Environmental Quality: Monitoring and Analysis
Prof. Ravi Krishna
Department of Chemical Engineering
Indian Institute of Technology, Madras

Lecture-38
Transport of Pollutants - Gaussian Dispersion Model

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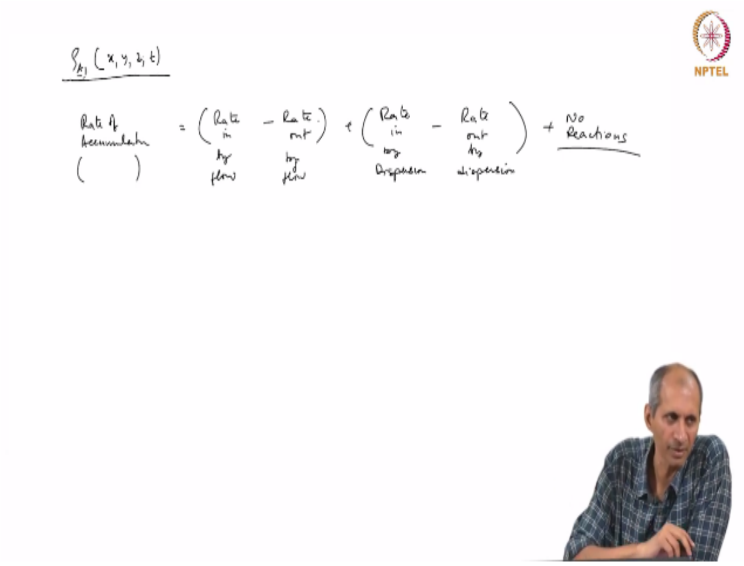

So, our goal is to model the system to predict rho A1 as a function of x, y, z and time, this is our general prediction. If you are trying to invoke the mass balance and trying to develop mathematical models. So, here two things may be happening, its rate in and rate out. So rate of accumulation equals rate in minus rate out plus there is no reaction (we are assuming no reactions here). This is an assumption because if you add reactions, then things will become very different. And we are also considering only rho A1. This is just vapor phase concentration. We are not looking at particulate matter and all that. If you add that, then that is a different thing.

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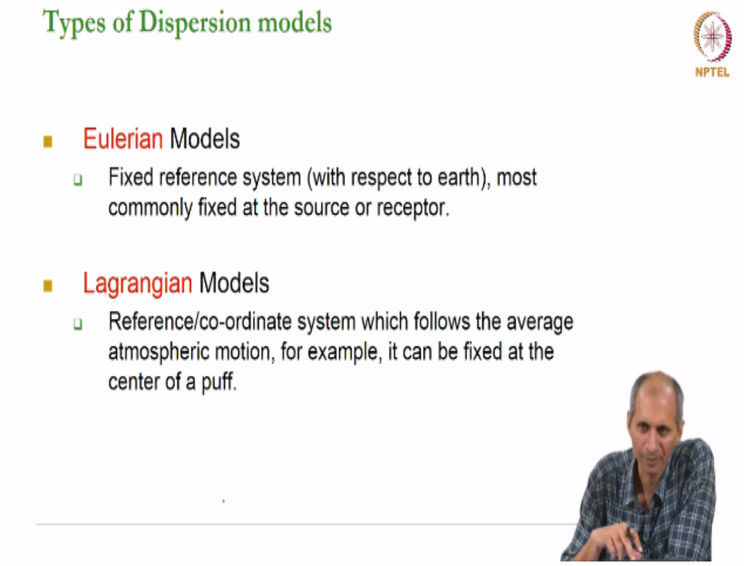

So dispersion models can be of two different kinds. One is, what is called as an Eulerian model, which is a fixed reference frame. What this means is, if I am modeling this room here. I am watching from here. So x equals 0 begins at that end goes to this end start from here and here. Lagrangian model on the other hand is that you are moving with the fluid that is the the frame of reference is that body of fluid.

So here dispersion model is set up as the most common commonly used model. This thing is what is called the Lagrangian model.

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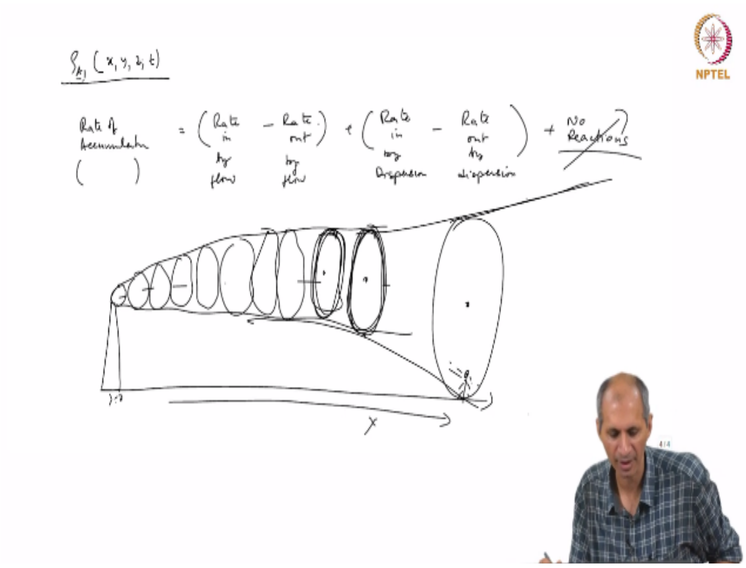

We are now looking at the plume (we are looking at the entire system), but we are also seeing that when we are talking about the z and y and all that and the dispersion it is the reference to this particular, it is not with the reference to a fixed reference frame. This is the fixed reference frame where x equal to 0, the spreading itself is happening in each of these volumes. So, if you imagine one puff that is going out and this as a series of puffs that is coming out because there is rate at which this is emission is happening. If you are burning something, there is a rate at every second there is a mass of exhaust that is coming out. So the concentration that you are going to be exposed to, is a concentration within this puff. So it is irrelevant, what if you model everything around here. We are trying to do what will be more useful in modeling what is inside this particular puff alone.

So if this puff becomes very large at some point and then there are also issues of this puff, spreading very wide. Which means that this puff will now occupy a big volume and therefore if there is a receptor standing here, this receptor is exposed to a certain concentration. So the goal is to find out what will be the concentration at a particular distance x and at a particular height?

But the plume behavior itself is modeled. It is with reference to this particular plume with respect to this particular puff only. You have to write down these equations, so we have no reactions here.

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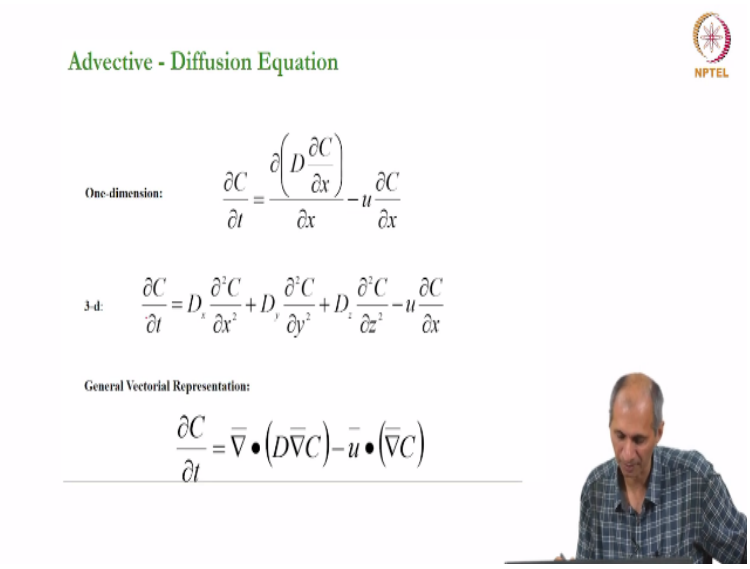

So when we write the general equation, we have written in down dC, here I will change it to zero. I will derive this equation for you in a minute.

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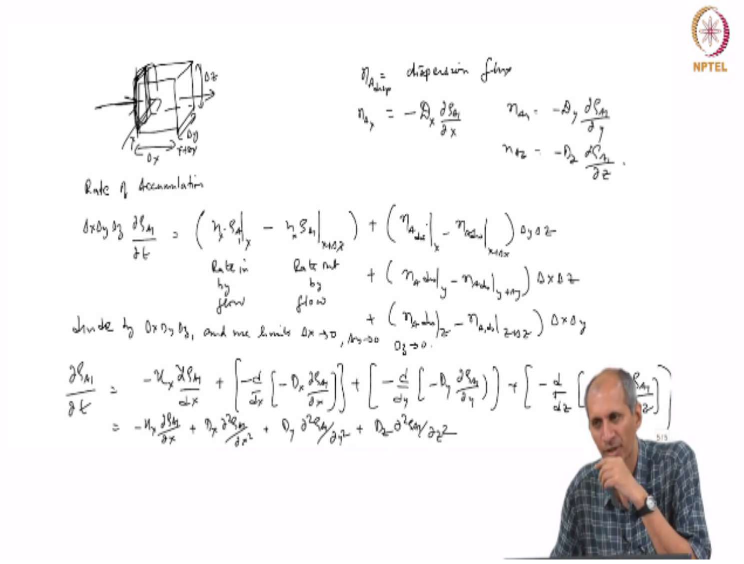

Let us say that we have a small volume. We take a three-dimensional volume. This is delta x, this is delta y, this is delta z. This is the volume inside one of the gas pollutants inside the plume, somewhere inside the plume we are trying to find out how it is moving and all that.

Now this is the direction in which this flow is happening. There is no flow in the y and z direction. We are looking at average velocities. Now the y, z component and their fluctuations, all that is now captured as dispersion. So by rate in flow we are talking about x and this x plus delta x. So rate in by flow is u at x is multiplied by rho A1.

Sorry this is ux delta at x, this is ux x + delta x. In all the 3 directions, you can have dispersion. So you the terms that we use for dispersion, that is we have D x Dx, we are only taking the main components of Dx in x direction multiplied by the general terms. So we are looking at a flux multiplied by area, that is we look at the dispersive flux.

We use the term n_A dispersive dispersion flux. This in the x-direction and the area of this is delta y delta z. So the x-direction flux is like this and the area it goes through this which is delta x delta y delta z plus we have another term which is y - n_A dispersion, y + delta y which is in this direction which is delta x delta z + n_A dispersion z + delta z this is delta x, delta y, this is delta, yeah, delta y. If there are no reactions here, if I divide everything by delta x delta y delta z and take the limits delta x tends to 0, delta y tends to 0 and delta z tends to 0, we will get dau rho A1 by dau t equals this first term this first will become minus of u ux d rho A1 by dx, ok. The rest of the terms nA, if you use basis for what we call a Fick's law model, this will become a dispersion term multiplied by rho x, this is the format we use so nAx is this, n Ay will be minus of Dy dau rho A by dau y and n Az will be - Dz dau rho A1 by dau z. So if you use all these three terms here the other terms now become, this minus of minus it will become - d by d z d by dx of -Dx into dau rho A1 of by dau x - d by d y of - d y dau rho A1 by dau y - d by dz - dz of dau rho A1 by dau z. So this equation will cancel out then, this equation essentially will become minus of ux dau this is dau here dau rho A1 by dau x + D x dau square rho A1 by dau x square + D y dau square rho A1 by d y square + Dz dau square dau A1 by dau z square. In the absence of nA so if you have reaction or something you can add those terms here.

This is a three dimensional equation in unsteady state including everything, this is the derivation of the full thing.

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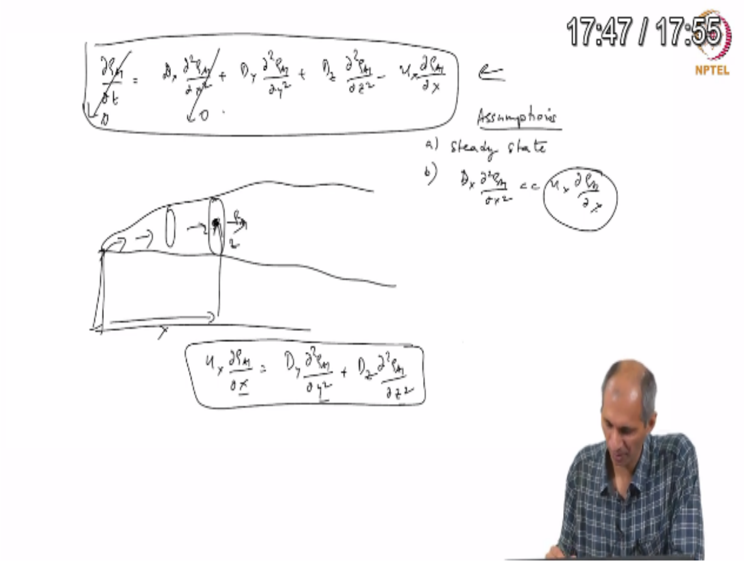

Now so you can write this equation like this dau rho A1 dau t equals Dx dau square rho A1 by dau x square + Dy dau y square + Dz - u of x. So here this is a general equation now, you can solve this with appropriate boundary conditions.

When will concentration change? Let us say if I have a plume here, so I am measuring concentration at this point. I would like to find out what is the concentration at this location which has a certain particular z, particular x and some y. And at this point if I want to measure concentration it will only be an unsteady state. When it will be unsteady state? What are the conditions under which this will be unsteady?

So when will it change with respect to time? Or let us put the reverse question, when do you expect it not to change with respect to the time? If it is already in equilibrium. There is nothing to do with the equilibrium in a steady state, but steady state need not be equilibrium, there is a difference. So in this case we are not talking about equilibrium there is only one phase, we are talking purely about transport, when can you make an assumption of steady state here which means that nothing is changing with time? What we essentially are saying is rho A1 is not changing with time at this location.

Zero turbulence. Turbulence? when you say something not changing with time what is else should not change with time? Environmental conditions? Environmental conditions should not change with time and anything else. Source? Source should not change with time it means you have a constant source of emission and for a given retime period of time, nothing is changing environmental wind speed is all the same.

So now this being a general case we want to use this but solution of this is quite complicated which means that you must have parameter 'u' as a function of time. We make various assumptions to simplify this equation.

This is general approach in transport phenomena, we take a very general model and then we cut several terms out we make assumptions, one is we assume steady state. So which means whatever we are going to do is valid for constant source emission systems, so there are several of them, for example. I can take industries their routine is known we know that they are going to be producing this amount of waste.

For a given period of time, we know that you can predict where the plume is going because we know for that period of time, we might be able to predict the wind speed and everything and we can then do the same calculation for a different set of conditions and we can run it. So it is easier to do it that way it is like the box model. Except that we are doing it in time now we are taking it in time and we are saying ok this.

So when we say steady state there is a constant flow coming here, right. So whatever is coming here is leaving here, so at this point the concentration is always the same it will be different from whatever it is here but it will be the same with reference to time, it doesn't change so that is the idea of steady state. This term Dx is a dispersion in the x direction is much smaller the term.

The Dx dau square rho A1 by dx square is much smaller than the contribution of d rho A 1 by dau x, the wind is moving in that direction and the distance by the amount of spreading by x dispersion is much smaller compared to the wind itself. So this term is negligible so what we do is these two terms goes to 0.

Which is essentially the equation becomes now u x dau rho A1 by dau y dau x equals Dy dau square rho A1 by dau y square + Dz dou square rho A1 by dau z square. So this is a steady state equation that we need to solve. So this equation leaves a few boundary conditions now which are related to x and y and z.

Three normal boundary conditions corresponding to x, y and z, that all of them are needed in this case. There's second order equation, ok, so I will stop here we will start from the derivation of what we call as a Gaussian dispersion model from this point.