Environmental Quality: Monitoring and Analysis

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Lecture – 49
Overall Mass Transfer Coefficient

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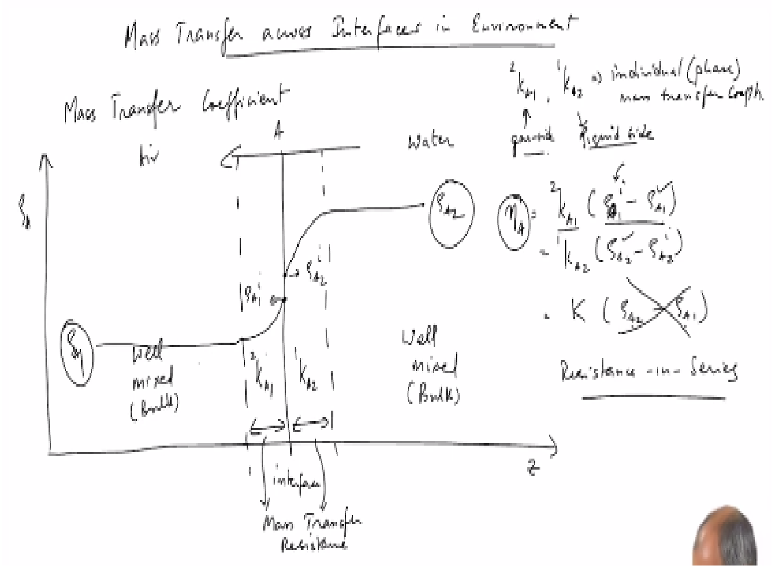

So, we were discussing mass transfer across interfaces. We were discussing the general principles of mass transfer coefficient. So we defined something called as mass transfer coefficient and its dependency on the flow, the properties of the fluid and properties of the solute itself to some extent. So we stopped at a point where we were looking at transfer across an interface. So we will just recap that. So let us say there is an interface between two phases, let us say this is water and this is air.

If mass transfer is happening from water to air, the transfer of A is going from water to air, which means there is a gradient from water to air. So, we also discussed that close to the fluid interface, it is convenient for us to assume that there is a region of mass transfer resistance. So the rest of the region is considered as well mixed or we call it as a bulk. So by definition, when we say well mixed, the concentration here is ρ_{A2}.

It is straight, it does not change. So you are drawing the scale of concentration on this and on the x axis is some kind of a length scale. It is notional, it is not an accurate scale. We are not going to give any numbers to it at this point, and then there is a gradient that applies within this region and so this point, we call it as ρ_{A2}^{i} interface, there is another number here on the air side and there is a gradient that goes in to this region. So, what this suggest is all the mass transfer resistance is in this phase.

So, if there is mass transfer resistance, there is also a mass transfer coefficient ${}^{2}k_{A1}$ and then there is a mass transfer coefficient ${}^{1}k_{A2}$. This is the definition of the mass transfer coefficients. It is individual phase mass transfer coefficient or individual mass transfer coefficient. So ${}^{2}k_{A1}$ is the gas side and ${}^{1}k_{A2}$ is the liquid side individual mass transfer coefficients. So we also defined that we said the flux can be defined as

$$\eta_A = {}^{2}k_{A1}\left(\rho_{A1}^{i} - \rho_{A1}\right) = {}^{1}k_{A2}\left(\rho_{A2} - \rho_{A2}^{i}\right)$$

So, this is a quantity of our interest. So, if we want to predict what is the flux, we need to know ${}^{2}k_{A1}$ and $\left(\rho_{A1}^{i} - \rho_{A1}\right)$ and the problem sometimes is we cannot, we do not know what ρ_{A1}^{i} is interface, not sometimes, always interfaces. When there are 2 liquid phases or 2 gas and liquid phase, this interface concentration is very difficult to estimate or unreliable. So, we need something. What we can measure though are these (ρ_{A1}, ρ_{A2}) two numbers . So, what we would like to do is something like this

$$\eta_A = K(\rho_{A2} - \rho_{A1})$$

Equivalent mass transfer coefficient (K) for this, but this is an illegal representation because transport, when we are talking about these kind of things, there is a continuum and there is a discontinuity in here because there is a phase in between, there is a phase boundary in between, that is the discontinuity. So, you cannot write these equations without invoking the equilibrium in between. So, I cannot simply write $(\rho_{A2} - \rho_{A1})$, as long as I can do this only if it is in one phase, there is a continuum. Because there is no continuum, there is a break there.

So therefore, we cannot do this. So how do we work around this one? So we invoke what is called as a resistance in series approach to do this, okay. So the resistance in series, what it says is I am trying to relate ρ_{A2} to ρ_{A1} through an equation in the system.

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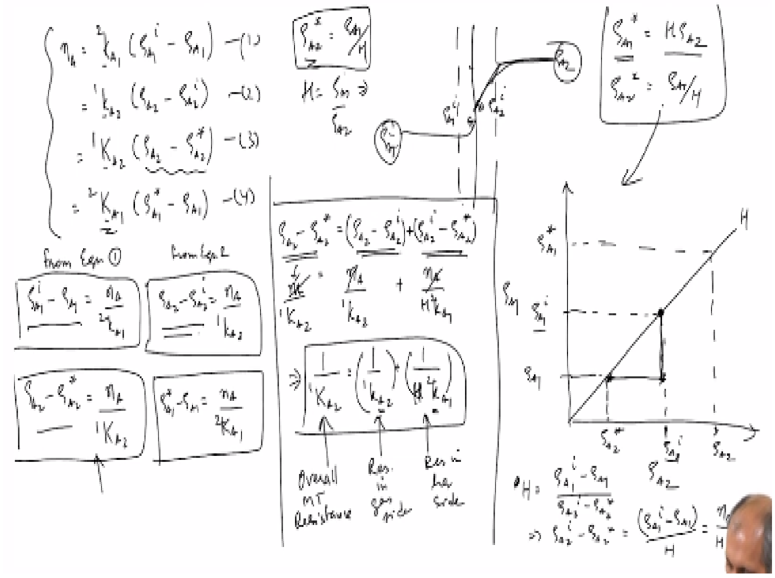

So I am going to write the terms here. So, what we are saying is that if we cannot write the two terms directly, what we will write instead is something like this we will write

$$\eta_A = {}^{2}k_{A1}\left(\rho_{A1}^{i} - \rho_{A1}\right) \longrightarrow \quad (1)$$

$$= {}^{1}k_{A2}\left(\rho_{A2} - \rho_{A2}^{i}\right) \longrightarrow \quad (2)$$

$$= {}^{1}K_{A2}\left(\rho_{A2} - \rho_{A2}^{*}\right) \longrightarrow \quad (3)$$

A driving force, which is essentially ρ_{A2} minus an equivalent concentration liquid state concentration which represents the gas state concentration and similarly I can write

$$= {}^{2}K_{A1}\left(\rho_{A1}^{*} - \rho_{A1}\right) \longrightarrow \quad (4)$$

So, notice that this is small k, this is individual mass transfer coefficient. So, here what is ρ_{A2}^{*}? We know that $\rho_{A1}^{i}, \rho_{A2}^{i}$ are in equilibrium. Yesterday's class we said that $\rho_{A1}^{i}\ and\ \rho_{A2}^{i}$ are in equilibrium with each other is the assumption that we can make. If we can extend assumption to ρ_{A1}, so what we are saying is that representation of ρ_{A2}^{*} in terms of ρ_{A1} can be through the equilibrium. So one relationship that we have is say the Henry's constant okay.

Henry's constant equals $\frac{\rho_{A1}}{\rho_{A2}}$ which are in equilibrium with each other, right. So, this implies that

$$\rho_{A2}^{*} = \frac{\rho_{A1}}{H}$$

by invoking the equilibrium relationship. So what this says is there is a number, an imaginary number ρ_{A2}^*, which is in equilibrium with ρ_{A1} A1 okay. So what we are saying is there is some number somewhere, we do not know where, we cannot mark it on this plot because it does not exist in reality, but this number represents the gas phase concentration.

Similarly, other side we are talking about ρ_{A1}^*. It is $H \times \rho_{A2}$, should be looking at the other side, okay. So, there is a equivalent concentration on the gas side that is in equilibrium with whatever is there in the bulk liquid side. So, $\rho_{A1}, \ \rho_{A2}$ are the bulk concentrations. So, what we are doing is we are circumventing the interface concentrations by attempting to do this okay. So here we have

$$\rho_{A2}^* = \frac{\rho_{A1}}{H}$$

$$\rho_{A1}^* = H \times \rho_{A2}$$

these two are the relationships, we will keep it aside for the time being. So what this means graphically is the following. I am going to draw this here. Henry's constant is we are assuming its constant for this temperature, this is linear relationship. So, we mark 3 concentrations here. We will mark ρ_{A2}, which is the bulk phase concentration here and then we also mark ρ_{A1}, which is a very low concentration on the gas side.

We also mark ρ_{A2}^i and correspondingly if you mark ρ_{A2}^i we mark on this graph, we mark ρ_{A1}^i because we have already established that $\rho_{A1}^i \ and \ \rho_{A2}^i$ are in equilibrium. So, the intersection, this point lies on the equilibrium curve. What we are now saying is that corresponding to ρ_{A2}, there is an equivalent concentration here which is called as ρ_{A1}^* and corresponding to ρ_{A1} there is an equivalent concentration which is called as ρ_{A2}^*.

This is what it means, what we have said here this $\rho_{A2}^* \ and \ \rho_{A1}^*$ are expressed on this curve here, okay. So now we can use these relationships. So based on this, see we have equation 1, 2, 3 and 4, four equations here. So, from equation 1, we get

$$\left(\rho_{A1}^i - \rho_{A1}\right) = \frac{\eta_A}{{}^2k_{A1}}$$

and from equation 2, we get

$$\left(\rho_{A2} - \rho_{A2}^{i}\right) = \frac{\eta_A}{{}^{1}k_{A2}}$$

Then from equation 3, we get

$$\left(\rho_{A2} - \rho_{A2}^{*}\right) = \frac{\eta_A}{{}^{1}K_{A2}}$$

and likewise we get the last equation here,

$$\left(\rho_{A1}^{*} - \rho_{A1}\right) = \frac{\eta_A}{{}^{2}K_{A1}}$$

So we will pick one of these equations. Let us pick the second one here. So the derivation of this is like this,

$$\left(\rho_{A2} - \rho_{A2}^{*}\right) = \left(\rho_{A2} - \rho_{A2}^{i}\right) + \left(\rho_{A2}^{i} - \rho_{A2}^{*}\right)$$

$$\frac{\eta_A}{{}^{1}K_{A2}} = \frac{\eta_A}{{}^{1}k_{A2}} + \left(\rho_{A2}^{i} - \rho_{A2}^{*}\right)$$

From the graph, Henry's constant can be written as

$$H = \frac{\left(\rho_{A1}^{i} - \rho_{A1}\right)}{\left(\rho_{A2}^{i} - \rho_{A2}^{*}\right)}$$

$$\Rightarrow \left(\rho_{A2}^{i} - \rho_{A2}^{*}\right) = \frac{\left(\rho_{A1}^{i} - \rho_{A1}\right)}{H}$$

So now,

$$\frac{\eta_A}{{}^{1}K_{A2}} = \frac{\eta_A}{{}^{1}k_{A2}} + \frac{\left(\rho_{A1}^{i} - \rho_{A1}\right)}{H}$$

$$\frac{\eta_A}{{}^{1}K_{A2}} = \frac{\eta_A}{{}^{1}k_{A2}} + \frac{\eta_A}{H\,.\,{}^{2}k_{A1}}$$

Now you cross out all the n's, now we get

$$\frac{1}{{}^1K_{A2}} = \frac{1}{{}^1k_{A2}} + \frac{1}{H\,.\,{}^2k_{A1}}$$

$\frac{1}{{}^1K_{A2}}$ is called as the overall mass transfer resistance is the sum of this resistance ($\frac{1}{{}^1k_{A2}}$) which is in the liquid phase plus this resistance ($\frac{1}{H\,.{}^2k_{A1}}$) in the gas phase.

($\frac{1}{{}^1k_{A2}}$) is the individual resistance in the liquid phase and ($\frac{1}{H\,.{}^2k_{A1}}$) is individual resistance in the gas phase, but there is an equilibrium term sitting here, so this takes care of that okay. Correspondingly, you can also write an equation for ${}^2K_{A1}$, you can derive it in the same manner and you will get a similar equation okay. This is a very useful representation. So, this resistance is a series. Because whenever you have 2 different mediums or 2 different mediums for different properties, you can add up the resistance in order to get the mass transfer flux. So, it is like in heat transfer, we have different slabs with different conductivities, you have different transport rates. So, it is a composite resistances in series, but you have to understand that all of this is a steady state system okay, which means that is the reason why this is true. Because it is steady state, they are all equal.

What it means is that the overall rate of transport is a combination of these 2 individual resistances. So if one of them is smaller, that controls the rate at which the overall process is moving.

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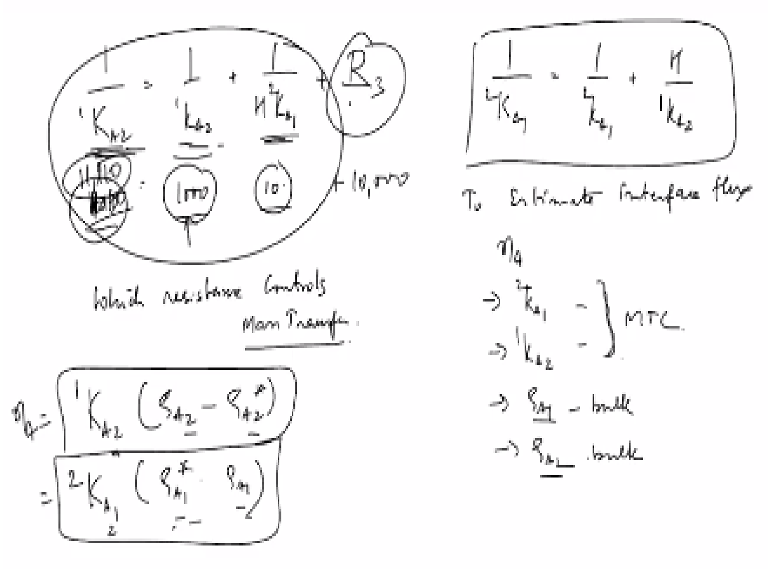

For example, in this case if I calculate

$$\frac{1}{{}^1K_{A2}} = \frac{1}{{}^1k_{A2}} + \frac{1}{H\,.\,{}^2k_{A1}}$$

it turns out that $\frac{1}{{}^1k_{A2}}$ resistance is 1000 and $\frac{1}{H\,.{}^2k_{A1}}$ resistance is 10. The overall resistance is 1010. So it is predominantly dependent on $\frac{1}{{}^1k_{A2}}$, what this means is that in the gas side, there is no resistance, resistance is very small, but it doesn't still matter because $\frac{1}{{}^1k_{A2}}$ is controlling how much is going across from one end to the other end, okay, yeah. So there are different analogies that you can think of and I would like you to think of different physical analogies that this is true and vice versa and so.

Sometimes it is possible that the resistances are almost equal. So, what this means is this analysis is very useful because now you can determine which is the controlling, which resistance controls mass transfer because that will tell you if you need to engineer the situation somehow, you need to reduce the mass transfer flux or increase. In our case in the environmental problems, we are always interested in reducing mass transfer, we do not want chemicals to get across the interface from one or the other.

In your chemical engineering application, sometimes we want to enhance mass transfer. So, you would like to do something to increase this resistance or decrease the resistance. So, here we would like to do is increase the resistance. One way to increase the resistance, I will add one more term here. I can add another term, which means that there is an addition, another term with another resistance, I will call it R3. Let us say the R3 resistance is 10,000, it is more than these two phases, which means that this now controls the overall mass transfer.

This now becomes 11,010. So this is useful in this case.
Student: sir, what is the difference between ${}^1K_{A2}\ and\ {}^2K_{A1}$.
Professor: Yeah, it is the same difference, if you are using capital ${}^1K_{A2}$, the flux you will write it as

$$= {}^1K_{A2}(\rho_{A2} - \rho^*_{A2})$$

If you are using ${}^2K_{A1}$, you will write it as

$$= {}^2K_{A1}(\rho^*_{A1} - \rho_{A1})$$

Depending on this number K, the concentrations that you are using in the driving force changes. Yeah, we will do some examples. This is a representation of, you can use either of these equations, you can use any one of these depending on what is convenient. I mean, both of them should be the same. Now, clear? It is just the way we have defined it, depending on which number we can take, if you want to take the equivalent of the liquid phase or the gas side.

So this is also true for other systems okay. So, the other equation is ${}^{2}K_{A1}$, if you derive it in the same manner in which we did, it will be

$$\frac{1}{{}^{2}K_{A1}} = \frac{1}{{}^{2}k_{A1}} + \frac{H}{{}^{1}k_{A2}}$$

it will turn out that way. So, this is the equation for ${}^{2}K_{A1}$. So, this is the preferred method when you have an interface, because then all you need to estimate interface flux which is n_A, you need to calculate ${}^{2}k_{A1}$, ${}^{1}k_{A2}$, ρ_{A1}, ρ_{A2} . ρ_{A1}, ρ_{A2} are the bulk phase concentrations and ${}^{2}k_{A1}$, ${}^{1}k_{A2}$ are estimate of the mass transfer coefficients.

So, the information that you will need is whatever is needed to calculate the mass transfer coefficient. Now, how do we estimate the mass transfer coefficient? So, this is the problem and if we want to calculate n, we need these 4 numbers. These numbers may be given to you in the problem saying that you already know by measurement that you know say there is a lake which is polluted with some chemical, you measure it and you know the concentration and then you measure the air, ambient air, and you know what is the concentration of this chemical there?

So, you know both these numbers okay. Then you need to estimate the mass transfer coefficients, then you can get whatever is the flux across in this scenario. How do you get the mass transfer coefficient? So, the mass transfer coefficient as we have seen already is a proportionality constant, so there is no a priory except for one or two different systems. There is no any derived equation for mass transfer coefficients.