Hydraulic Engineering
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Lecture-33
Introduction to Open Channel Flow and Uniform Flow (Contind.)

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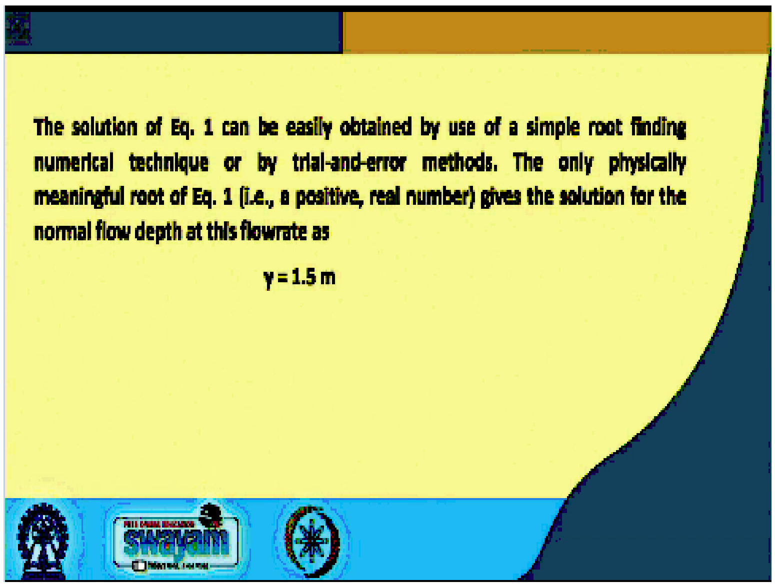

Welcome back. So, last lecture at the end of last week, we saw that the normal depth. We found out in this particular question was 1.5 meter. Although the problem in itself was very simple and easy, but the calculation was very complex, but I hope this has given you a good idea. Now, we are going to solve another question, at the beginning of this lecture itself.

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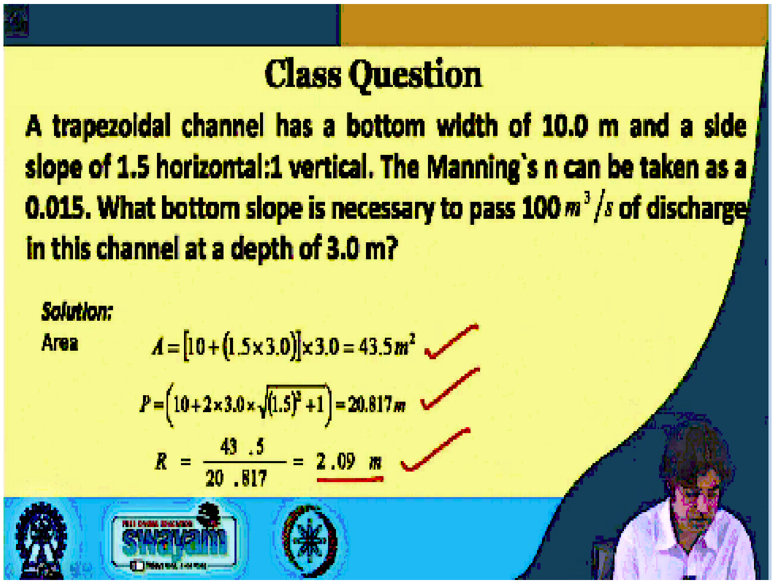

And it says a trapezoidal channel has a bottom width of 10 meter and a side slope of 1.5 horizontal is to 1 vertical. The Manning's n is also given as 0.015. Now, the question is, what bottom slope is necessary to pass 100 meter cube per second of discharge in this channel at a depth of 3 meter. See, in all these questions, the formula is Q, the whatever is 1 / n area into R h to the power 2 / 3. You see what all things are given, n is given.

You have been given area also because you have been given a bottom width, side slope, so area is known. So, everything is known, so hydraulic parameters. So, in this particular question, the it is about finding S 0, this is a broader, you know, I mean, a simpler way to look at the problem So, what exactly is being asked? So, we start doing this question, so a trapezoidal channel, very simple. So, this is 10, this is 3, this is 4.5, so area is 10 into 3 plus half, base is 4.5 into 3, for this one, and plus same is for this one, into 2.

So, area is going to be 43.5 meters square. So, what is given is correct. And the wetted parameter, the same thing, this is 10 + 2 into whatever this length is. So, this is 3 and this is 4.5. So, this length we can find out, because I think they have taken it in terms of the slope. So, that is where this, but this will come 10 + 2 into wetted parameters under root 3 square + 4.5 whole square and that will come to be 20.81 meter, perimeter is also correct.

Therefore, hydraulic radius is A / P, so 43.5 / 20 that is 2.09 meter. So, we have been able to find out area, we have been able to find out perimeter, we have been able to find out the hydraulic radius.

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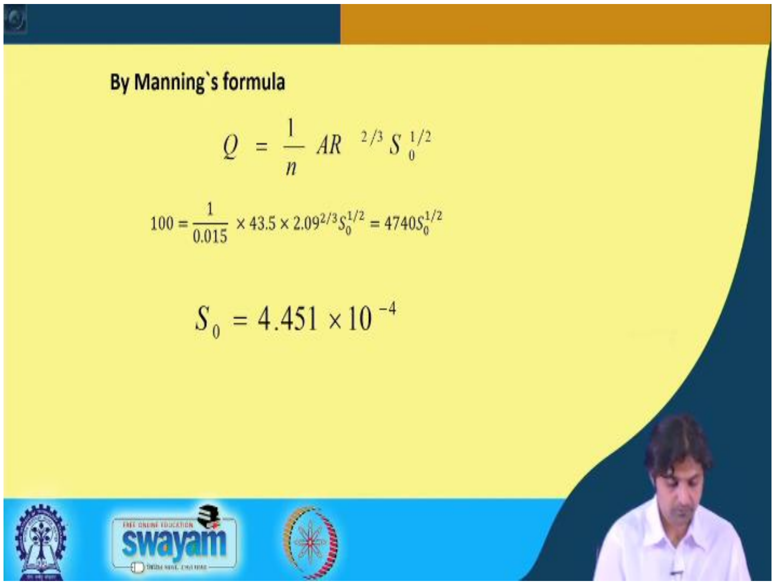

So, by Manning's formula, Q is 1 / n AR to the power 2 / 3 S 0 to the power half. Q we already know, n it is already been told to us, area we have already find out, hydraulic radius we already found out 2.09 to the power 2 / 3 by dividing A / R, sorry, A by perimeter, S 0 is something we do not know. So, this comes out to be 100 is equal to 4740 into S 0 to the power half, and if you calculate, S 0 will come out to be 4.451 into 10 to the power minus 4.

So, this is one question. So, we have seen, in Manning's equation we have seen, we were given everything we were asked to calculate Q in one problem. In the other problem we were not given y, so we were asked to calculate y, here we have been asked to calculate S 0. But the procedure is remains the same, first you find out the area, you have to find out the wetted parameter

P is wetted parameter. In assignments, I will give you the type of questions that will be expected in the exam, as shorter one basically, not taking too much, you know, I mean, complex problems. And then when you know A and P then you find out R h, in every problem, you look up the tables or if n is given, you should be able to find out S 0 from either the questions or if sometimes it is given.

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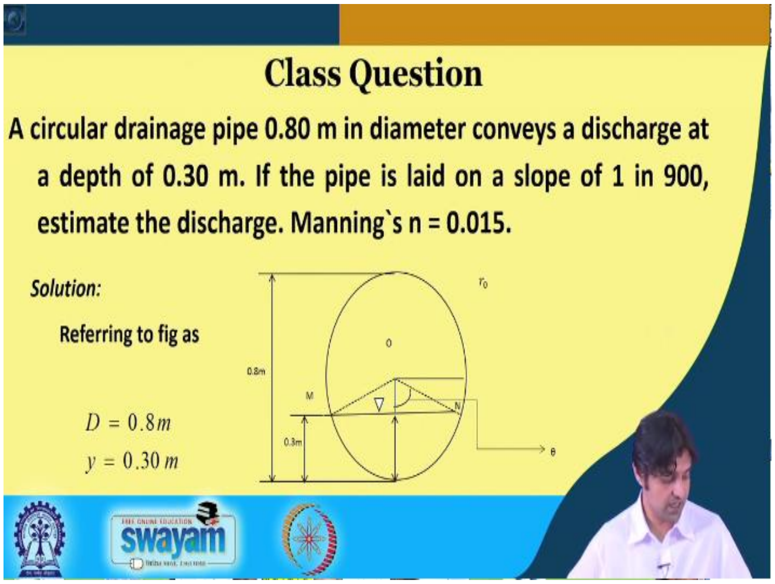

So, we move to the next question. So, a circular drainage pipe of 0.80 meter in diameter conveys discharge at a depth of 0.30 meter. If a pipe is laid on a slope of 1 in 900, so we have already been given a slope, estimate the discharge. We have already been given n, now the complexity of this problem is that our pipe is no longer, our channel is no longer rectangular, it is circular. So, how to deal this problem? So, most importantly, you have to always find out the area and the wetted parameter.

This is a circular drainage and it will look something like this. So, the D is 0.8 meter and y is 0.30 meter. So, the area of the flow section shown here, you see, this area of the flow section has to be calculated in terms of this angle theta. And this will be the area of the sector O M N minus area of the triangle O M N. So, if we, for this area if we find out the area of this, which we can easily find out in terms of this angle theta and subtract this area of the triangle, we should be fine.

So, this is how and the same is with the perimeter, perimeter we will be able to, this is the wetted part and this is the wetted perimeter. Now, the question is, how to be able to find that.

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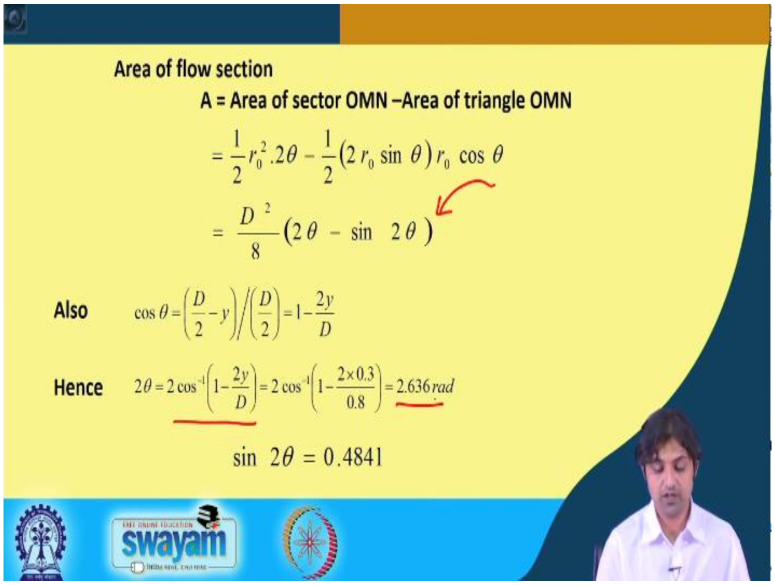

So, as I said, area of the sector O M N minus area of the triangle O M N. So, this sector, you know, area of the sector O M N is very simple, half r square into 2 theta. If the, this is theta, so what is 2 theta? So, if this is theta, this one, so this is 2 theta. So, this is what it has been used here, minus area of the triangle is half base into height. So, you see base, this is base into height, if this is r. So, this is r sin theta and this is theta and this is r cos theta.

So, we get, area as D square / A 2 theta - sin 2 theta. Also, we know that cos theta can be written in terms of D and y, D is the diameter, for example. So, here we use r 0, but actually we have finally turned this in terms of diameter D, cos theta is simply, D / 2 - y. So, if this is y and this is D / 2, so D / 2 - y will be this length, divided by D / 2, this is cos theta. Hence 2 theta can also be written as 2 cos, inverse of this value. So, this becomes, because y we know, D we know, so we are able to calculate actually 2 theta.

So, sin 2 theta is going to be 0.4841. So, we substitute the value, found out the value of theta and sin 2 theta also we are able to find out

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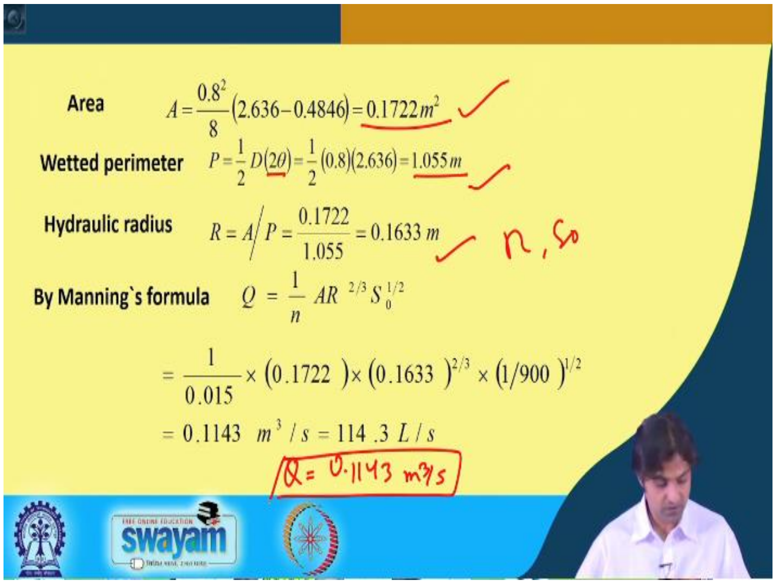

Area therefore, we are able to calculate for this one, it comes out to be 0.1722 meter square. Now, what the wetted perimeter is going out to be? It is simple, it is wetted parameter is half D 2 theta; it is so r 2 theta. This is 2 theta, we could write, r 2 theta or D / 2 r 2 theta. So, therefore, it is going to be 1.055 meters and the hydraulic radius is going to be A / P, which is 0.1633 meters. So, hydraulic radius we found out, wetted perimeter we found out, area we found out, n we already know, S 0 we already know.

So, by Manning's formula, Q is equal to 1 / n A into R to the power 2 / 3 and S 0 to the power half. If we substitute in the values, we can get Q as 0.1143 meter cube per second, very simple, just that the configuration. So, the catch in this problem was that the configuration was a little different, other things were fine. So, we proceed to our next part.

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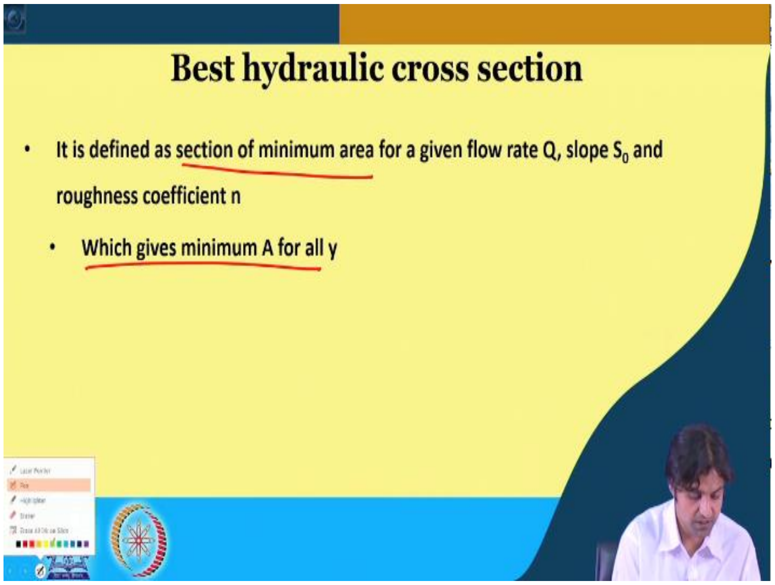

So, there is something called Best hydraulic cross section. What is best hydraulic cross section? It is defined as section of minimum area for a given flow rate Q, slope S and roughness coefficient. So, best hydraulic cross section is defined as a section of minimum area. So, if we have been given a Q slope, S 0 and roughness coefficient n, it is the best hydraulic cross section is the section of the minimum area, that has minimum area, which gives minimum A for all y, for any depth it gives minimum area. So, this is the definition of the best hydraulic cross section.

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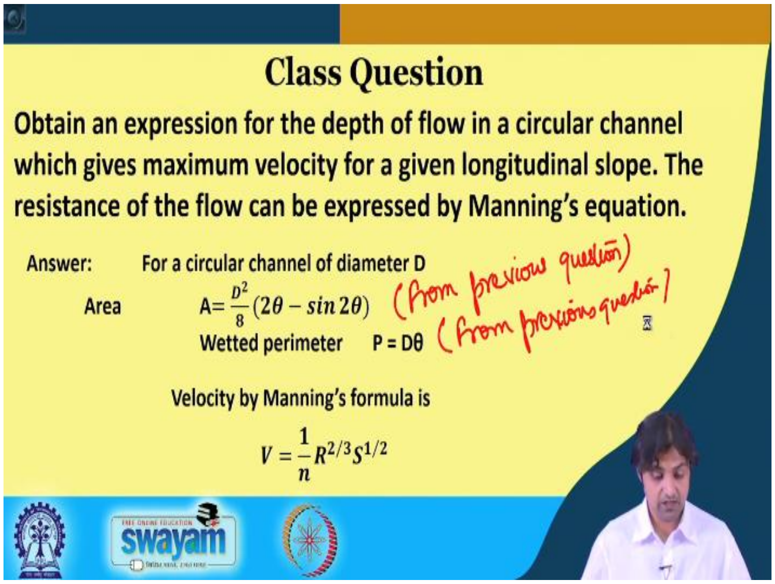

We will see some questions as well. So, question is, the best is to go for this concept through solving a problem. So, question is, obtain an expression for the depth of flow in a circular channel which gives maximum velocity for a given longitudinal slope. The resistance of the flow can be expressed as Manning coefficient. So, we already know that, you know, about the circular channel because we have already derived it in the last question.

So, for a circular channel of diameter D, area is in terms of diameter and theta is given as, D square / 8 2 theta, from previous question. Wetted perimeter is simply, D into theta or half D into 2 theta, it does not matter the way you write, but it comes out to be from. So, velocity by Manning formula is simply, 1 / n R to the power 2 / 3 S to the power half, same formula.

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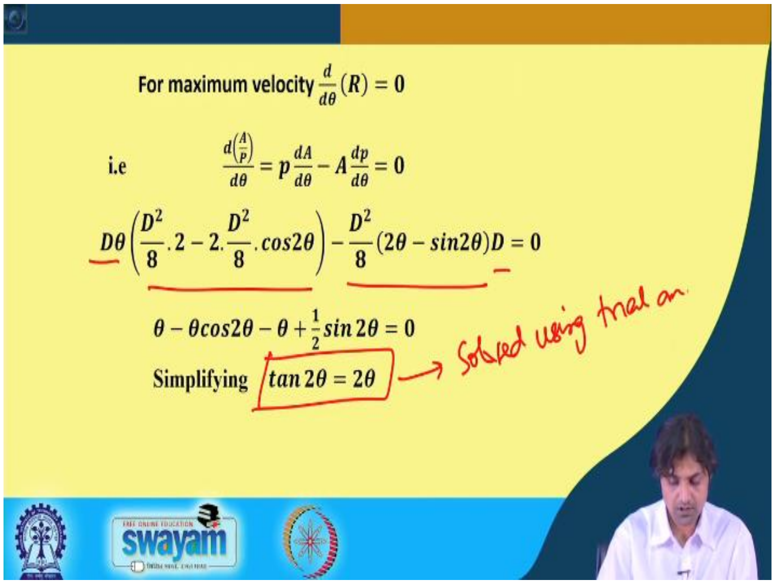

For maximum velocity, we have to do d d theta of R is equal to 0 hydraulic, you know, radius and that means, D of A / P / d theta or P dA d theta - A dP d theta. We can actually find A / P but it becomes more complex, so this is much better this is, you know. So, P we simply say, D theta and dA d theta will come out to be this, because A we have already found out, A we simply write down and dP d theta will be D and if we solve this, this will give, theta - theta cos 2 theta - theta + half sin theta, this finally will give us, simplification that tan 2 theta is equal to 2 theta. Now, until this point it is simple, but the solution for this is solved using trial and errors

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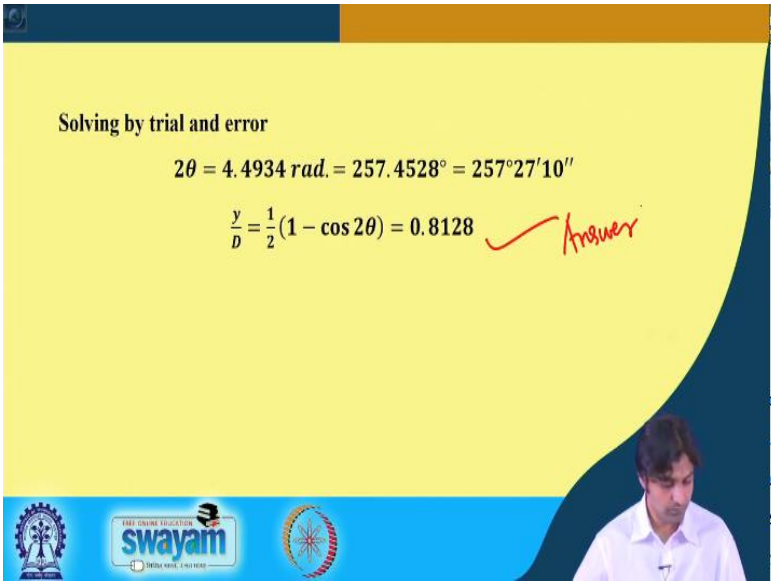

And if you do that, theta will come out to be, 2 theta come will come out to be 4.4934 rad. I do not expect you to be able to do solve, you know, trial and error but this is the way it is to be done and if you find out theta, then y / D is given as, half 1 - cos 2 theta. So, this is and this was what was required. So, this is one question.

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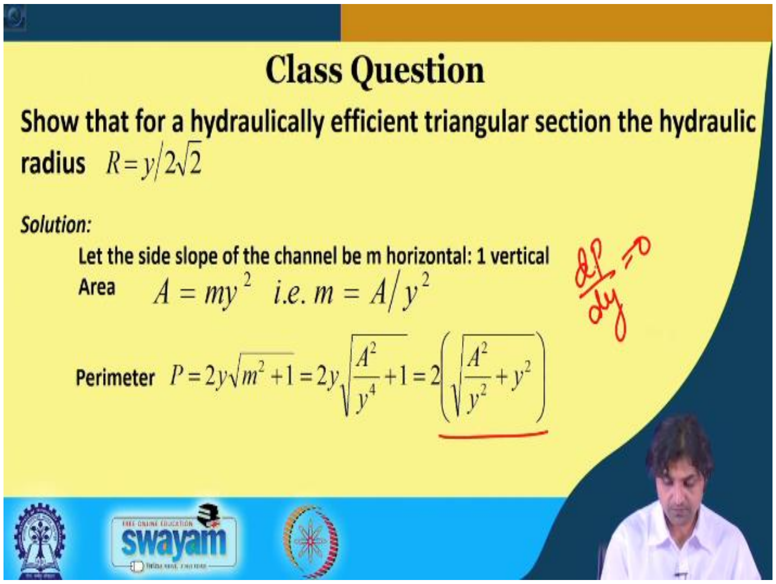

So, another question for the hydraulically efficient triangular section, we you have to prove that the hydraulic radius is y times, so this is what we need to prove. So, the solution is, let the side slope of the channel be m horizontal and one vertical. So, area is going to be for a triangular channel. So, area is going to be, what it is going to be area. So, area is going to be, half base into height, 2, 2 cancelled, area will be m y square. That is what is written here and the perimeter is going to be very simple, sorry, so perimeter will be this whole.

So, it will be 2y. So, if this is m y and this is y, it will be y under root m square plus one Pythagoras theorem and the total will be this plus this, so it will be 2y, which is written and what we have done instead of m, m here, we have used this equation from area. So, our area comes out to be, A / y square and perimeter comes out also in terms of A and y. So, our unknown here is in A and y. For an efficient section what do we need to do? We need to have the dP / dy is equal to 0. So, to be able to do that we already have this equation, in terms of area.

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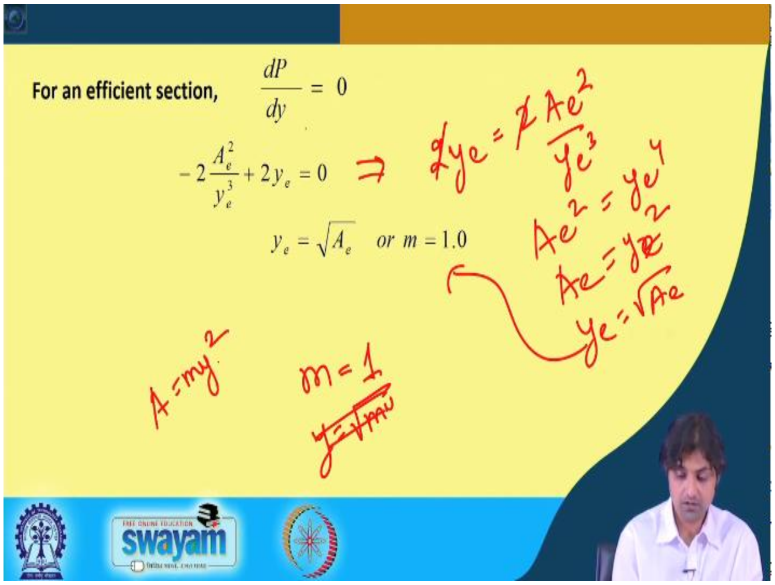

So, we do dP / dy and we obtain minus 2 into A e whole square divided by Y e whole cube + 2 y e is equal to 0 and on solving this we get, y e is equal to under root A e, this very simple you know. So, 2y is equal to 2A e whole square divided by y e cube, so 2 2 gets canceled. So, A e square is equal to y e to the power 4 so A e will be y e whole square or y e will be under root A e, which is shown here. So, this also means, for efficient channel m is equal to one because we got y is equal to under root m. Sorry, we got, A is equal to m y square. This will hold true when m is equal to 1.

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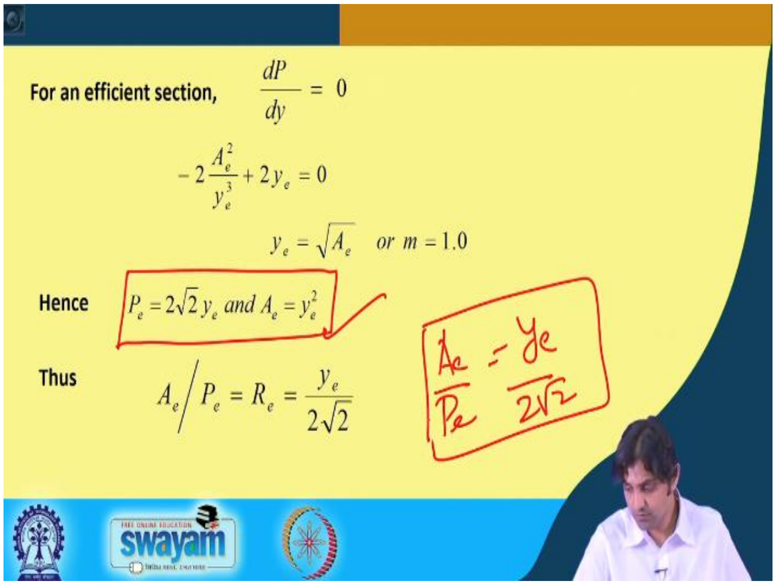

Hence, P e is going to be, 2 under root 2 y e or A e equal to y e whole square, because y e is under root A e or A e is y e whole square. So, in terms of perimeter, you go back. So, this perimeter is, A square is, so what we got was, A e is y e whole square. So, y e whole square divided by y e whole square plus, so it is to the power 4 and this will give us, 2 under root y e, 2 under root 2y e and A e is y e square, this is what we got.

So, 2 under root 2y e square or 2 under root 2 y e. So, this is what we derive. So, A e / P e, that is, hydraulic radius is A e by P e. So, these questions are quite simple, if you follow step by step you should be able to solve these questions very easily.

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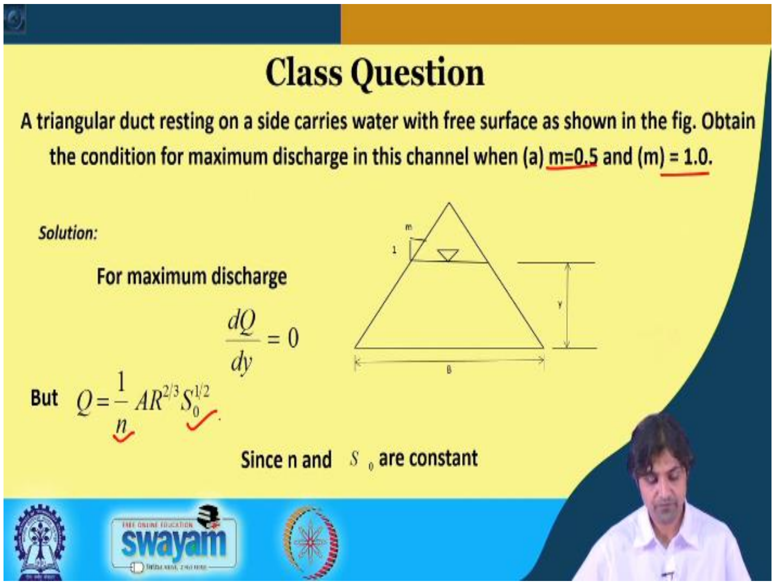

So, this is another question, it sayss that the triangle duct resting on a side carries water with 3 surface as shown in the figure. Obtain the condition for maximum discharge. So, just telling you for the maximum discharge dQ / dy is equal to 0. So, everything is actually given, so we are going to go a little bit quick and, so this is also actually the calculation for this is difficult, but the procedure is very simple.

But our objective in solving is not the difficulty of the calculation for tests or anything; the calculations are not going to be so complex. I want to do it because for this Manning's equation and the coefficient, you know, you should be able to handle different type of cross section. So, this is the triangular, you know, a different type of cross section. For different m is equal to 0.5 and different m is equal to 1.0, we need to find out, you know, maximum discharge.

So, the solution is for maximum discharge, dQ by dy should be equal to 0. But, Q is, we know, 1 / n AR to the power 2 / 3 and S 0 to the power half. We know that n and S 0 is constant, we know that.

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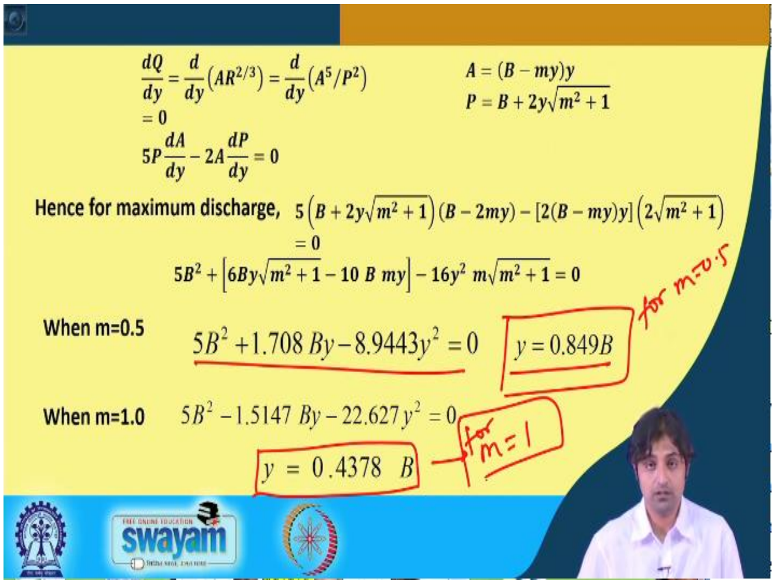

So, simply, dQ / dy is going to be d dy of AR to the power 2 / 3, because n and S 0 are constant or in terms of A and P, if you write, R to the power 2 / 3 as, A square to power 2 / 3 and so A into R to the power 2 / 3, let us, so d dy is the same as, d dy of AR to the power 2 / 3 to whole cube. So, it becomes, A cube multiplied by A square / P square or d dy of A to the power 5 / P square, same as what we have written here just I.

So, where the area is B – m y into y, if you go and look at, you know, it is very simple. This is the area of submerged and this is the perimeter as well. So, I will just, this is the area and this is simple trigonometry. So, you can find out m, this is the area. I will take this out and this is the perimeter, wetted perimeters, so this one. So, area is B – m y into y, perimeter is B + 2y under root m square. You can verify it yourself.

So, if we put this here we will get, so from this we get, 5P into dA dy into A to the power 4 - 2A dP / dy is equal to 0. It is a little more complex actually. And for maximum discharge, if you use this, A and P is this, now the calculation becomes very cumbersome. But anyways, if you put the value of 1s, when you put in this equation value of m as 0.5 and m as 1, you can see for yourself that this will turn out to be equation like this, 5B square + one point, very complex, but yeah and you solve, this is y is equal to 0.849B.

So, it is a quadratic. So, not so complex actually but it can be complex, when m is equal to 0.1, we get equation like this and y will be 0.4378B. For, m is equal to 0.5, m is equal to, for m is equal to 1. So, see this question actually is more towards tackling all the possible problems for uniform flow, Manning's equation, wherever they are required. And I think, that should be the core thing that you must be practicing this week as well.

We are going to start with a newer topic, gradually varied flow and rapidly varied flow, from our next lecture. So, we will spend at least 4 at least 3 to 4 or maybe 5 lectures that depends how fast we go. So, this is enough for today's first lecture of this week, from next lecture onword. For this week we will be dealing with gradually varied flow. Thank you for listening and see you in the next class.