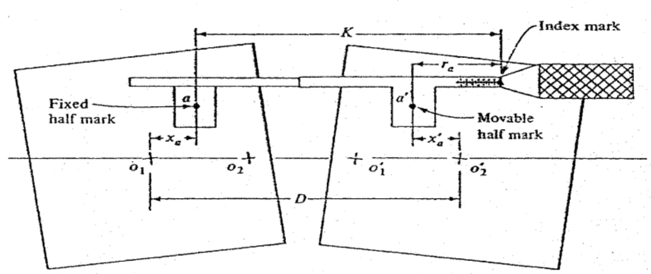


Figure 4.19 Parallax bar measurements (Garg, 2019)

The points/objects on stereo-pair are selected whose parallax bar readings are to be taken for the computation of heights. The oriented stereo-pair is now viewed in stereo mode, and the floating marks of graticules are kept at a point/object (left mark on left image and right mark on right image). To merge the two images of the floating marks, the separation of two floating marks can be changed by the rotation of micrometer screw having a least count as 0.01mm. It means one complete rotation of micrometer drum which has 100 graduations, will change the reading on main graduated rod by 1 mm. The floating marks on clear glass will either move away or come closer. While further rotating micrometer screw, at a given instant, these marks start to fuse together into a single mark. If micrometer screw is further rotated, these marks will appear as one mark which will now rise up or go down vertically (and appears to be floating in the air). When the two floating marks appear to be a single mark, resting exactly on the terrain point, main scale on graduated rod and micrometer screw readings both are read and added together. The combined reading is called *parallax bar reading* at that point (in this figure, it is K + micrometer reading). Several parallax bar readings at that point may be taken and average value used in the computation. Similarly, the parallax bar readings of other points/objects are taken.

4.12.3 Measurement of absolute parallax

The absolute parallax of a point on a stereo-pair is determined as the algebraic difference between two distances which are measured from the corresponding principal points, parallel to the direction of flight (air base). It is also called *x-parallax*. Figure 4.20 shows a stereo-pair where locations of two points *a* and *b* are marked with their x-coordinates. Let the distance on left photo be x_a and x_b and on right photo be x'_a and x'_b .

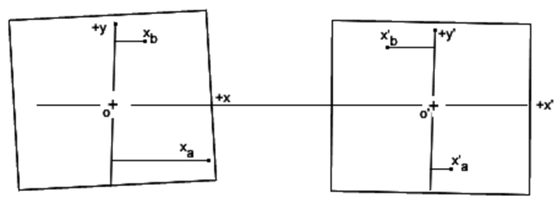


Figure 4.20 Measurements on stereo-pair

Since, the absolute parallax of a point is determined as the algebraic difference between two distances of a point, it is computed as given below.

$$p_a = x_a - x'_a$$
$$p_b = x_b - (-x'_b) = x_b + x'_b \quad (4.19)$$

The distances on the left and right photos are measured with the help of parallax scale, which is shown in Figure 4.21. This scale has a better least count that the normal scale (ruler) available for manual measurements. It has a main scale and a vernier scale and the addition of both the readings is the final distance. The main scale is graduated in mm while the vernier scale has 11 fine graduations in 10 mm space., so one graduation reads as 1.1mm. The distance is measured in such a way that the main scale and vernier scale both are read and added.

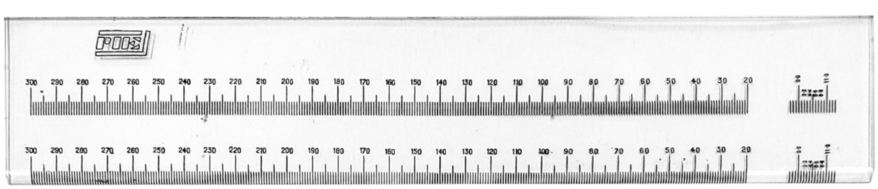


Figure 4.21 Parallax scale

4.12.4 Height determination

Let A and B are two points whose images are a_1 and b_1 on left photo and a_2 and b_2 on right photo (Figure 4.22). The stereo images are taken from L_1 and L_2 with B as air base distance and H as the flying height. Let h_a and h_b be the heights of these point with respect to a datum. From L_2 draw two lines parallel to $L_1a'_1$ and $L_1b'_1$ to cut the photographic plane at a'_1 and b'_1.

If the average parallax bar readings at two points *a* and *b* are P_A and P_B, respectively, the difference in absolute parallax (Δp) of p_a and p_b can be related with the difference in parallax bar readings.

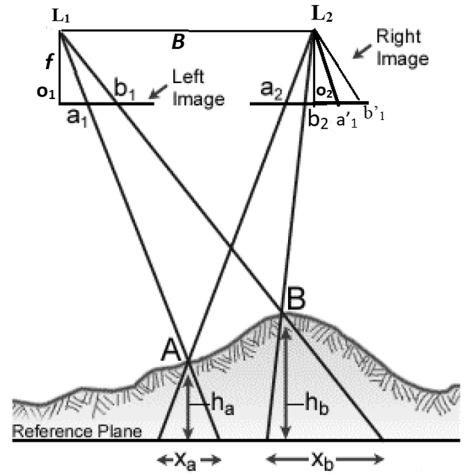


Figure 4.22 Height observations on a stereo-pair

From Figure 4.22, the parallax p_a at point *A* is related as follows.

Triangles L_1L_2A and $L_2a_2a'_1$ are similar triangle, so

$$\frac{a2a'1}{B} = \frac{f}{H - h_a}$$

where *B* is air base, *f* is the focal length of camera lens, *H* is the flying height, h_a is the elevation of point A.

But $p_a = a2a'1$, So

$$p_a = \frac{fB}{H - ha} \qquad (4.20)$$

Similarly, from similar triangles L_1L_2B and $L_2b_2b'_1$, the absolute parallax p_b is computed as -

$$p_b = \frac{fB}{H - h_b} \qquad (4.21)$$

Parallax difference is-

$$\Delta p = p_b - p_a \qquad (4.22)$$

$$= \frac{fB}{H - h_b} - \frac{fB}{H - h_a}$$

$$= \frac{fB(H - h_a) - fB(H - h_b)}{(H - h_a)(H - h_b)}$$

$$= \frac{fB(h_b - h_a)}{(H - ha)(H - h_b)}$$

$$= \frac{p_b(h_b - h_a)}{H - h_a}$$

$$h_b - h_a = \frac{\Delta p(H - h_a)}{p_b}$$

$$\Delta p = p_b - p_a$$

$$h_b = h_a + \frac{\Delta p(H - h_a)}{p_a + \Delta p} \qquad (4.23)$$

$$\Delta p = p_b - p_a$$

$\Delta p = (x_b + x'_b) - (x_a - x'_a)$ (Refer to Figure 4.20)

Δp is the difference of parallax bar readings at *a* and *b* ($P_A \sim P_B$)

Since the elevation of a ground point *A* (h_a) is known from reference map or field observation or contour map, the elevation of unknown point *B* can be determined (say h_b) using parallax bar measurements.

Since, it is difficult to measure the coordinates of all the objects on the photos, so these coordinates can be replaced by the parallax bar readings (P_A and P_B) at points *a* and *b*.

Using the relationship in equation 4.23, the elevation of point *B* (h_b) can be computed.

In a similar way, elevation of another unknown point *C* (h_c) can be computed using the relationship given below.

$$h_c = h_a + \frac{\Delta p(H - h_a)}{p_a + \Delta p} \qquad (4.24)$$

Where $\Delta p = P_A \sim P_C$

It is evident from above relationship that the elevations of various points can be computed by knowing the elevation of at least one ground control point. This method, however, determines the approximate height of the points, so care must be taken to minimise the errors. The errors in the determination of height may be due to several reasons –

1. Locating and marking the flight line on photos.
2. Orienting the stereo-pair for parallax measurement.
3. Measurements in parallax and photo-distances.
4. Shrinkage and expansion of photographs.
5. Difference in flying height of stereo-photographs.
6. Tilt present on photos, if any.
7. Error in the height of known ground control point.

4.13 Tilted Photographs

Tilted photographs are those taken with the camera axis making more than 3^0 angle with the vertical line. These photographs are mainly used by army and military. Titled photographs can’t be used directly for the measurement or mapping purpose because of inherent distortions present. Therefore, we need to understand the geometry of tilted photographs before they are used for engineering purpose.

4.13.1 Scale of a tilted photograph

The scale of a tilted photograph changes throughout the photograph. It is too small on the high side and too large on the low side (Figure 4.23). The scale also changes along the principal line (i.e., in the direction of tilt), however it does not change along any line perpendicular to the principal line. In Figure 4.23, all the standard notations have been used to represent various points in a tilted photograph. The *o* is the principal point and *n* is the nadir point on the photo. Here, *y* -axis is rotated till the new *y* -axis or the *y* '-axis coincides with the principal line having a positive direction (*no)*.

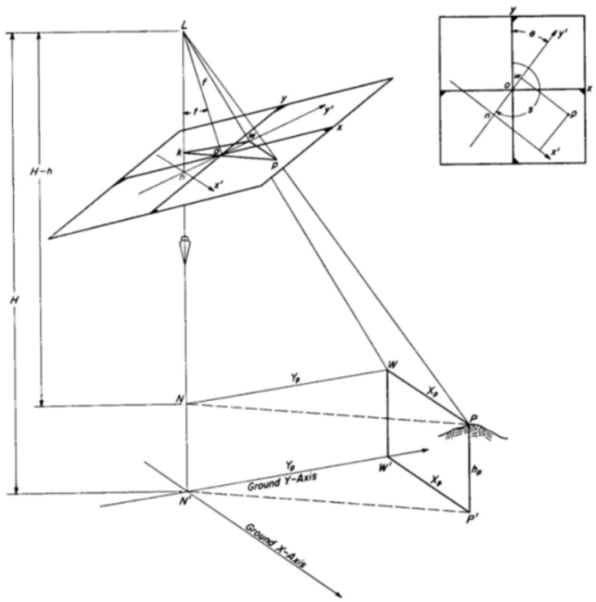


Figure 4.23 Geometry of a tilted photograph (*Mikhail et al., 2001*)

Let this angle of rotation be θ=180°- s, where s=swing angle, *P* is the point lying at an elevation h_p above the datum, *p* is its image on the tilted photo. Its photo-coordinates are (*x, y*) and measured coordinates with respect to axes defined are (*x′, y′)* by collimation marks. Since θ *is* measured clockwise, it must be negative. From *p* point a perpendicular *pw* is dropped to the principal line. Co-ordinates in rotated system can now be derived. On translating *x′* axis from *o* to *w,* the co-ordinates of point *p* on a tilted photo after the translation are-

$$\left.\begin{aligned} x' &= x\cos\theta + y\sin\theta \\ y' &= -x\sin\theta + y\cos\theta + on \\ &= -x\sin\theta + y\cos\theta + f\tan t \end{aligned}\right\} \qquad (4.25)$$

$$on = f\tan t$$

A perpendicular line is dropped from *w* as *wk* on *Ln.* In fact, this denotes a horizontal line; *Ln* being a vertical line). Since *wp* is perpendicular to the principal line, it is also a horizontal line. Therefore, the plane made by *kwp* points is a horizontal plane. Also, *PP′= WW′= NN′=* h_p (from construction on the ground), and therefore, the plane formed by *NWP* points is also horizontal.

From similar Δ^s Lkp and LNP, we can write-

$$\frac{kp}{Np} = \frac{Lk}{LN} = \frac{Lk}{H-h}$$

But *Lk = Ln - kn = f* sec *t - y′* sin *t.*

Therefore,

$$\frac{kp}{NP} = \frac{f \sec t - y' \sin t}{H - h}$$ (4.26)

But kp / NP = scale for a point lying in a plane *kwp*. Since *p* lies on this plane, so scale of tilted photo (S_t) at *p* can be written as-

$$S_t = \frac{f \sec t - y' \sin t}{H - h} \quad (4.27)$$

Where, S_t is the scale of a tilted photograph at a point whose elevation is *h, f* is focal length, *t* is the tilt angle, *H* is the flying height above datum, and *y′* is the *y*- co-ordinate of the point with respect to a set of axes whose origin is at the nadir point and whose *y′* axis coincides with the principal line. This equation shows that scale is a function of tilt only, and scale variation occurs on a tilted photo even when the terrain is flat.

4.13.2 Tilt displacement

On a tilted photo, relief displacement is considered to be radial from the nadir point (*n*). Compared to an equivalent relief displacement on vertical photo, the relied displacement on a tilted photo will be (i) less on the half of the photograph upward from the axis of the tilt, (ii) greater on downward half of the photo, and (iii) identical for points lying on the axis of the tilt. The amount of relief displacement depends upon: (i) flying height, (ii) distance from nadir point to image, (iii) elevation of ground point, and (iv) position of point with respect to principal line and to the axis of the tilt. In tilted photos, the radial distance should be measured from the nadir point and not from the principal point.

The tilted photo and its equivalent vertical photo are identical along the isometric parallel, where they intersect. At any other position on the tilted photo, the image of a point will be displaced either outward or inward with respect to its equivalent position on the vertical photo. The characteristics of a tilted photograph are-

1. Displacement due to tilt is zero on a truly vertical photo, but increases proportionally as tilt angle increases.
2. Tilt displacement on slightly tilted photos is usually less in magnitude than displacement from elevation differences, but tilt is much more difficult to detect, calculate, and correct.
3. Images on “up side” of a tilted photo are displaced toward photo center
4. Images on “down side” of titled photo displace away from photo center

In Figure 4.24
t is the angle of tilt
p is the principal point
i is the isocenter
n is the nadir point
o is the perspective center (lens)
j is the focal length of photograph
$\alpha = \angle pOa$ = angle at the perspective center *O* measured counter-clockwise from the photograph perpendicular to any point a on the upper side of the photograph
$\beta = \angle pOb$ = angle at the perspective center *O* measured clockwise from the photograph perpendicular to any point b on the lower side of the photograph
a, b are the images of ground points in the principal plane of the photograph
a′, b′ are the corresponding points on the equivalent vertical photograph

$+r$ is the distance *ia* = distance *ic*
$-r$ is the distance *ib* = distance *ie*
d is the displacement for an image, parallel to the principal line and due to tilt, on the upper side of a photograph
d' is the displacement for an image, parallel to the principal line and due to tilt, on the lower side of a photograph

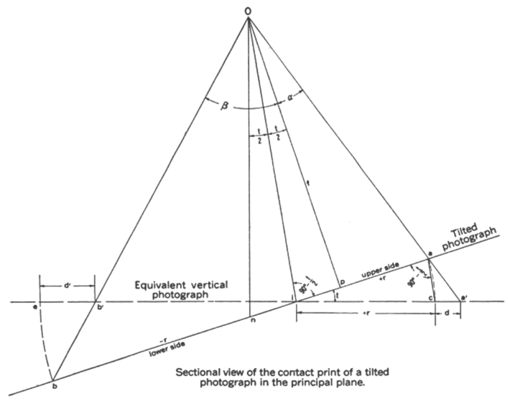


Figure 4.24 Tilt displacement (*Kraus, 1994*)

So
$\tan(t/2) = (pi)/f$
$(pi) = f \tan(t/2)$
$\tan(t) = (pn)/f$
$(pn) = f \tan(t)$

The displacement is computed as:

$$d = ia' - ic \tag{4.28}$$
$$= ia' - (ia) \quad = ia' - (ip + pa)$$
$$d' = ie - ib'$$
$$= ib - ib' \quad = (pb - pi) - ib'$$
$$d = f\left[\tan(\alpha + t) - \tan\alpha - 2\tan\frac{t}{2}\right]$$
$$d' = f\left[\tan\beta - \tan(\beta - t) - 2\tan\frac{t}{2}\right] \tag{4.29}$$

In somewhat simpler equations, the image displacement can also be computed as:

$$d = \frac{(ia)^2}{\dfrac{f}{\sin t} - ia} \tag{4.30}$$

$$d' = \frac{(ib)^2}{\dfrac{f}{\sin t} - ib} \tag{4.31}$$

Considering the isocenter, *i*, as the center of rotation, an arc is drawn from point *a* on the tilted photograph so as to intersect the equivalent vertical photograph at a point *c*. The distances *ia* and *ic* are therefore equal and the isosceles triangle *iac* is thereby formed having two sides equal to $+r$.

$$\angle\ Oip = 90^o - \frac{t}{2}$$

From the isosceles triangle *iac*,

$$\angle\ iac = 90^o - \frac{t}{2}$$

Therefore, lines *Oi* and *ac* are parallel (corresponding opposite interior angles) and, $\Delta\ Oia'$ is therefore similar to $\Delta\ aca'$. Therefore -

$$\frac{d}{r+d} = \frac{ac}{Oi} \quad (4.32)$$

$$ac = 2r \sin\frac{t}{2}$$

$$Oi = \frac{f}{\cos\frac{t}{2}}$$

Substituting the values in equation 4.32, we get-

$$\frac{d}{r+d} = \frac{2r\sin\frac{t}{2}}{\frac{f}{\cos\frac{t}{2}}}$$

$$d = \frac{2r\sin\frac{t}{2}\cos\frac{t}{2}}{f}(r+d)$$

Using the property of trigonometry-

$$2\sin\frac{t}{2}\cos\frac{t}{2} = \sin t$$

Therefore, we can write as-

$$d = \frac{r\sin t(r+d)}{f} \quad (4.33)$$

$$df = r^2\sin t + dr\sin t$$

$$d(f - r\sin t) = r^2\sin t$$

$$d = \frac{r^2\sin t}{f - r\sin t} \quad (4.34)$$

Dividing the numerator and the denominator by *sin t*, we get-

$$d = \frac{r^2}{\frac{f}{\sin t} - r} \quad (4.35)$$

The equation 4.35 applies to both sides of the photograph when the algebraic sign of r is observed. The equation 4.35 is usually expressed in an approximate form when computing the value of d for either side of a single lens photograph having a tilt angle of less than 3°.

$$d = \frac{r^2 \sin t}{f} \tag{4.36}$$

4.14 Aerial Triangulation

Aerial triangulation in photogrammetry is done to determine and calculate 3-D coordinates of points by using series of stereo-photographs, thereby reducing the field work. The results of aerial triangulation are the orientation elements of all photographs or stereo-models and the 3-D coordinates of points in ground coordinate system. It is used in different mapping tasks, such as DEM and orthophoto generation, 3D extraction and object reconstruction, surveying and cadastral purposes (3rd and 4th order networks).

Aerial triangulation is used extensively for many purposes, such as extending or densifying ground control through strips or blocks of photos for use in subsequent photogrammetric operation. Establishment of control points are required for compilation of topographic maps with stereo-plotters, locating the property corners for cadastral mapping, and developing the DTM. Determining ground coordinates of points at various time intervals are also useful to monitor the displacement of dams or deformations in structures, and precise industrial measurement, such as determination of the relative position of large machine parts during fabrication. The aerial triangulation can provide controls for photogrammetric purposes for both small scale and large scale maps. For small scale mapping (1:50,000 or so), the required accuracy is 1-5 m, and for large scale mapping (1:1,000 - 1:10,000), the required accuracy is 0.1-1 m.

There are several benefits of aerial triangulation besides having an economic advantage over land surveying, such as; (i) minimizing delays due to adverse weather condition, (ii) Access to non-accessible ground within the project area, and (iii) eliminating field surveying in difficult areas, such as marshes, high slopes, hazardous rocks, etc. The workflow in aerial triangulation is shown in Figure 4.25.

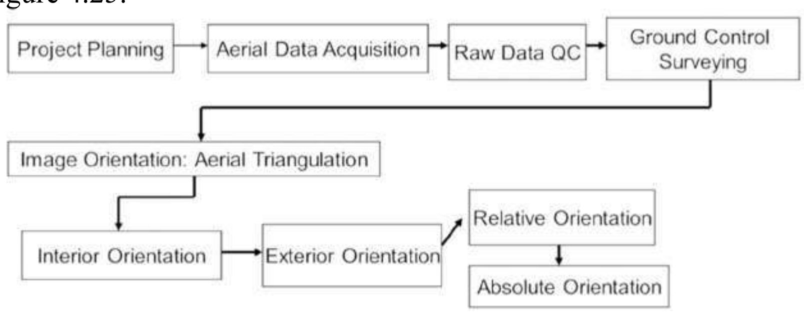


Figure 4.25 Basic steps in aerial triangulation (Phillipson, 1996)

4.14.1 Types of aerial triangulation

There are broadly two types of aerial triangulation; (a) Radial triangulation, (ii) Block triangulation.

(a) Radial Triangulation

Radial triangulation was used in 1950s in which stereo and slotted templates layouts provided photo-control for mapping purposes. A photograph is a perspective projection of the terrain on a plane, and the angles are true at the principal point, only when the optical axis of the camera has been exactly vertical at the time of exposure. In such a case, the principal point is usually taken as the radial centre (i.e., a point at which angles can be measured graphically in all directions) for radial line triangulation. The principal point is easily determined on the photograph and angles are measured at this point.

Radial triangulation is a graphical approach and based on the principle of radial line method that on a truly vertical photograph, the angles measured at the principal point in the plane of photograph to the images on the photograph are horizontal angles to the corresponding points on the ground. Here, the principal points of the neighboring photographs are transferred and the rays to various points are drawn for each photo. These planimetric bundles can be put together along a strip or in a block using two ground control points on each photo. The other points can be located by multiple intersections with 3-4 photos in a strip. Thus, the aerial photographs can be used for measuring the horizontal directions. Radial line methods can be utilized both for extension of planimetric control over large areas (radial triangulation) and for detailed plotting (radial plotting) and contouring.

Extension of planimetric control between known control points is carried out by radial triangulation using radial line principle. A minimum of 9 points per photograph, distributed as shown in middle photograph in Figure 4.26a, are provided. These points are called *minor control points* (MCPs) or *pass points*. The number of necessary ground control points can be reduced to a minimum by using the radial triangulation. Provision of ground control points constitutes a major part of survey cost, and therefore, by using radial triangulation methods, the time and cost of survey is reduced. The equipment used in the method is comparatively modest and the technique is simple in principle. The radial line methods are, therefore, preferred, particularly in developing countries. Although this simple technique provides the required accuracy, it is limited by the large physical space required for the layout of photographs.

The radial triangulation can be carried out by: (i) Graphical, (ii) Mechanical, and (iii) Analytical radial triangulation method. Graphical radial triangulation is the simplest of the above three methods, which has been explained below:

Arundel method of graphical radial triangulation

It is also known as the *Arundel method of radial triangulation*, as it was first tried in U.K. on an area near Arundel village. The basic problems in transforming the aerial photographs onto planimetric map are: (i) to obtain the control points necessary to establish the true position of principal point of each photograph, and (ii) to obtain positions of other image points with the help of which the detail plotting can be carried out. The minimum two ground control points (GCPs) per strip of photographs are necessary for bringing a strip of aerial photographs to a desired scale, and their correct position has to be provided by ground triangulation or traversing. The position of these GCPs are carefully located in the field and marked on the aerial photographs. A number of suitably located image points are chosen on either side of the principal points in order to develop a chain of radial triangulation. The principle in its simplest form is evident from Figure 4.26b.